Signaling Value through Assortment

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Abstract

Oftentimes, close competitors carry partially overlapping assortments in seeming contradiction to the principle of maximum differentiation. One of the justifications of such practice is that an overlapping assortment with competitive prices on the common products may prevent further consumer search and therefore could be useful even when profits from the common products do not justify the costs of carrying them. In this paper, we examine the validity of this intuition and show that such strategy may be optimal when consumers are uncertain about prices they might find elsewhere and have search costs for discovery of all prices. Specifically, we show that the (larger) assortment with the overlapping product(s) may signal that the price of the relatively unique product(s) is “competitive” as well and prevent consumers from searching for lower prices of the relatively unique products.

Keywords: game theory, retail assortment, consumer uncertainty, signaling, competitive strategy, search costs
1 Introduction

Once, quite some years ago, one of the authors was attracted by the window display of short-wave portable radio receivers in a small shop of San Francisco’s Chinatown. He was quickly approached by a salesman. After confirming the interest, the salesman proceeded to show one of the best radios they had. The features and reception seemed great. So was the style. But the price tag somewhat steep. The author wondered whether the same radio could be gotten elsewhere considerably cheaper.

To the author’s surprise, the salesman was very understanding of this concern and quick to respond. He pointed to one of the cheapest radios on his display, which he said was no good at all, and then pointed to another electronics store - just across the street - which carried a wide selection of electronics with “discounted prices” (according to a sign in the window) but not so many short-wave radio receivers. It did though carry the radio pointed out by the salesman in his window display and at the same price. “As you see, we are very competitive,” – the salesman assured the author – “but a high quality radio just costs much more.”

Although the argument seemed convincing, the author suspected that the first store carried the low-end receiver only to make this argument. He wondered if it was indeed a valid inference or a deceptive sales trick. Later, the author has also came across a discussion of similar strategy in the “Tweeter, etc.” HBS case study.

This paper explores the possibility that an overlapping assortment has a value to the retailer due to the information it provides to consumers, and given a rational consumer inference. More formally, we explore the validity of the following thread of consumer inference. Consumers rationally expect that different retailers face correlated product costs, for example, because all costs are a function of the same (or similar) wholesale price. Therefore, having observed the price at one retailer, the consumer forms belief about the benefit of further search for a better price at other retailers as a function of what he/she believes about the margin of this retailer. The consumer beliefs about the margin are, in turn, deduced from the equilibrium retail strategies in assortment and pricing. If the retailer is not as concerned about cannibalizing the sales of the unique product(s) by the common product(s), then the margin on the unique product(s) must not be very high (margins on the common product(s) must be low due to competition). Therefore, carrying overlapping assortment may be a credible signal that the margin on the unique product(s) is not too high.
Note that consumers are facing the above informational problem if the following market conditions are present. First, the retail product costs are not known to the consumer and these costs are correlated across retailers (e.g., due to correlated-across-retailers and unknown-to-consumers wholesale prices). Second, consumers observe product line overlap at some retailers, i.e., consumers are informed about product selection and prices at some retailers. And finally, consumers face a non-trivial decision of whether to search for the price(s) at other retailers, i.e., consumers are not informed about prices of all retailers. The latter is the case when consumer costs of exhaustive search are not negligible and when other retailers carrying the same products possibly exist. For example, some retailers may be conveniently located near a given consumer, but other retailers further away also exist. Given the prevalence of consumer shopping at multiple retailers as well as a high number of retailers consumers can potentially reach, it is likely that the above conditions are satisfied in many markets.

To capture the above market characteristics, we develop a model of competitive market with multiple competitors and consumers, who have easy access to some competitors (the two “local” retailers) and have a search cost when looking for prices at other competitors (the “second marketplace”). For parsimony, we restrict the product selection to two products – a low- and a high-quality one, – and assume a uniform distribution of preference for quality across consumers. Thus, the local retailers face two competitive constraints: first, as consumers are familiar with the products at the two local retailers, they choose which products to buy on the basis of preferences for the products and the willingness to pay; second, if consumers expect that the deal they could get in the second marketplace justifies the search costs, they may leave. The question of interest for us is how consumer expectation of the outside deal depends on the assortment and pricing of the local retailers, and how the retailers adopt their pricing and product line choices in view of their effect on consumer expectations. The underlying uncertainty consumers face is whether the product cost is high or low. This is relevant for the decision to search (given the prices observed) because the prices not observed are expected to be positively correlated with costs.  

If search costs are high enough, the second marketplace is of no relevance, and we have the

1Note that in this setting, the retailer would like to signal that the costs are high. This could be counter-intuitive if the marketing environment we consider is not clear. Specifically, if retail costs are independent across retailers and consumers do not observe the price of a given retailer (e.g., the environment considered by Simester 1995), then the retailer may want to signal that its costs are low because a lower cost implies it should have lower price (given a downward sloping demand). In our setting, the consumer observes the price at the retailer she considers buying from. Therefore, this retailer would like to convince consumers that the price at the other retailer is high, which would be the case if it convinces consumers that the (common-across-retailers) costs are high.
standard result that it is optimal for local retailers to differentiate and choose non-overlapping product assortments, i.e., one retailer chooses only the low quality product and the other chooses only the high quality product to sell. The consumer cost uncertainty does not play a role in this case because consumers observe all relevant prices and do not directly care about costs.

If search costs are low enough, consumers have a non-trivial decision of whether to search at the second marketplace. What would reduce the consumer expectation of the benefit of search and reduce the importance of the consumer search constraint? For parsimony, we concentrate on the cost uncertainty of the high quality product only (the search constraint for better price of the low quality product is likely to be less restrictive because the observed price is lower to start with), and therefore on the strategy of the local retailer carrying the high-quality product.

The results are the following. When consumer search costs are low enough so that consumer search is a relevant concern, but not too low, the high-end product’s price becomes constrained by consumer search (as opposed to by competition with the local low-end retailer) first when the retail cost is low. This is intuitive: retail margins decrease when costs increase because the optimal price is between the (unchanged) consumer value and decreased costs. As a result, somewhat counter-intuitively, the price is distorted downward to prevent search only if the price is low to start with (i.e., only if the cost is low).

When consumer search costs are even lower, the retailer margins in the case of low retail costs get squeezed so much that it becomes optimal for the retailer facing low costs to imitate facing the high-cost and raise price. Due to such imitation, the price level that is optimal in the absence of consumer search possibility is no longer perfectly informative about the high cost. If consumers doubt whether the cost is high, they see more benefit of searching and will search. Therefore, when the marginal cost is high, the high-end retailer wants to convince consumers that it is facing high costs as opposed to that it is facing low costs and imitating the decisions under low costs. Thus, for some range of search costs, we obtain that the high-end retailer can, and in equilibrium does, signal high cost and prevent consumer search by raising price above the level it would set when consumer search costs are prohibitively high. In other words, lower consumer search costs may imply probabilistically higher prices and higher price dispersion. This is because relative to the case of higher consumer search cost, the high-end product’s price is higher if the product’s cost is high (to signal high cost) and the price is lower when the product’s

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2 Again, note that one may find it somewhat counter-intuitive that the low-cost retailer prevents search by increasing the price.
cost is low (to prevent consumer search).

Of course, when search costs are low enough, signalling high cost by raising the price is not possible because the price is already constrained by the consumer search even under the assumption that consumers are convinced that the product’s cost is high. This is where the opportunity to signal through assortment becomes attractive. If the high-end retailer expands its product line to also include the low-end product sold by the other local competitor, the direct competition for consumers with low preference for quality leads to the low-end product being sold at cost by both retailers. Thus, the addition of this product to the high-end retailer’s assortment results in no profit contribution from this product to either retailer. But since the low-end product becomes more attractive to consumers, some consumers switch from the high-end to the low-end product. This is the negative effect of having overlapping products. However, if consumers infer high cost from overlapping assortment, the question is whether the loss of demand due to some consumers switching to lower end product is compensated by preventing other consumers interested in the high-end product from searching for it at the second marketplace. Since high-cost retailer is more concerned about the ability to keep price up than about having higher sales of the high-end product, it is more willing to sacrifice the demand for the high-end product by expanding the assortment to include the low-end product. We thus obtain our main result that carrying overlapping assortment (with the closest competitor) may be optimal for a retailer to prevent consumer search for better prices at more retailers. Again, the intuition for this result is that the assortment overlapping with that of a close competitor signals to consumers that the prices of the retailer adopting such assortment are driven more by the common-across-retailers product costs than by the retailer’s margins.

2 Related Literature

This paper is mainly related to the streams of marketing literature on firms’ product (line) decisions and firm-to-consumer signaling.

The existing literature on product choice and assortment decisions considers product number and product line design in the presence of cannibalization (e.g., Mussa and Rosen 1978, Moorthy 1984, Desai 2001), shelf space allocation (e.g., Marx and Shafer 2010), assortment affecting costs (e.g., Dukes, Geylani, and Srinivasan 2009), difficulty to communicate about products (e.g., Villas-Boas 2004) and the incentives to differentiate (e.g., d’Aspermont et al. 1979, Shaked and
Sutton 1982, Hauser and Shugan 1983, Moorthy 1988, Kuksov 2004). Although when products and stores are differentiated, the relationship between assortment size and competitive intensity may not be monotonic (Hamilton and Richards 2009), the cannibalization-margin tradeoff cannot explain carrying identical products on which margin reduces to zero. In the case of consumer uncertainty about fit and costly search, Liu and Dukes (2013) argued that larger assortment may reduce competition through a reduction of consumer search across stores (when consumer search within a store is also costly). There is also literature on optimality of max-min differentiation, which suggest that minimum differentiation may be optimal along dimensions less important to the consumer if enough differentiation is achieved on one of the more important dimensions (see, e.g., Irmen and Thisse 1998 and Lauga and Ofek 2011). Alternatively, we consider the marketing environment where fit is known and the two retailers in question are not differentiated except for assortment, but a comprehensive price search is costly.³

Complementing the above literature, our study links the assortment management problem with consumer inference. In this respect, there is some literature on consumer inference of product attributes (Kuksov and Villas-Boas 2010, Kamenica 2008) and price (Villas-Boas 2009) from the assortment length. In particular, Villas-Boas (2009) argues that consumers may infer better value from smaller assortment in a monopoly setting where prior to search, consumers observe the number of products but not the price of the monopolist, and Kuksov and Villas-Boas (2010) show that the effective consumer search costs for a product with a given fit reduces when the monopoly retailer reduces the number of products, which also creates an incentive for the monopoly retailer to reduce the product line (when fit is uncertain and being searched for). Alternatively, in the current paper, we show that consumer inference about competitors’ prices is a force toward a larger assortment of a given retailer (once the fit and the price at this retailer are known to the consumer). Note that our result that consumers may be prevented from further search by the addition of an “irrelevant” alternative may be interpreted as that consumers behave as if they are attracted to retailers with large assortments even when they do not per se like the extra choices offered. This is consistent with some findings in consumer behavior literature. For example, Iyengar and Lepper (2000) find that more people are attracted to a jam tasting display when the display has more jam choices, even as conditional on trying at least one sample, people do not taste more jams when the selection is larger.

³There is also a large empirical literature on the affect of assortment on consumer choice. For example, Briesch, Chintagunta, and Fox (2009) estimate that assortment has a significant positive influence on driving consumers to the store (see also their literature review on other empirical findings related to assortment).
This paper also relates to an extensive literature in marketing and economics on signaling (Spence 1973, Cho and Krepps 1987). Most of this literature concentrated on signaling of quality (e.g., Kihlstrom and Riordan 1984, Milgrom and Roberts 1986, Bagwell and Riordan 1991, Balachander and Srinivasan, Moorthy and Srinivasan 1995, Soberman 2003, Miklos-Thal and Zhang 2013) or demand (Desai and Srinivasan 1995, Desai 2000), some literature considered how sales signs (or advertising) can signal good value within the store either on other products (Simester 1995, Shin 2005) or due to future (un)availability (Anderson and Simester 1998). Alternatively, we consider how a retailer can signal high price at the other stores and consider assortment size as a possible signal.

The remainder of the paper is organized as follows. Section 3 formally defines the model we use, which is then analyzed in Section 4. Section 5 discusses robustness of the results to relaxing various assumptions of the model. Section 6 concludes with a further discussion of the results, their relevance to practice, and other relevant issues in retail assortment not formally considered in this paper. The formal proofs are completed in Appendix A.

3 Model Setup

A product category consists of two products: a low-end one with quality \( q_1(>0) \), and a high-end one with quality \( q_2(>q_1) \). A unit mass of consumers each having a single-unit demand have the following utility from the purchase:

\[
U = \theta q - p, \tag{1}
\]

where \( q \) is the product’s quality, \( p \) is the price paid, and \( \theta \) is the preference parameter uniformly distributed on \([0, 1]\) across consumers.\(^4\)

Several firms are competing for the above consumers who differ in the ease of access to these firms. For parsimony, we split the firms into two sets: “local,” to which consumers have free access and “outside,” for access to which a consumer has to pay the search cost \( s \). We further assume that the local marketplace consists of two retailers with Retailer 1 carrying the low-end product and the other retailer (Retailer 2) deciding whether to carry one or both products.\(^5\)

\(^4\)The assumption of the market being vertically differentiated is not essential to our analysis. What we aim to capture is that consumers trade off between different products, and consequently, that intensifying competition for one product has a negative effect on the demand for the other.

\(^5\)Allowing both retailers to choose assortment does not invalidate the equilibria presented in this paper, but adds an equilibrium with mixed strategies in assortment (if retailers fail to “coordinate” on who is to offer the high-end product) and an equilibrium where both retailers carry both products and price both at cost. Restricting the model analysis to the product line selection by Retailer 2 only (and given that the postulated product line
We further normalize the marginal cost (e.g., the wholesale price) of $q_1$ to 0, and assume that the marginal cost $c$ of $q_2$ could be either low ($c = c_L \geq 0$) or high ($c = c_H > c_L$) with equal probability. Retailers know the cost of the products, but consumers only know the above cost distribution.

An important assumption about the outside market is that although shopping there comes with a cost, incurring this cost brings a potential benefit of finding a lower price, i.e., at least probabilistically, the price in the outside market could be lower. This could be due to, for example, different valuation distribution of the consumers local to the outside market or a larger number of firms there. For the most parsimonious model, we assume that the outside market is perfectly competitive, i.e., consumers can find both products available there at the price equal to cost, and that all consumers face the same cost $s$ of accessing and learning the prices in that market.

The supply side specification above captures the reality of retailing through the assumption that different firms (retailers) have access to the same products at the same costs. This is a reasonable assumption if the retailers have access to the same manufacturers, but may not hold if the retailers are independently producing the products they sell.

The timing of decisions in the above game is as follows: First, nature draws the realization of the underlying uncertainty – in our case, the cost of the high-end product. Retailers observe this cost while consumers never observe it directly. Having observed the costs, Retailer 2 decides on the assortment to carry. Next, both retailers set prices (conceptually, this is when the prices are set in the outside market as well, although technically, we simplified the outside market by postulating that these prices are set at cost). Then, each consumer observes the prices and assortments in the local market (i.e., at Retailers 1 and 2) and decides whether to incur the search cost $s$ and visit the outside market or not. Finally, consumers decide whether and which product to buy. To fully define consumer preferences, we assume that if prices in the local market are such that a consumer is indifferent between searching further and not, she will not search further.

We look for a perfect Bayesian equilibrium satisfying Intuitive Criterion (henceforth, IC)
where consumers consider choices of Retailer 2 as potential messages about the cost of $q_2$. To reduce the number of cases to consider, we assume that the quality difference between products ($\Delta = q_2 - q_1$) is high enough and that the cost $c$ of $q_2$ is not very high, in particular, so that the high quality product ends up having a higher absolute margin than the low quality one.\footnote{These assumptions are not essential but simplify the analysis, since they allow us to concentrate on the case when the price of the low-quality product is not constrained by search, thereby reducing the number of cases to consider.} As it is useful to consider pricing decisions given assortment choice before considering the assortment choice, it is useful to define a \textit{pricing game} as the continuation game starting from the pricing stage in which the assortment choice is exogenously fixed (i.e., Retailer 2 either has the single-product $q_2$ or the full assortment).

4 Analysis

We aim to understand how assortment decision is affected by the concern about the possible consumer search for lower prices and consumer uncertainty about the potential prices they can find with further search due to uncertainty about retailers’ costs. Consequently, we first solve the benchmark model where consumers do not have uncertainty about costs, i.e., observe them directly (Section 4.1). Note that when search in the outside market is too costly, consumers have no use for the cost information and therefore consumer knowledge of costs can only be potentially affecting the results when the consumer search decision is non-trivial. Then, we solve the full model and compare its pricing and assortment outcomes with the outcomes of the benchmark models. This allows us to understand how the consumer uncertainty about costs and the potential for search affect the retail pricing and assortment choices.

4.1 Benchmark: Perfect Information

In this case, facing no uncertainty, consumers correctly anticipate the price they can obtain in the outside market. The outside market prices are 0 and $c$ for products $q_1$ and $q_2$, respectively. Therefore, consumer search decision is straightforward. Consumers search and buy from the outside market if and only if the best product choice in the outside market provides them with utility more than $s$ higher than the best choice in the local market. Adapting to such consumer strategy, it is optimal for the local retailers to keep the prices at or below this constraint.

Let us first consider the case of search costs high enough, so that the above-discussed constraint is not binding. If Retailer 2 chooses to carry only the high-end product, we have the
standard quality-differentiated competition which results in the following prices and profits:

\[
\begin{align*}
    p_{1s}^n &= \frac{q_1(\Delta + c)}{4q_2 - q_1}, & \pi_1 &= \frac{q_1q_2(\Delta + c)^2}{(4q_2 - q_1)^2\Delta}, \\
    p_{2s}^n &= \frac{2(\Delta + c)q_2}{4q_2 - q_1}, & \pi_2 &= \frac{(2q_2(\Delta - c) + cq_1)^2}{(4q_2 - q_1)^2\Delta},
\end{align*}
\]

where \(c = c_L\) or \(c_H\) is the cost of the high-end product, \(\Delta = q_2 - q_1\), and the subscript on price indices the retailers (and coincides with product sold). If Retailer 2 chooses to carry both products, the equilibrium prices and profits are as follows:

\[
\begin{align*}
    p_{11}^{ns} &= 0, & \pi_1 &= 0, \\
    p_{21}^{ns} &= 0, & p_{22}^{ns} &= \frac{\Delta + c}{2}, & \pi_2 &= \frac{(\Delta - c)^2}{4\Delta},
\end{align*}
\]

where the first subscript indices the retailers and the second subscript on the price indices the product sold: 1 for \(q_1\) and 2 for \(q_2\) (Retailer 1 does not carry \(q_2\)). Retailer 2 choosing to only carry product \(q_1\) results in perfect competition and zero profits for both retailers.

Comparing retailer profits under different assortment choices, we see that retailers strictly prefers to differentiate in the product choice and not have any product overlap. This is intuitive because the effect of a retailer carrying the overlapping product is that the margin of this product is completely eroded by competition and the lower competitive price of this product further erodes sales of the other product. In other words, although Retailer 2 could possibly gain market share (increase sales) by increasing assortment, its profits unambiguously decrease if it chooses to carry the product that Retailer 1 sells.

It may be useful to clarify the result of the perfect-competition outcome in the equilibrium pricing of the overlapping product \((q_1)\). The reason for this is that for any positive-margin price of \(q_1\) Retailer 2 can choose, it is optimal for Retailer 1 to set its price a little bit lower. Then, given costless search across the two retailers, the relevant comparison for consumers is between the high-end product at Retailer 2 and the low-end product at Retailer 1. Thus, even if Retailer 2 does not reduce the price of the low-end product to zero, it cannot possibly have a positive profit from its sales. Furthermore, if Retailer 1 is the only one with positive sales of the low-end product, and it achieves this at a positive price, Retailer 2 strictly benefits from just undercutting Retailer 1 on the price of the low-end product: the effect of slightly reducing the relevant price of the low-end product (from the price at Retailer 1 to the new price at Retailer 2) on the sales of the high-end product are negligible, but the profit gained from sales of the low-end product is strictly positive (now, all the demand for the low-end product is shifted from
Retailer 1 to Retailer 2). Of course, the optimal response of Retailer 1 to such undercutting by Retailer 2 is to undercut Retailer 2’s new price (as far as it allows for a positive margin). Thus, the only equilibrium is for the price of the low-end product to be zero.

Note that the above argument applies for any overlapping products between the two retailers, regardless of whether they are vertically or horizontally differentiated from the other products and regardless of the total number of products (or retailers in the local market).

Putting together the optimal pricing in the absence of the above-discussed consumer no-search constraint with the constraint, we have that if Retailer 2 chooses to carry only product \(q_2\), the equilibrium prices are

\[
\begin{align*}
&\begin{cases}
  p_j^f = p_j^{ns}, & \text{for } s > \frac{2\Delta(q_2 - c) - q_1c}{4q_2 - q_1}; \\
  p_1^f = \min \left\{ \frac{(c + s)q_1}{2q_2}, s \right\}, & p_2^f = c + s, & \text{for } s \leq \frac{2\Delta(q_2 - c) - q_1c}{4q_2 - q_1}.
\end{cases}
\end{align*}
\]

(4)

Note that given the assumption that \(\Delta\) is sufficiently high, we have that the profit margin of \(q_2\) is higher than that of \(q_1\). Therefore, the price of \(q_2\) is constrained by the necessity to prevent consumer search for a wider range of \(s\) than the price of \(q_1\). However, when the price of \(q_2\) is lowered due to the no-search constraint, it becomes optimal for Retailer 1 to respond by also lowering its price. Of course for small enough \(s\), both prices become constrained by the need to prevent consumer search.

If Retailer 2 chooses to carry the full product line (both products \(q_1\) and \(q_2\)), the equilibrium prices and profits are as follows:

\[
\begin{align*}
&\begin{cases}
  p_{11}^f = p_{21}^f = 0, & \pi_1 = 0, \\
  p_{22}^f = \min \left\{ \frac{\Delta + c}{2}, c + s \right\}, & \pi_2 = \min \left\{ \frac{(\Delta - c)^2}{4\Delta}, \frac{\Delta - c - s}{\Delta} \right\}
\end{cases}
\end{align*}
\]

(5)

Therefore, we again have that Retailer 2 gains no profit margin on product \(q_1\) but the strictly lower price on this product creates better outside option for consumers choosing between \(q_1\) and \(q_2\). Also, the presence of product \(q_1\) in Retailer 2’s assortment (or its lower price) does not relax the constraint on price placed by the outside market (the prices are still bounded from above by the same \(c + s\)). Thus, it is clear that single-product assortment of product \(q_2\) only is optimal for Retailer 2 in the case consumers have perfect information about costs.

4.2 Full Model Analysis

We now turn to the equilibrium analysis of the full model with consumer uncertainty about the cost of \(q_2\) and therefore, uncertainty about its price in the outside market. In this case, consumers
need to form expectations about the cost $c$ of $q_2$ because this cost affects their expectation of the price in the outside market, which in turn affects their expected benefit of search. A-priori, consumers know that $c$ is equally likely to be either $c_L$ and $c_H$. However, they also know that costs affect the local retailers’ decisions and therefore they should update their expectations of costs given the observed (assortment and price) decisions made by the local retailers. In this context, there are two clear extremes of consumer inference.

One possibility is that the sets of equilibrium actions taken by the retailers in the cases of low and high costs are disjoint. In this case, called the separating outcome, consumers (in equilibrium) infer the cost from the retailers’ actions precisely. The other extreme is that the actions taken by the retailers in the case of high and low costs are exactly the same. In this case, called the pooling outcome, consumers fall back on their prior beliefs, in our case, that the cost is equally likely to be either high or low.

In the market environment we consider, if consumers may potentially search, Retailer 2 carrying $q_2$ potentially benefits from consumer belief that the cost $c$ of $q_2$ is high, because such belief discourages consumer search. This gives an incentive for Retailer 2 in the case of low cost to mimic the decision it would make in the case of high cost. Therefore, pooling may be an equilibrium (in which price is not affected by the cost) even though in the absence of consumer inference, it would not be (since when demand is downward sloping, costs would have a one-to-one mapping into prices). On the other hand, in the case of high cost, Retailer 2 is interested in preventing this from happening, so that consumers could infer from its actions that the cost is high. A distortion generated by such actions is the signalling distortion.

When search costs are sufficiently high, the equilibrium is separating (the cost can be inferred from the price with certainty). This is because nobody stands to benefit from affecting consumer beliefs if search is too costly to be undertaken regardless of the consumer beliefs. To analyze the model, we therefore first consider the question of when the (separating) strategies defined by the system of Equations (4) stop being an equilibrium of the game with consumer uncertainty about costs.

To answer this question, we need to consider when Retailer 2 in the case of low cost of $q_2$ would be willing to imitate the pricing it would make under the high cost instead of pricing as in Equation (4). Note that as $s$ decreases from a high value, the consumer search becomes a constraint first in the case of low cost. This is intuitive since the benefit of lower cost is shared between the retailer and the consumers and so the retailer’s margins are higher when costs are
lower. Comparing the profits of Retailer 2 in the case of low cost when following the strategy in Equation (4) under $c = c_L$ with the alternative of pricing as if $c = c_H$, we obtain that the latter becomes a profitable deviation when consumer search cost is small enough. The exact condition under which Retailer 2 facing low cost prefers to imitate the perfect-information price it would set if the cost would be high is

$$s < s_1 \equiv \frac{2q_2 (2q_2 \Delta - (2q_2 - q_1) c_H)}{(4q_2 - q_1) (2q_2 - q_1)}. \quad (6)$$

An interesting observation is that at the above critical value ($s_1$) of the consumer search costs $s$, the price of Retailer 2 in the case of high cost (and consumer belief that the cost is high) may still be below the search-constrained price of $c_H + s$. This has two consequences. First, at search costs close to this value, Retailer 2 prices optimally in the case of high cost and therefore (by the envelop theorem), the effect of slight price change on its profits is of second order in the price change. Second, the effect of Retailer 2 slightly changing price on its profits given low costs is first-order in the price change (with price increase having the negative effect). Therefore, by the Intuitive Criterion, a slight increase of price by Retailer 2 would signal that the cost is high (since Retailer 2 would be willing to do this to affect consumer beliefs only if it actually faced the high costs). Thus, for some range of $s$ below the above critical value $s_1$, the perfect Bayesian equilibrium satisfying the IC must be such that Retailer 2 prices as in Equation (4) with $c = c_L$ when in fact $c = c_L$ and prices higher than Equation (4) suggests for $c = c_H$ when in fact $c = c_H$. In the latter case, Retailer 2 facing $c = c_H$ needs to price at the level where Retailer 2 facing $c = c_L$ is indifferent between pricing at that point with consumers not searching and pricing at $c_L + s$ (and consumers not searching). However, this price distortion in the case of high cost can only be sustained as far as it does not cross the $p = c_H + s$ boundary, since above that level, sales would revert to zero as consumers would search even if convinced that the cost is high. The latter constraint defines the minimum $s$ for which the pricing game where Retailer 2 only carries $q_2$ has a separating perfect Bayesian equilibrium, and is (see Appendix):

$$s > s_2 \equiv \frac{2q_2 (\Delta - c_H) + q_1 c_L}{4q_2 - q_1}. \quad (7)$$

The above considerations lead us to the following proposition (the result that for $s > s_2$, the equilibrium of the full game involves single-product assortment will be proven below, but for completeness, we report it immediately here):

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$^9$Note that this is for a given the product, i.e., given consumer value. The margin of the lower quality product is lower than that of the high quality product even though the cost of the low quality product is lower.
Proposition 1. For \( s_2 < s < s_1 \) (defined by Equations (6) and (7); this range is not empty for small enough \( c_H \)), in the unique perfect Bayesian equilibrium satisfying IC, Retailer 2’s pricing strategy is:

\[
p_2(c_L) = c_L + s \quad \text{and} \quad p_2(c_H) = p_2^{\text{Sig}} = \Delta - s + \frac{q_1(c_L + s)}{2q_2}.
\] (8)

Furthermore, when \( c = c_H \) and \( s \in (s_2, s_1) \), the price \( p_2 \) is distorted upwards from the perfect-information price. For \( s > s_1 \), the full-information pricing reported in Equation (4) is the unique perfect Bayesian equilibrium of the full game. For \( s < s_2 \), there is no separating perfect Bayesian equilibrium where Retailer 2 chooses a single-product assortment.

As we will show below, for \( s < s_2 \) (but not too small), Retailer 2 choosing full product assortment could signal high cost and therefore, could be beneficial for Retailer 2. However, before we turn to that case, it is worthwhile to point out the following implication of the above proposition: For a range of consumer search costs \( (s \in (s_2, s_1)) \), Retailer 2’s full-information price in the case of high cost does not depend on consumer search cost but its equilibrium price under consumer uncertainty increases when consumer search cost decreases. Furthermore, since the equilibrium price of Retailer 2 facing low costs decreases when consumer search costs decrease, the ex-ante price dispersion is decreasing in consumer search costs (with ex-post price decreasing in the search cost when the cost is high). Note that this possibility is obtained in Kuksov (2004) through the consideration of the endogenous product choice decision, while here, it is obtained from the price signaling alone. Note also that when price increases, consumer surplus decreases. Therefore, the expected consumer surplus (across different cost realizations) increases when \( s \) decreases, in the high cost case, consumer surplus decreases when search costs decrease within the range \( s \in (s_2, s_1) \).

To understand the potential benefits the retailer may gain from increasing product assortment, it is important to understand what can happen in equilibrium with single-product assortments for \( s < s_2 \). While we know from Proposition 1 that separating equilibrium is not possible in this case, one can show that pooling is an equilibrium for low enough \( s \). Specifically, let

\[
s_3 = \frac{4q_2(\Delta - \Delta_c) - (c_L + c_H)(2q_2 - q_1)}{2(4q_2 - q_1)},
\] (9)

where \( \Delta_c = c_H - c_L \). Then, the following proposition states when a pooling equilibrium exists:

\(^{10}\) An astute reader will note that there is a range of search costs where neither pooling nor separating equilibrium exist. Due to the difficulty of analytical derivations of mixed strategy equilibria, we omit consideration of this range of search costs and concentrate on the range of parameters where pure strategy equilibria exist (pooling and separating equilibria are in pure strategies; semi-separating (hybrid) equilibria necessarily include a mixed strategy).
Proposition 2. In the pricing game with Retailer 2 carrying only product \( q_2 \), the following pricing is the outcome of a pooling perfect Bayesian equilibrium satisfying IC when \( s < s_3 \):

\[
p_1 = \min \left\{ s, \frac{(c+s)q_1}{2q_2} \right\}, \quad p_2 = \frac{c_L + c_H}{2} + s.
\]

Furthermore, a pooling perfect Bayesian equilibrium satisfying IC does not exist for \( s > s_3 \).

Let us now turn to the equilibrium consideration in the pricing game where Retailer 2 has the full product line. Similar analysis to the above reveals that when Retailer 2 chooses overlapping assortment, the full-information pricing derived in Section 4.2 defines a unique separating perfect Bayesian equilibrium satisfying IC for \( s > s_4 \equiv \Delta - \frac{c_H}{2} \).

Note that this full-information pricing constitutes a separating equilibrium, since different costs correspond to different prices. Thus, for \( s_4 < s < s_3 \), Retailer 2 can signal high cost through overlapping assortment. Would it be willing to do so? The answer depends on the comparison of the profits in the case of high cost between the single-product and full (overlapping) assortment cases. It turns out that for

\[
s < s^* \equiv \frac{2q_2\Delta - \sqrt{[2q_2\Delta^2 - c_H(2q_2 - q_1)]q_1}}{2(2q_2 - q_1)} - \frac{c_L}{2},
\]

the equilibrium profit of Retailer 2 is higher under the (separating) equilibrium of the full assortment choice then under the pooling equilibrium of the single-product choice. Therefore, for \( s \in (s_4, s^*) \), when the cost of \( q_2 \) is high, Retailer 2 is able and willing to signal high cost by choosing overlapping assortment.

Getting back to the pricing game with Retailer 2 having full assortment, we have that for \( s < s_4 \), Retailer 2 in the low cost case would like to pretend that the cost is high by imitating the decision it would make in the high cost case if consumers are convinced by such action that the cost is high. Furthermore, unlike in the case of single-product assortments, Retailer 2’s price when facing high cost in the full information case is already constrained by search (i.e., \( p_{22} = c_H + s \)) when \( s < s_4 \). That is, when \( s < s_4 \), Retailer 2 cannot profitably signal high cost by raising price (i.e., the equivalent of \( s_1 \) and \( s_2 \) for the full assortment case happen to be equal). Because of this, one might expect that no separating equilibrium exists when \( s < s_4 \). However,\footnote{Incidentally, this is also the exact cut-off when the price of Retailer 2 becomes constrained by consumer search in the case of the high cost (i.e., for \( c = c_H \)).}
considering the high-assortment continuation game of the full game (with assortment choice), one has to ask: why are we in the situation with high assortment? If consumers expect pooling equilibrium with single-product assortment, under what conditions would Retailer 2 possibly benefit from increasing assortment?

It turns out that when consumers expect pooling equilibrium with single-product assortment but would be convinced that the cost is high by high assortment, Retailer 2 would benefit from increasing assortment (and pricing at $c_H + s$) if the cost of $q_2$ is actually high and

$$s < s_5 \equiv \frac{\sqrt{2q_2(2q_2 - q_1)\Delta_c^2 + 4q_1q_2\Delta_c + q_1^2c_H^2} - 2q_2\Delta_c - q_1c_L}{2q_1}, \quad (13)$$

where $\Delta_c = c_H - c_L$; while if the cost is low, Retailer 2 may only benefit when

$$s < s_6 \equiv \frac{\sqrt{2q_2(2q_2 - q_1)\Delta_c^2 + 4q_1q_2\Delta_c + q_1^2c_L^2} - 2q_2\Delta_c - q_1c_H}{2q_1}. \quad (14)$$

Comparing the two expressions above, one can see that clearly $s_6 < s_5$ (when $q_2 > 2q_1$, we also have that the equilibrium $p_1$ is not constrained by search). Therefore, the Intuitive Criterion rules out the single-assortment pooling equilibrium for $s \in (s_6, s_5)$, and within this range of $s$, we have a separating perfect Bayesian equilibrium (satisfying IC) in which Retailer 2 chooses to carry the full assortment if and only if it faces the high cost of $q_2$. Note that in that equilibrium, if Retailer 2 facing low cost would happen to have high assortment, it would imitate the pricing decision under high cost condition. However, when observing full assortment (and $s \in (s_6, s_5)$), consumers believe the cost is high for sure because Retailer 2 would not have chosen full assortment to start with if it faced low cost. Therefore, we obtain the following proposition:12

**Proposition 3.** For $s_6 < s < \min\{s_4, s_5\}$ and $s_4 < s < \min\{s^*, s_3\}$, in the separating perfect Bayesian equilibrium satisfying the IC, Retailer 2 chooses full assortment when $c = c_H$ and single-product assortment when $c = c_L$. On the other hand, a separating perfect Bayesian equilibrium in which Retailer 2 chooses full assortment when facing $c = c_L$ never exists.

Since, as we have seen in Section 4.2, Retailer 2 never chooses an assortment overlapping with the product carried by Retailer 1 under perfect information, the above proposition implies

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12Note that such separating strategy (Retailer 2 choosing full assortment if and only if it faces high cost of $q_2$) is a part of the separating equilibrium satisfying IC for a wider range of $s$ if consumers believe low cost implies low cost (as opposed to equal chance of high and low cost implied by the pooling equilibrium in the single-assortments game). However, for such $s$, pooling equilibrium with low assortment also satisfies IC. Therefore, due to this multiplicity of equilibria, we leave this range out of consideration.
that carrying overlapping assortments signals high cost (and therefore low benefit of searching for better price) on the relatively unique products. Furthermore, such signaling indeed occurs in equilibrium for some parameter range.

Figure 1 illustrates the cut-off values of $s$ and the equilibrium signaling strategy of Retailer 2 facing high cost. In particular, it shows that depending on the other parameter values, each of the ranges in Proposition 3 ($s_6 < s < \min\{s_4, s_5\}$ and $s_4 < s < \min\{s^*, s_3\}$) may be not empty (when both not empty, the ranges always touch each other).

Another implication of the above analysis is that given consumer uncertainty about costs, and therefore, about prices they may find with further search, introduction of common products in the retail assortments (on which the retailers obtain zero margin) may result in higher prices of some products. The reason for this is that the assortments with overlapping products may cause consumers to update their beliefs about the cost of the relatively unique product(s) upwards, which in turn allows the retailer to raise prices on such products. The outcome is that the price of the products that became common decreases while the price of the unique products increases relative to the case of assortments that do not overlap (note that in the absence of consumer belief updating or in the absence of outside market, prices on both common and unique products would decrease with higher assortment overlap). This has an overall beneficial impact on the retailer expanding its product line. The intuition is that although the competition with the local retailer increases due to the larger assortment, the competition with the further-away retailers (the outside market) decreases due to consumers updating their beliefs about the prices available at those retailers.

Although we have omitted some parameter ranges from full consideration, it is important to note that the reverse of the main result, i.e., high assortment being negatively correlated with costs and benefit of searching, is not possible in any perfect Bayesian equilibrium, since without the benefit of the larger assortment preventing search, it is always optimal for the retailers to not carry overlapping assortments. Thus, we have the following corollary:

**Corollary 1.** Under some parametric conditions, in the perfect Bayesian equilibrium satisfying IC, Retailer 2 uses overlapping assortment to prevent consumer search. Furthermore, on the equilibrium path of any perfect Bayesian equilibrium (under any parameter values), Retailer 2’s
(equilibrium) use of overlapping assortment always decreases consumer search (as compared to the amount of search that would happen if Retailer 2 drops \( q_1 \) from its assortment).

The following section discusses robustness of this result to relaxing various assumptions we made in the formal model.

5 Robustness

The stylized model we formally considered has a number of simplifying assumptions that one could consider restrictive. We have considered a market where consumers can search without cost across some retailers and have a search costs at others. That some retailers are easier to search at than others is probably a realistic assumption. But the interpretation which retailers are likely to have lower search costs could be more complicated. We called the set of retailers with low search costs “local.” It is important to note that such definition implies that “local” does not necessarily reflect the physical proximity. In offline shopping, neighborhood markets (e.g., gas stations, drugstores) may be physically closer and more convenient to shop at than superstores. When online shopping is included, at least for some customers, online shopping may represent the low search cost and therefore “local.” Note also that the assumption of zero search cost in the local market is not important as far as search costs are such that consumers do shop across some stores.

An important market characteristic for our result is that the prices in the “outside” market could be lower and that they are positively correlated with costs. Note that while our operationalization of these assumptions specified perfectly competitive outside market, such strong assumption is in no way necessary for the conceptual results. Consumers have a potential benefit of search even if the prices in the outside market are in expectation higher than in the local market as far as they could be lower in some realizations. From this point of view, it is realistic that the potential benefit of search exists no matter what type of the stores are in the outside market. Of course, if the expectation is that the outside market is likely to be less competitive than the local one, the benefit of search is lower. But this just means that the critical values of the search cost parameter would be scaled down by a multiplicative factor.

Likewise, it is important that the “local” retailers have an incentive to signal that the costs are high. For that to hold, it is not important that prices are perfectly correlated with costs in the outside market, but only that the correlation is positive, which is likely a realistic assumption.
The assumption of perfectly competitive outside market also implied that there is no strategic response of the outside retailers to the strategies of the local retailers. If margins in the outside market were positive, the competition would be more strategic. However, as far as in equilibrium, the local retailer(s) convince their (local) customers not to search further, the outside retailers, expecting such equilibrium, would not react strategically. Even if they did, the important condition we need is that whatever the local retailers do that intensifies competition has a stronger effect on the local market than on the outside market, likely a realistic assumption.

For simplicity, we have also considered cost uncertainty of the high-quality product only. It is easy to extend the model to the case when consumers have uncertainty about costs of both products. Remind that as far as preventing search is not a binding constraint on the local retailers’ strategies, consumers do not care about costs (and prices end up revealing costs). Therefore, the results would hold if uncertainty about costs of one product is high enough, while the uncertainty about the cost of the other is not high enough to justify further search. Since low quality products have lower costs (and margins), it is quite likely that consumers would actively consider search for lower price of the higher quality product, but not for a lower price of the low quality one. As far as the search constraint is not binding for the low quality product, the same issues as present in the main model would be present in the case with consumer uncertainty about both costs.\(^{13}\)

Another simplifying assumption one could like to relax is that we considered product choice decision by one retailer only. What would happen if both retailers were choosing assortments? Without the outside market, the standard result of retailers differentiating in quality holds (Mussa and Rosen 1978, Moorthy 1988). Once the outside market and the consumer uncertainty about costs is introduced to the model, the retailer for whom consumer search is a binding constraint would want to convince its customers that the cost is high, and will consider expanding the assortment to have an overlap with the other retailer. Within our model (with consumers not having sufficient uncertainty about costs of the low quality product), the retailer carrying the high quality product is the only one who then considers increasing the product assortment. Note that the product choice of the retailer carrying low quality product (Retailer 1) remains optimal regardless of whether the other retailer chooses high quality product only or the full assortment. Thus, the main conceptual result – that overlapping assortment increases consumer

\(^{13}\)Although we have no reason to believe the results would not hold in case both constraints are binding, the formal consideration of uncertainty in multiple dimensions would be very complicated.
expectation of costs and therefore, reduces their incentive to search – still holds.

Finally, note that while we considered vertically differentiated market, the same intuition should apply to a horizontally differentiated market as well: product overlap increases competition and results in loss of sales of the unique products (keeping their prices constant), which is less costly for the retailer if the margin on the unique product(s) are smaller.

6 Discussion and Conclusion

When we think about retail product assortment decision, we are used to think about the trade-off between costs of a larger assortment – such as the costs of shelf space, inventory management, consumer search and confusion, or cannibalization, – and the benefits of serving a heterogeneous and/or variety-seeking customer base. Competitive considerations add the incentives to adapt assortment for differentiation or to gain consumer loyalty by never giving a reason for a consumer to doubt that she can find the best fit at a given store. In particular, the idea of product differentiation is a cornerstone of the mainstream advice in competitive strategy. In the context of a retailer’s strategy, where a major part of product decision is the selection of products to carry from the common pool of products offered by the manufacturers, product differentiation strategy implies carrying assortment that has little or no overlap with that of the close-by competitors.

Although some retailers (e.g., Trader Joe’s or Whole Foods in the supermarket context) do seem to follow the above differentiation strategy, most seem not to. For example, most supermarket chains have an amazing amount of overlap in their product assortments. Certainly, consumer heterogeneity and variety seeking as well as consumer search for the best fit (or rather, a retailer’s incentive to prevent such consumer search across stores) could partially explain such assortment choice. However, these explanations imply that assortments should overlap less in categories where consumers are more familiar with the products (i.e., in frequently purchased and relatively stable categories) and where consumers have less propensity for variety seeking. But arguably, supermarkets have more overlapping assortment than apparel retailers even though they have a more stable selection of products in many categories and carry products that are more frequently purchased (e.g., packaged and canned foods).

It is possible that the basket-purchase nature of supermarket trips together with consumer heterogeneity may help explain product overlap in all categories even though consumers search for fit more in some categories than others. It is also possible that some retailers are sufficiently
differentiated in some dimension, so that they do not need to differentiate through their assortments. But in this paper, we revisited the problem of competitive retail assortment from a somewhat different perspective. Undoubtedly, the issue of consumer heterogeneity and fit are important. But equally important is the consumer desire not to pay more than necessary for a given product. Given the numerous shopping alternatives consumers face in the modern world, a consumer’s trade-off is often not as much between how well the product satisfies her preferences and the price she needs to pay, but between the price of a particular chosen product at a given store and the price of the exactly same product obtained through a more exhaustive search across stores. This trade-off brings forward the question of how consumers form beliefs about what prices they may find elsewhere.

Note that the uncertainty about price at the retailers not yet visited is, in a sense, a more pervasive characteristic of contemporary markets than the uncertainty about product quality or fit. This is because the uncertainty about product fit in some categories could be resolved either over time (in case of frequently purchased categories) or through search at the first few retailers, while the potential for consumer uncertainty about prices at stores not yet visited remains due to the ease (and empirically observed frequency) of price changes, even if past prices at all retailers have been observed and remembered.

The rationale explored in this paper is that an assortment overlap with the closest competitor can signal that the retailer’s price on the relatively unique products (i.e., those not carried by the closest competitor) is due to the underlying cost of the product (presumably similar between all competitors) and thus prevent consumers from searching further for a better deal. The analysis shows that such signaling, although costly due to both intensifying competition with the closest competitor and some consumers down-trading to the low-end product, could be preferable to the alternative option of reducing price sufficiently to prevent consumer search, i.e., due to reducing competition with the further away competitors. Although we show that in this model, high cost may, under some conditions, be also signaled by raising price (a result similar to low-cost signaling through low price in the models of extant literature), such signaling through price is not always possible as high price encourages consumer search irrespective of their belief about costs. The alternative available mechanism is then to increase assortment in the direction of products commonly available at close-by retailers. The intuition is that such action shows consumers the retailer’s willingness to sacrifice sales similarly to how a higher price would, but without the effect of encouraging consumer search.
As another example of the marketing environment we aim to study, consider the following situation adapted from the one described in the introduction to a more contemporary setting. Imagine a consumer looking to buy a juice extractor. She starts with visiting one retailer and finding that this retailer only carries a basic model which will not suit her unique needs. We will call this retailer the low-end one. Then, visiting another retailer, she finds a more advanced model which can perform all the functions she needs. We will call this retailer the high-end one since it carries the more advanced model.

Having decided to buy this advanced juice extractor, the consumer still has an important uncertainty left in her mind: does the high-end retailer offer a good price on this particular product? Certainly, it is reasonable that the price of the more advanced model is higher than the price of the basic model (found at the first retailer), but is it too much higher? In other words, this consumer is concerned that the current retailer may be selling this advanced model at a high margin, in which case further search for price could be beneficial: it could be possible to find another retailer where the advanced model is offered at a lower price. If this uncertainty is not resolved, the higher-end retailer in question could lose the demand from this consumer even though it offers the exactly right product for the customer in terms of the product features.

As we have argued, a possible and indeed observed in practice action a salesperson at the higher-end retailer could use is to bring to the consumer’s attention that the retailer also carries other juice-extractor models, in particular the lower end models carried by the lower-end retailer, and that its price on such models is competitive with the other retailers.

In order to validate that such consumer inference is not too far-fetched, we have conducted an in-lab experiment replicating the setting of the above example. The subjects were 50 undergraduate students at a major North American university participating in the experiment for a partial course credit. The one-factor (assortment) two-level (one or two products at the focal retailer) between-subjects experiment design was as follows. First, all of the participants are asked to imagine that they are shopping for a juice extractor (see Appendix B for the details). Available products consisted of only two items: a basic model carried by a low-end/discount retailer, and an advanced model that can be found in a specialty retailer with a higher price compared to the basic model. Each participant was randomly assigned into one of two groups. Participants

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14 We choose to illustrate the marketing environment on a non-frequently purchased item both to illustrate how the search for fit may be separated from further search for price and to avoid considering product basket decisions.

15 As noted above, the HBS case “Tweeter, etc.” explores this strategy by the specialty audio-video equipment retailer.
in the “control group” were told that the specialty retailer only carries the advanced model in question, while participants in the “experiment” group were told that the specialty retailer carries both the advanced model and the basic model, which is priced at the same level as it is at the discount retailer. It was made clear to the subjects that the basic model will not serve their needs and they are only interested in the advanced model. After the participants were presented with the situation to imagine themselves in, they were asked to complete a series of questions. The one of interest to us was to state their willingness to search elsewhere (on a 10 point scale) for a lower price on the advanced model. The participants observing the overlapping assortment carried by the specialty retailer had the average willingness to search of 7.5/10 for a lower price of the advanced model elsewhere. The participants observing non-overlapping assortment had the average willingness to search of 8.6. The difference is in the hypothesized direction and statistically significant at a 5% level. This short laboratory study therefore provides some support for the applicability of our analytical analysis to a practical marketing environment.
References


Appendix A: Derivation of Benchmark Results and Proofs of Propositions

Proof of Results in the Benchmark Model

Consumer utility defined by Eq. (1) implies that the consumer indifferent between buying from Retailer 1 and not buying at all has \( \theta = \theta_1 \equiv \frac{p_1}{q_1} \), while the consumer indifferent between buying \(q_2\) from Retailer 2 and \(q_1\) from Retailer 1 has \( \theta = \theta_2 \equiv \frac{p_2 - p_1}{q_2 - q_1} \). Therefore, in the absence of the outside market and when retailers carry one product each, the demand for Retailer 1 comes from consumers with \( \theta \in [\theta_1, \theta_2] \) while demand for Retailer 2 comes from consumers with \( \theta \in [\theta_2, 1] \).

Hence, the retailers’ profit functions can be written as:

\[
\pi_1 = p_1 \left( \frac{p_2 - p_1}{q_2 - q_1} - \frac{p_1}{q_1} \right), \quad \pi_2 = (p_2 - c) \left( 1 - \frac{p_2 - p_1}{q_2 - q_1} \right).
\] (A1)

Simultaneous profit maximization of the two retailers with respect to \(p_1\) and \(p_2\) yields the results in Equation (2). Note that \( p_2^{ns} - c > p_1^{ns} - 0 \) if

\[
c < \frac{\Delta (2q_2 - q_1)}{2q_2}
\] (A2)

which is satisfied when \( \Delta \) sufficiently large. The margin of \(q_2\) is higher than the margin of \(q_1\) when (A2) is satisfied.

When Retailer 2 carries both products, as argued in the main text, the direct competition with Retailer 1 for \(q_1\) results in the price of \(q_1\) equal to zero at both retailers. Therefore, in this case, the consumer indifferent between buying \(q_2\) at Retailer 2 and \(q_1\) at either retailer has \( \theta = \theta_3 = \frac{p_2}{q_2 - q_1} \) and the demand for Retailer 2’s product \(q_2\) comes from consumers with \( \theta \in [\theta_3, 1] \).

Retailer 2 sets \(p_2\) to maximize the following profit function:

\[
\pi_2 (q_2) = (p_2 - c) \left( 1 - \frac{p_2}{q_2 - q_1} \right), \quad (A3)
\]

which yields the results in Equation (3) and completes the derivation of the results in the case when consumer search in the outside market is not a binding constraint.

Consideration of the outside market adds the following. When consumers have perfect information about costs, they also have full information about prices in the outside market. Therefore, the simultaneous maximization of profits in (A1) is subject to the constraints

\[
p_1 \leq s \quad \text{and} \quad p_2 \leq c + s.
\] (A4)
Consider single-product assortments. In this case, the constraint \( p_2 \leq c + s \) is not binding if

\[
p_{2s}^n - c = \frac{2\Delta (q_2 - c) - q_1 c}{4q_2 - q_1} < s
\]

which is satisfied for \( s > \frac{2\Delta (q_2 - c) - q_1 c}{4q_2 - q_1} \). The condition \( (p_{2s}^n - c) > (p_1^n - 0) \) also ensures that the constraint \( p_1 \leq s \) is not binding. As a result, \( p_j^f = p_j^n, j = 1, 2 \), when the above constraint on \( s \) is satisfied.

If \( s \leq \frac{2\Delta (q_2 - c) - q_1 c}{4q_2 - q_1} \), we have \( p_2^f = c + s \). Expecting this \( p_2 \), Retailer 1 finds its optimal price to be \( p_1 = \frac{(c+s)q_1}{2q_2} \), which is smaller than \( s \) if

\[
s > \frac{cq_1}{2q_2 - q_1}.
\]

If Retailer 2 carries both products, following similar logic as in Benchmark 1, one can see that the competition for sales of \( q_1 \) drive its price to cost at both retailers, i.e., \( p_{11}^f = p_{21}^f = 0 \). Then the results in Equation (5) follow from the no-search constraint similarly to the single-product assortments case. This completes the derivation of Benchmark results.

Proofs of Results and Propositions in the Full Model

1. Derivation of \( s_1 \) (Equation (6))

We have that when Retailer 1’s price of \( q_1 \) is not constrained by consumer search and consumers know (or correctly expect) the cost \( c \) of \( q_2 \), the price of \( q_2 \) at Retailer 2, \( p_2^{ns}(c_H) \) is constrained by search cost if \( s < s_{\text{min}} = \frac{\Delta q_2 + q_1 c_H - 2q_2 c_H}{4q_2 - q_1} \), while \( p_2^{ns}(c_L) \) is constrained by search cost if \( s < s_{\text{max}} = \frac{\Delta q_2 + q_1 c_L - 2q_2 c_L}{4q_2 - q_1} \). As one can easily see, \( s_{\text{max}} - s_{\text{min}} > 0 \). This implies that \( p_2^{ns} \) is constrained by consumer search for a larger range of \( s \) when \( c = c_L \) as opposed to when \( c = c_H \). In the absence of consumer search constraint on Retailer 1’s pricing, Retailer 1’s profit maximization (see Equation (A1)) implies

\[
p_1 = \frac{q_1 - p_2^*}{2q_2}
\]

where \( p_2^* \) is the Retailer 1’s belief about Retailer 2’s price. Note that although Retailer 1 knows Retailer 2’s cost, \( p_2^* \) depends on whether Retailer 1 believes Retailer 2 is pooling or separating.

To show that Equation (4) constitutes the outcome of a PBE satisfying IC if and only if \( s > s_1 \), consider the following. First note that this pricing, if an equilibrium one, implies a separating equilibrium, i.e., when Retailer 2 follows the equilibrium strategy, rational expectations imply that consumers can infer the cost \( c \) from the price \( p_2 \) precisely. Therefore, we are in a situation
similar to Benchmark 2 (full information). When none of the prices are constrained by consumer search, clearly, all agents behave optimally and cannot gain from deviation regardless of how the deviation may change consumer beliefs. Therefore, when no price is constrained by consumer search, Equation (4) is an outcome of PBE satisfying IC. (To fully define PBE, define the consumer off-equilibrium beliefs about $c$ to be $c = c_L$ with probability 1).

Consider now Retailer 2’s pricing in the range where according to Equation (4), $p_2(c_L)$ is constrained by search but $p_2(c_H)$ is not. That is, we have $p_2(c_L) = c_L + s$, while $p_2(c_H)$ is the optimal price unconstrained by consumer search $p^{ns}(c_H)$. Clearly, Retailer 1 and Retailer 2 in the high-cost case cannot benefit from deviation as these retailers price optimally given consumer expectations and have no potential benefit of changing consumer expectations of $c$ (since prices are not constrained by consumer search). But Retailer 2 under the low cost ($c_L$) may benefit if it takes an action that convinces consumers that the cost is high (since it would then possibly not face the consumer search constraint). Substituting the candidate equilibrium price $p_2^*$ from Equation (4) for $p_2^*$ in Equation (A7), we derive that the candidate equilibrium profit of Retailer 2 under low cost ($c_L$) is

$$
\pi_{2\text{Sep}}(c_L) = \frac{(c_L q_1 - 2c_L q_2 + sq_1 - 2sq_2 + 2q_2^2 - 2q_1 q_2) s}{2q_2 (q_2 - q_1)}.
$$

(A8)

Alternatively, given the separating equilibrium beliefs, Retailer 2 can ensure that consumers expect $c = c_H$ by mimicking the equilibrium price of Retailer 2 under high cost ($c_H$). Note that if Retailer 2 deviates to this price when $c = c_L$, Retailer 1 still sets price $p_1 = \frac{q_1}{2q_2} p_2^* = \frac{q_1}{2q_2} (c_L + s)$, since by definition, deviations are not expected in the equilibrium (by assumption, Retailer 1 sets price before observing Retailer 2’s price but observes $c$ directly). Substituting this $p_1$ as well as the true cost $c = c_L$ into Retailer 2’s profit function, we obtain that Retailer 2’s profit given low cost and the above deviation is

$$
\pi_{2\text{Dev}}(c_L) = \frac{(-2q_2^2 + 2q_1 q_2 - 2q_2 c_H - q_1 c_L + 4q_2 c_L) (-2q_2^2 + 2q_1 q_2 - q_1 c_H + 2q_2 c_H)}{(q_1 - 4q_2)^2 (q_2 - q_1)}.
$$

(A9)

The deviation is strictly preferable (and thus the above defined pricing cannot be the outcome of a separating equilibrium) when $\pi_{2\text{Sep}}(c_L) < \pi_{2\text{Dev}}(c_L)$. Solving this for $s$, we have that this inequality is equivalent to $s < s_1$ as specified in Equation (6). Thus, we have proven that pricing defined by Equation (4) is an equilibrium outcome and it satisfies IC if and only if $s < s_1$. The proof that this pricing is a unique outcome of a PBE satisfying IC is completed in the proof of Proposition 1.
2. Derivation of $s_2$ (Equation (7))

Consider the possibility of separating equilibrium with Retailer 2 pricing at $c + s$. If in this case Retailer 2 under low cost mimics the separating pricing strategy of high cost constrained by search, i.e., if $p_2 = c_H + s$ while the cost is $c_L$, Retailer 2’s profit is:

$$\pi^{Dev2}_2(c_L) = \frac{(c_H - c_L + s)(sq_1 - 2sq_2 + 2q_2^2 - 2q_1q_2 - 2q_2c_H + q_1c_L)}{2q_2(q_2 - q_1)}.$$  \hfill (A10)

Alternatively, it can set $p_2 = c_L + s$ and achieve profit of $\pi^{Sep}_2$ defined in Equation (A8) (note that $p_1$ is defined by Equation (A7) with $p_2^* = c + s$ since Retailer 1 knows $c$ and expects Retailer 2 to follow the equilibrium strategy). We have that $\pi^{Sep}_2(c_L) - \pi^{Dev2}_2(c_L) > 0$ if and only if $s > s_2$ as specified in Equation (7). Thus, Equation (4) defines a PBE pricing if and only if $s > s_2$.

3. Proof of Proposition 1

First note that in any separating equilibrium, Retailer 2 in the case of low cost sets price optimal given that consumers correctly recognize that the cost is low. This is because in separating equilibrium, when the cost is low, consumers recognize that the cost is low from the equilibrium strategy and in any deviation, if consumers were to change their beliefs, then Retailer 2 could only be better off. Therefore, a strategy not optimal for the low cost Retailer 2 under perfect information cannot be an equilibrium strategy for the low cost Retailer 2 in a separating equilibrium. Thus, we have that in any separating equilibrium, Retailer 2’s price follows Equation (4) when $c = c_L$. The same holds for Retailer 1’s pricing for any $c$ since its price is not constrained by consumer search. It remains to consider Retailer 2’s price for $c = c_H$.

Setting $p_2$ according to Equation (4) is strictly optimal for Retailer 2 with high cost as far as consumers believe it has high cost. By definition of $s_1$, Retailer 2 with low cost does not want to price as the high cost one in Equation (4) to be (incorrectly) recognized as the high cost one when $s > s_1$. Therefore, for $s > s_1$, Equation (4) defines the unique pricing outcome of a PBE satisfying IC.

When $s < s_2$, we have that Retailer 2 in the case of low cost prefers $p_2 = c_H + s$ to $p_2 = c_L + s$ if consumers then believe its cost is high (implying no search). Therefore, it also prefers any $p_2 \in (c_L + s, c_H + s)$ to $p_2 = c_L + s$ if consumers then believe its cost is high. Thus, no price $p_2 \in [c_L + s, c_H + s]$ for $c = c_H$ could be the outcome of a separating equilibrium. Price $p_2 > c_H + s$ cannot be optimal either as it guarantees zero sales regardless of consumer beliefs. Price $p_2 = p_2' < c_L + s$ for $c = c_H$ cannot be an equilibrium one since $p_2 = \frac{p_2' + c_L + s}{2}$
(with consumers then not searching regardless of their belief about cost) would result in strictly higher profits. Therefore, we have proven the claim of Proposition 1 that for \( s < s_2 \), there is no separating PBE with positive sales of Retailer 2 with any \( c \).

It remains to consider \( s \in (s_2, s_1) \). Price \( p_2 \) for \( c = c_H \) in Equation (8) is derived from the condition that Retailer 2 facing \( c = c_L \) is indifferent between pricing at \( c_L + s \) and that price (assuming consumers don’t search). Specifically, if Retailer 2 prices at the level \( p_2^{Sig} \) when \( c = c_L \), its profit will be

\[
\pi_2^{Dev3} (c_L) = \frac{(p_2^{Sig} - c_L) \left( s q_1 + 2 q_2^2 - 2 p_2^{Sig} q_2 - 2 q_1 q_2 + q_1 c_L \right)}{2 q_2 (q_2 - q_1)} \tag{A11}
\]

Equating the profits in Equations (A11) and (A8) and solving for \( p_2^{Sig} \) yields the expression for \( p_2 \) reported in Equation (8) for \( c = c_H \). Note that this \( p_2^{Sig} \) satisfies \( p_2^{f}(c_H) < p_2^{Sig} < c_H + s \) when \( s_2 < s < s_1 \) and becomes equal to \( c_H + s \) when \( s = s_2 \) (the latter by definition of \( s_2 \)). Note that this pricing strategy reported in Proposition 1 for \( s \in (s_2, s_1) \) constitutes a unique PBE equilibrium satisfying IC since it is derived as the least cost deviation by the high-cost retailer that makes the low-cost one indifferent between following the equilibrium and deviating when deviation results in the belief that the cost is high (Cho and Kreps, 1987). To fully define a PBE satisfying IC, define consumer beliefs as \( c = c_L \) if \( p_2 < p_2^{Sig} \) and \( c = c_H \) if \( p_2 = p_2^{Sig} \).

The claim that \( p_2^{Sig} \) increases when \( s \) decreases and that it is higher than the full-information equilibrium price (Equation (4)) are straightforward to check. This completes the proof of Proposition 1.

4. Proof of Proposition 2

The value of \( s_3 \) (in Equation (9)) is defined by the Retailer 2 indifference when \( c = c_L \) between pricing at \( p_2^{Pool} = c_L + c_H \) \( \frac{c_L + c_H}{2} \) + \( s \) and pricing at \( c_H + s \) (with consumers not searching): the profit with the former pricing is

\[
\pi_2^{Pool} (c_L) = \left( -2s - c_H + c_L \right) \left( 2 s q_1 - 4 q_2^2 + 4 q_2 q_1 c_H - 2 q_2 c_H + q_1 c_L - 2 q_2 c_L \right) \frac{8 q_2 (q_1 - q_2)}{8 q_2 (q_1 - q_2)} \tag{A12}
\]

while with the latter one (and consumer not searching), it is

\[
\pi_2^{PoolD} (c_L) = \frac{(2 s q_1 - 4 q_2^2 - 4 q_1 q_2 + q_1 c_H - 4 q_2 c_H + q_1 c_L) (-s - c_H + c_L)}{4 q_2 (q_1 - q_2)} \tag{A13}
\]

We then have that \( \pi_2^{PoolD} (c_L) - \pi_2^{Pool} (c_L) > 0 \) if and only if \( s < s_3 \). Thus, we have that for \( s < s_3 \), Retailer 2 facing either \( c \) prefers to set \( p_2 = c_H + s \) if consumers won’t search, while for
\( s \in (s_3, s_2) \), only the high-cost Retailer 2 prefers \( p_2 = c_H + s \) to \( p_2 = p_2^{Pool} \) (since \( c_H + s > c_L + s \), we have \( s_3 < s_2 \)). In other words, by definition of \( s_3 \), a pooling equilibrium with pricing defined by Equation (10) is ruled out by the Intuitive Criterion for \( s > s_3 \) and satisfies IC for \( s < s_3 \). To fully define the PBE satisfying IC, define consumer beliefs to be \( c = c_L \) for any deviation (As shown above, rules out belief \( c = c_L \) with any probability when \( p_2 \) is slightly below \( c_H + s \) when \( s \in (s_3, s_1) \)). This concludes the proof of Proposition 2.

### 5. Proof of Proposition 3

When Retailer 2 carries full assortment (i.e. both \( q_1 \) and \( q_2 \)), according to Equation (5), we have that \( p_2 \) becomes constrained by consumer search when \( \Delta + c > c + s \), i.e., when

\[
\frac{s}{2} < \frac{\Delta - c}{2}.
\]  

(A14)

Again, the search constraint becomes binding first when the cost is low (\( c = c_L \)). The equilibrium derivation in this case is similar to one in the single-product assortment case, with the difference that now the price of \( q_1 \) at both retailers is \( p_{11} = p_{21} = 0 \).

**Case 1:** The search constraint is binding for Retailer 2 when \( c = c_L \) but not when \( c = c_H \).

In this case, according to Equation (5), if \( c = c_L \), Retailer 2 has the following profit when following the full-information pricing reported in Equation (5):

\[
\pi_2^{HS-Sep}(c_L) = \frac{\Delta - c_L - s}{\Delta} s
\]  

(A15)

Substituting \( p_2 = \frac{\Delta + c_H}{2} \) into the profit of Retailer 2 when \( c = c_L \) (deviation to mimic the price under high cost), we obtain

\[
\pi_2^{HS-D}(c_L) = \frac{1}{4} (\Delta + 2q_1 - 2q_2 + c_H) \frac{\Delta + c_H - 2c_L}{q_1 - q_2}
\]  

(A16)

Given \( s < \frac{\Delta - c_L}{2} \) and \( c_H > c_L \), we have \( \pi_2^{HS-Sep}(c_L) > \pi_2^{HS-D}(c_L) \), i.e., Retailer 2 facing low cost never wants to mimic its optimal pricing under the high cost in the current case. Thus the full-information pricing is a separating equilibrium. Similarly to the single-product assortment, it is easy to see that this is a unique PBE satisfying IC of any continuation game where Retailer 2 chose full assortment.

Going back to the equilibrium of the full game, the above consideration implies that it is strictly suboptimal for Retailer 2 to choose full assortment when \( c = c_L \) and \( s > s_4 \). Indeed, if it were to chose full assortment, it would still optimally chose to reveal low cost through
its pricing; but given this, single-product assortment is optimal. Therefore, if Retailer 2 can possibly benefit from choosing full assortment when $s > s_4$, Intuitive Criterion dictates that consumers should believe Retailer 2’s cost is high if it chose full assortment. Similarly to the derivations in the single-product assortment case, it is easy to derive that Retailer 2 prefers choosing full assortment and pricing $q_2$ at $c_H + s$ (given that consumers believe it has high cost and don’t search) to the pooling equilibrium outcome of single-assortment choice when $s < s^*$ (by definition of $s^*$) reported in Equation (12). Therefore, when $s \in (s_4, s^*)$, the Intuitive Criterion rules out the pooling single-assortment equilibrium in favor of the separating equilibrium with Retailer 2 choosing full assortment if and only if the cost is high. Note that such equilibrium separation by assortment choice can be also supported for some $s > s^*$ by the consumer belief that single-product assortment implies $c = c_L$ (since then the comparison of full assortment choice would need to be done with pricing at $p_2 = c_L + s$ instead of $p_2 = c_H + c_L + s$ in the single-product assortment choice). But in that wider range, both pooling (to single-product assortment) and separating in assortment (with the above-defined consumer beliefs) satisfy the Intuitive Criterion.

Case 2: The search constraint is binding for Retailer 2 under full information for all $c$.

This is the case when $s < s_4$. The only difference in consideration of this case is that when Retailer 2 chooses full assortment and consumers believe that $c = c_h$, Retailer 2 needs to price $q_2$ at $c_H + s$. Similar analysis to the above shows that when $c = c_H$, Retailer 2 prefers to choose full assortment and price $q_2$ at $c_H + s$ instead of being in the single-assortment pooling equilibrium when $s < s_5$, while when $c = c_L$, it prefers this only when $s < s_6$ (where $s_5$ and $s_6$ are reported in Equations (13) and (14), respectively). Thus, the Intuitive Criterion rules out the single-product assortment pooling equilibrium when $s \in (s_6, \min\{s_5, s_4\})$.

It remains to note that there is no separating equilibrium where Retailer 2 chooses full assortment if and only if the cost is low because in such an equilibrium, Retailer 2 facing low cost would strictly prefer to deviate to single assortment full-information pricing. This completes the proof of Proposition 3.
Appendix B: Situation Description for the Lab Study

[Underlined part indicates the difference between the two conditions, but was not emphasized for the subjects.]

a) “Control” Group

“You have decided to try a special diet that requires consuming only a combination of vegetable and fruit juices except for one solid meal a day. As a result, you plan to buy a juice extractor which can handle a variety of fruits and produce. You went to a local Wal-Mart, but they only had a simple juicer that can handle fruits. This juicer would not be able to handle leafy vegetables and wheatgrass essential to your diet. The price of the juicer (item 1) was $50. Following the advice from the sales associate, you then went to a nearby specialty health store to look for a more advanced juice extractor. The specialty store did not have the simple juicer, but had a juice extractor (item 2) with $250 price. The salesperson at the specialty store assured you that this juice extractor (item 2) is very good at extracting juice from fruits, root and leafy vegetables, as well as wheatgrass. You were convinced that this is the juice extractor you should buy, whether at this store or elsewhere. However, the question you still have standing in front of this juicer in the store is whether to buy now or search for a lower price for item 2 elsewhere.”

b) “Experiment” Group

“You have decided to try a special diet that requires consuming only a combination of vegetable and fruit juices except for one solid meal a day. As a result, you plan to buy a juice extractor which can handle a variety of fruits and produce. You went to a local Wal-Mart, but they only had a simple juicer that can handle fruits. This juicer would not be able to handle leafy vegetables and wheatgrass essential to your diet. The price of the juicer (item 1) was $50. Following the advice from the sales associate, you then went to a nearby specialty health store to look for a more advanced juice extractor. While the specialty store had a similar simple juicer with $50 price tag, it also had another juice extractor (item 2) with $250 price. The salesperson at the specialty store assured you that this juice extractor (item 2) is very good at extracting juice from fruits, root and leafy vegetables, as well as wheatgrass. You were convinced that this is the juice extractor you should buy, whether at this store or elsewhere. However, the question you still have standing in front of this juicer in the store is whether to buy now or search for a lower price for item 2 elsewhere.”
Figure 1: Illustration of the Equilibrium Strategy (as a function of $s$) of Retailer 2 facing high cost.

Top figure: $q_1 = 1, q_2 = 2.85, c_L = 0.8, c_H = 1.2$;
Bottom figure: $q_1 = 1, q_2 = 2, c_L = 0, c_H = 0.3$.

Notes: Figures are not to scale. Variable values out of range of where they are applicable are grayed out. These are: In the top figure: $s_6$ (because the assumption that $p_1$ is not constrained by search does not hold for $s < s_7$) and $s_5$ (because it is relevant only when $p_2$ in the overlapping assortment case is constrained by consumer search, i.e., when $s < s_4$); In the bottom figure: $s^*$ (because it is relevant only when $p_2$ in the overlapping assortment case is NOT constrained by consumer search, i.e., when $s > s_4$).