Optimal Thresholds in Accounting Recognition Standards

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Abstract

This paper investigates the design of recognition thresholds in accounting standards. In statistics, a threshold classifies evidence to balance two types of recognition errors weighted by their respective costs to a decision maker. In accounting recognition standards, a threshold induces firms to respond strategically and thus affects the very distribution of evidence the threshold classifies. With this strategic effect, the optimal recognition threshold is determined by not only the decision maker’s loss function but also the transaction’s features. We compare the optimal threshold’s properties under the statistical and strategic approaches, provide their respective empirical predictions, and discuss the limitations of using a statistical approach to guide accounting standard setting.

**JEL recognition:** M41, M45, G28, G38

**Key Words:** Thresholds, Evidence Management, Accounting Standard Setting, Statistical Inference

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1 Introduction

This paper presents a model to study the design of recognition thresholds in accounting standards when firms can influence accounting evidence. Many recognition standards involve a choice from two treatments, such as capitalizing v.s. expensing, sales v.s. secured borrowing, operating leases v.s. capital leases, off-balance-sheet v.s. on-balance-sheet, and so on. Recognition thresholds specify the strength of evidence required for a preferred accounting treatment. To make the research question concrete, consider revenue recognition as an example. Accountants collect and analyze evidence to assess the probability that the firm has earned revenue from a transaction. This probability is then compared with a threshold $P$ prescribed by the revenue recognition standard to determine whether revenue is recognized. This paper is interested in the optimal design of threshold $P$. What economic forces determine $P$? Is 50% a reasonable benchmark? If it becomes more costly for firms to influence evidence, how does the optimal level of $P$ change?

These seem to be basic questions in setting recognition standards, yet they have received only limited attention in the prior literature. An explanation for the lack of research on this seemingly important question is not obvious. One possibility is that threshold design has been perceived as an immediate application of the well-established statistical approach and thus does not require further research in accounting. For example, one might liken the thresholds that qualify a lease transaction as an operating or capital lease to thresholds in a clinical test that confirm or refute the diagnosis of a disease. Under this statistical approach, a higher threshold reduces the undue optimism error at the expense of increasing the false alarm error. The optimal threshold balances these two types of recognition errors according to their respective costs to the decision maker.

However, threshold design in accounting standards differs from its counterpart in clinical tests in at least one crucial aspect: the threshold choice in an accounting standard influences the very distribution of accounting evidence the threshold actually classifies. A change in the threshold for diagnosing a cancer does not affect the protein content of cells of patients. In contrast, a change in the term requirements for recognizing capital leases, say from 75% to 60%, is likely to induce lessees and lessors to structure the lease terms in their contracts
differently, resulting in a different distribution of lease terms. Empirical evidence about such strategic responses has been compiled in the accounting literature, including, among others, Imhoff and Thomas (1988) (for leases), Lys and Vincent (1995) (for merger and acquisition), Engel, Erickson, and Maydew (1999) (for hybrid securities), and Dechow and Shakespear (2009) (for securitization). Moreover, the use of accounting-motivated transactions has long been a major concern for standard setters and regulators. For example, in its report to the Congress on the financial reporting of off-balance sheet arrangements, the first recommendation the Securities and Exchange Commission (the SEC) makes is "to discourage transactions and transaction structures primarily motivated by accounting and reporting concerns, rather than economics" (see SEC (2005)). We use the broad term "evidence management" to refer to firms’ activities that influence a transaction’s accounting evidence without changing its economic substance.

In this paper we examine the optimal design of recognition thresholds in the presence of evidence management. The paper’s main result is that the optimal threshold in a strategic approach, which takes into account the firm’s strategic response through evidence management, differs markedly from its counterpart in a statistical approach. The optimal probability threshold in a statistical approach is determined solely by the costs of recognition errors to the decision maker, while its counterpart in a strategic approach is also affected by the fine details of the transaction under consideration. To illustrate, consider a revenue recognition standard when the decision-making costs of premature and delayed recognition of revenue are the same. The statistical approach predicts that the probability threshold $P$ for recognizing revenue is 50%, but the strategic approach predicts that it is different from 50% in general. Moreover, it can be either higher or, perhaps surprisingly, lower than 50%, even though the manager has a one-sided incentive to inflate evidence in the model. The exact conditions for each case to prevail depend on the transaction’s details, such as the link between the transaction’s economic substance and its accounting evidence and this link’s vulnerability to the manager’s influence.

The intuition for the main result lies in the two distinct effects a threshold has on recognition errors. On one hand, for any given evidence, a higher threshold reduces a report’s undue optimism error and increases the false alarm error simultaneously. This statistical effect is
well-understood in the vast statistical literature. On the other hand, a threshold affects evidence management that, in turn, affects the distribution of evidence and thus recognition errors. This strategic effect involves the specifics of the transaction under consideration and behaves differently from the statistical effect. Since the optimal threshold takes into account both the statistical and strategic effects, it is qualitatively different from its statistical benchmark.

The intuition for the result that the optimal probability threshold in the presence of inflationary evidence management can be either higher or lower than its statistical benchmark is as follows. In the model, since evidence management reduces the report’s value for decision-making, the threshold is designed to discourage evidence management. Whether this entails a higher or lower threshold, however, depends on the sign of the strategic effect (the threshold’s effect on evidence management). It turns out that the strategic effect is not monotonic: a higher threshold initially increases and then reduces evidence management. Therefore, the optimal threshold is higher than the statistical benchmark if, and only if, a higher threshold reduces evidence management.

The non-monotonicity of the threshold’s effect on evidence management can be demonstrated by considering two extremes of setting a threshold. If the threshold is set so low that everyone can clear it without evidence management, there will be no costly evidence management. On the other hand, if the threshold is set so high that no one can clear it even with costly evidence management, there will be no evidence management, either. Therefore, whenever the threshold affects evidence management, the relation is not monotonic.

To further understand the interaction of threshold design and recognition errors, we study the threshold design when the standard setter is concerned about maximizing the accounting report’s \textit{ex post} value to the decision maker, that is, the value assessed after evidence is presented. The standard setter takes the evidence as given at the time of choosing the threshold but has rational expectations about evidence management that has influenced the distribution of the observed evidence. The optimal probability threshold in this case is the same as in the statistical benchmark and does not depend on the details of the transaction. Therefore, it is the threshold’s effect on evidence management, rather than the rational expectations about evidence management, that distinguishes the strategic from the statistical
approach to threshold design.

The qualitative trade-off of the strategic and statistical effects in determining the optimal threshold is robust to a number of extensions. While it is necessary to assume the use of binary recognition in order to study threshold design, we show that full disclosure of evidence leads to a lower firm value and more evidence management than recognition in the model. This mitigates the concern that the main results are driven by the suboptimal imposition of recognition in the model. We also show that the qualitative trade-off of the dual effects is robust to a number of model specifications.

This paper's main antecedent is Dye (2002) who has formalized a tractable framework to study threshold design in the shadow of evidence management. The manager can influence the mapping from state to evidence and the standard setter chooses a threshold to influence the mapping from evidence to recognition. Using this representation, Dye (2002) demonstrates a clear distinction between an official and a shadow standard, describes conditions for a creep of official standards over time, and characterizes the dynamic evolution of both the official and shadow standards when the standard setter is sophisticated or naive and when the distribution of projects is exogenous or endogenous.

This paper extends the framework of threshold design in Dye (2002). It highlights a different channel through which threshold design interacts with evidence management and recognition errors. In Dye (2002), a threshold affects investors' rational interpretations of the report and their pricing, which in turn affects the manager’s benefit of receiving a preferred report and thus his evidence management. This channel is isolated in the baseline model of this paper by the assumption of an exogenous conflict of interest between the stakeholder and the manager. Instead, this paper focuses on a different channel. The threshold affects the classification of evidence, which in turn changes the manager’s probability of receiving a preferred report and thus his evidence management. This channel makes it possible to study this paper's main research question of comparing a strategic approach to threshold design with its statistical counterpart in a setting with minimum departure from the statistical literature. It generates the central trade-off of the threshold’s strategic and statistical effects and the main result that the optimal threshold is not monotonic in evidence management. In other words, this paper differs from Dye (2002) in both their main results and the economic
forces behind the results. In Section 4.2, we compare the two channels in the same model to shed more light on their interaction.

A few other papers have also studied threshold design. Magee (2006) studies threshold design in a setting in which the manager communicates information about the second moment of the distribution of his signal in the absence of opportunistic evidence management. Mittendorf (2010) studies the role of audit thresholds in the misreporting of private information. The commitment to tolerating misreporting within the materiality threshold makes the threat of punishing egregious misreporting above the threshold more effective. Fan and Zhang (2012) study a model in which managers could exert private efforts to improve the precision of evidence and use the model to provide justification for conservatism. Laux and Stocken (2013) show that standards and enforcement can either be complements or substitutes. In their model the board incorporates rational expectations about the manager’s evidence management in designing his compensation contract, which in turn affects the manager’s benefit of receiving a preferred treatment and thus his incentive of evidence management.

The issue of setting evidence thresholds is also studied in the legal literature in the context of designing the burden of proof (e.g., Kaplow (2011)). In that literature, the manipulation of evidence is a lesser concern, partly because of the adversary adjudication system. In addition, the evidence threshold works exclusively through the \textit{ex ante} incentive effect on primary activities in Kaplow (2011), whereas in this paper, the threshold’s dual effects on \textit{ex ante} evidence management and the \textit{ex post} statistical efficiency of decision-making are the key tension.

More broadly, this paper complements the literature on the real effects of accounting measurement (e.g., Kanodia and Lee (1998), Sapra (2002), and Kanodia, Sapra, and Venugopalan (2004), see Kanodia (2007) for a review). This literature emphasizes the two-way interactions between a measurement rule and the transactions the rule measures. That evidence management responds to threshold design is one example of the real effects accounting measurement rules have on a firm’s real decisions and transactions. In addition, the broad economic force in this paper, that controlling a manager’s ex ante incentive requires the inefficient use of information \textit{ex post}, is a recurring theme in the agency literature. As such, this paper could be viewed as an application of the agency theory to accounting rule design. Other papers
that apply a similar insight to explain different accounting issues include, among others, Arya, Glover, and Sivaramakrishnan (1997), Dye and Sridhar (2004), and Kanodia, Singh, and Spero (2005).

The two-step framework of standard design formalized in Dye (2002) has a deep root in accounting (e.g., Ijiri (1975), Watts and Zimmerman (1986), Ball (1989), Leuz (1998a)). The states of nature are not observable and their accounting measurements are used to implement state-contingent contracts or decisions. An accounting rule prescribes a certain accounting treatment to a transaction or event that serves as an imperfect, but verifiable indicator of the underlying state. Most studies along these lines have been done in contracting settings (e.g. Reichelstein (1997), Leuz (1998a), Dutta and Reichelstein (2002), and Liang (2004)) or a combination of contracting and decision-making settings (e.g., Gigler and Hemmer (2001), Chen, Hemmer, and Zhang (2007), and Drymiotes and Hemmer (2013)). Leuz (1998b) points out that in such settings the boundary between accounting rules and contractual terms is still not well understood. In this paper, we adopt a pure decision-making setting. Even though it is not immune from the boundary issue, it might be easier to justify the difficulty of the contractual information provision in capital markets with dispersed investors.

The rest of the paper proceeds as follows. Section 2 describes the model. Section 3 solves the model and characterizes the optimal threshold. Section 4 considers several extensions. Section 5 offers some empirical implications. Section 6 concludes.

2 Model

The baseline model has three players, a standard setter, a manager, and a stakeholder. Each moves once in the following sequences.

1. At date 0, the standard setter chooses a recognition threshold \( T \);

2. At date 1, the manager engages in unobservable evidence management \( \beta \) that influences a transaction’s accounting evidence \( t \) without affecting its economic substance \( \omega \). \( \omega \) is drawn by Nature but not observed by anyone.

3. At date 2, evidence \( t \), whose distribution is determined by both \( \omega \) and \( \beta \), is realized. A
recognition report $r$ is generated by comparing evidence $t$ with threshold $T$.

4. At date 3, the stakeholder observes report $r$ and makes a decision $d$. $\omega$ and $d$ jointly determine the payoffs of the stakeholder and the manager, $v(\omega, d)$ and $u(\omega, d)$, respectively.

The model is comprised of two parts. The first part depicts a simple decision-making setting that creates demand for information. The second part elaborates the recognition process that generates the information.

2.1 The demand for information

The state of nature, denoted as $\omega$, is a transaction’s economic substance. It is either good ($G$) with probability $q_G$ or bad ($B$) with probability $q_B = 1 - q_G$; i.e., $\omega \in \{G, B\}$. It is chosen by Nature at date 2 and not observable to anyone. At date 3, the stakeholder observes an accounting recognition report $r \in \{g, b\}$ about the state and makes a binary decision $d(r) \in \{0, 1\}$ to maximize her expected payoff $E_\omega [v(\omega, d)|r]$. Without loss of generality we assume

$$L_G \equiv v(G, 1) - v(G, 0) > 0, \text{ and } L_B \equiv v(B, 0) - v(B, 1) > 0. \quad (\text{Assumption 1})$$

Assumption 1 states that with complete information the stakeholder prefers decision $d = 1$ to $d = 0$ if and only if the state is good. With incomplete information about state $\omega$, the stakeholder uses report $r$ as an information source for her decision-making. $L_G$ and $L_B$ measure the value of this information to her.

The manager, however, does not always share the stakeholder’s interest. His payoff, denoted as $u(\omega, d)$, is always higher when the stakeholder chooses $d = 1$, which is equivalent to assuming

$$\delta_\omega \equiv u(\omega, 1) - u(\omega, 0) > 0, \text{ for } \omega \in \{G, B\}. \quad (\text{Assumption 2})$$

$\delta_\omega$ indexes the manager’s preference for $d = 1$ in state $\omega$.\footnote{The alternative assumption that the manager prefers $d = 0$ to $d = 1$ in both states does not change the main results qualitatively. What is necessary is the existence, not the direction, of the misalignment of interest.} This conflict of interest mo-
tivates the manager to engage in evidence management, as will be described in the next subsection. We assume that nothing is contractible in this setting so as to focus on the role of financial reporting in a decision-making setting.

Examples that fit this generic setting are common. The stakeholder’s decision \( d \) can be whether to finance a firm’s project \((d = 1)\) or not \((d = 0)\). \( L_G \) is the NPV of a viable project and \( L_B \) the absolute value of the negative NPV of an unprofitable project. For another example, \( d \) could be whether to foreclose a loan or not. \( L_G \) captures the cost of foreclosing a good loan and \( L_B \) of continuing a bad one. Yet another example of the binary decision can be whether to replace a manager or not. \( L_G \) is the cost of removing an able manager and \( L_B \) of keeping an incompetent one. While in all three examples information about the underlying state is useful for the stakeholder, the manager prefers financing the project, continuing the loan, and keeping the job, regardless of the underlying state. Thus, Assumption 1 and 2 are descriptive of these settings.

### 2.2 The supply of information through accounting recognition

We turn to the second part of the model and describe the accounting recognition process that relates report \( r \) to state \( \omega \). Had the standard setter been able to condition the recognition standard directly on \( \omega \), the ideal standard would simply be \( r(G) = g \) and \( r(B) = b \). That is, the state is recognized as it is. Unfortunately, \( \omega \) is not observable and only some evidence about \( \omega \) may exist.

Facing this constraint, the recognition process is decomposed to two steps: one from the state to the evidence and the other from the evidence to the report. The model’s key friction lies in the first step: evidence \( t \) depends on both the state and the manager’s evidence management. In the absence of the latter, evidence \( t \) is drawn from a differentiable density distribution function \( f^\omega(t) > 0 \) over \([t, \tilde{t}]\) with corresponding cumulative density function \( F^\omega(t) \). \( f^\omega(t) \) satisfies the strict monotone likelihood ratio property (MLRP) and thus a higher \( t \) is good news about the state (Milgrom (1981)).

However, at date 1 before knowing \( \omega \) the manager can take an unobservable action \( \beta \) to "improve" the distribution of evidence from \( f^\omega(t) \) to \( f^\omega(t; \beta) \) without changing \( \omega \). \( \beta \) captures all activities the manager engages in to influence evidence \( t \) and is broadly referred
to as evidence management. We parameterize \( \tilde{f}^\omega(t; \beta) \) as

\[
\tilde{f}^\omega(t; \beta) = \begin{cases} 
    f^G(t) & \text{if } \omega = G \\
    \beta f^G(t) + (1 - \beta) f^B(t) & \text{if } \omega = B, \beta \in [0, 1].
\end{cases}
\]  

(1)

It can be verified that for any \( \beta_1 > \beta_2 \), \( \tilde{F}^B(t; \beta_1) \) is (weakly) increasing in \( t \) and \( \tilde{F}^B(t; \beta_1) \) is better than \( \tilde{F}^B(t; \beta_2) \) in the sense of FOSD (first-order stochastic dominance). Thus, we say evidence management \( \beta \) improves the evidence distribution (without changing the economic substance). Note that \( \beta \) does not affect the evidence of the good state (i.e., \( \tilde{F}^G(t; \beta) = f^G(t) \)).

As it will become clear, this assumption stacks us against finding a non-monotonic relation between the optimal threshold and evidence management. A more general technology of evidence management will be studied in Section 4.3.

\( \beta \) costs the manager \( cC(\beta) \) privately. \( C(\beta) \) has the standard properties of a cost function. It is increasing and convex with \( C(0) = C'(0) = 0 \) and \( cC'(1) \) is sufficiently large to assure an interior \( \beta \) in equilibrium. Further, \( \frac{d}{d\beta} \left[ \frac{C'}{C''} \right] = \frac{(C'')^2 - C' C'''}{(C')^2} > 0 \), which sets a bound on the speed at which \( C'' \) increases.\(^2\) For example, the standard quadratic cost function \( cC(\beta) = \frac{\beta^2}{2} \) with \( c \) properly restricted satisfies these assumptions. \( c \) is a cost parameter we will use later for comparative statics.

We have assumed that the manager chooses \( \beta \) without knowing \( \omega \) or initial evidence, making it a model of signal-jamming (e.g., Holmstrom (1999) and Stein (1989)) rather than signaling (e.g., Spence (1973)). Of course, evidence management in practice can occur in all stages of a transaction, both before and after the transaction’s economic substance becomes clear to the manager. The \textit{ex post} evidence management is examined in Section 4.4.

The second step of the recognition process from the evidence to the report is the domain of standard design. A recognition standard is characterized by a threshold \( T \) such that

\[
r = g \text{ if and only if } t \geq T.
\]  

(2)

That is, a preferred treatment \( r = g \) is granted if and only if the evidence exceeds threshold

\(^2\)This assumption on the cost function eliminates the "boil them in oil" results, in which evidence management is prevented entirely with sufficiently large punishment for even small amount of evidence management. See Shleifer and Wolfenzon (2002) for more discussion of this assumption.
Finally, the standard setter chooses threshold $T$ to maximize the stakeholder’s expected payoff at date 0:

$$W(T) = E_{\omega}[v(\omega, d)].$$

This objective function is consistent with FASB’s proclamation that the objective of financial reporting is to provide information useful for stakeholders’ decision-making (FASB (2010), para OB2).$^3$ The manager’s expected payoff from the decision, i.e., $E_{\omega,d}[u(\omega, d)]$, is assumed to be a transfer from the stakeholder and thus does not enter the ex ante firm value separately. Otherwise the standard setter would have to worry about the stakeholder’s potential opportunistic behavior against the manager as well. Moreover, we do not include in $W(T)$ the direct cost of evidence management in order to focus on the real decision-making consequence of threshold design. An alternative objective function of $E_{\omega,d}[v(\omega, d)] - \lambda cC(\beta)$ with any $\lambda \geq 0$ does not affect the qualitative trade-off for threshold design in the model.

We call $W(T)$ the ex ante firm value. Ex ante and ex post refer to the observation of evidence at date 2. Denote the ex ante firm value in equilibrium as $W(T^*)$. $W(T^*)$ can be expressed as a function of the model’s parameters after the equilibrium is solved. We make one last assumption:

$$W(T^*) > \max \{ \sum_{\omega \in \{G,B\}} q_\omega v(\omega, 1), \sum_{\omega \in \{G,B\}} q_\omega v(\omega, 0) \}. \quad (Assumption \ 3)$$

Assumption 3 states that it is dominated for the stakeholder to take a constant decision that is not sensitive to the report in equilibrium.$^4$ In other words, the assumption focuses us on the informative equilibrium in which the stakeholder’s decision responds to the report. If this assumption is violated, the equilibrium report is too noisy to influence the stakeholder’s decision and threshold design becomes irrelevant.

$^3$To focus on the optimal design of accounting standards, both the political economy issues of standards setting and the implementation of standards are assumed away. Interested readers are referred to, for example, Bertomeu and Magee (2011) and Bertomeu and Cheynel (2013) for the former. The standard’s implementation is assumed to be administered frictionlessly by a neutral third party, such as the firm’s accountant.

$^4$The stakeholder receives $\sum_{\omega \in \{G,B\}} q_\omega v(\omega, 1)$ with a constant decision of $d = 1$ and $\sum_{\omega \in \{G,B\}} q_\omega v(\omega, 0)$ with a constant decision of $d = 0$. Hence the right hand side.
In sum, at date 0 the standard setter chooses threshold $T$ to maximize $W(T)$, at date 1 after observing $T$ the manager chooses $\beta(T)$ to maximize $E_{\omega,r}[u(\omega, d(r))] - cC(\beta)$, and at date 3 after observing $T$ and $r$ the stakeholder chooses $d(r)$ to maximize $E_{\omega}[v(\omega, d)]$ with respect to her belief about unobservable $\beta$. The solution concept is Perfect Bayesian Equilibrium. An (PBE) equilibrium consists of a triplet of decisions $(T, \beta, d)$ that maximize their respective objective functions and the stakeholder’s belief about $\beta$ that is consistent with the manager’s equilibrium choice.

To make this financial reporting process concrete, consider a stylized example of revenue recognition. The economic substance $\omega$ is whether revenue from a transaction has been earned (and realizable) or not. Revenue recognition is a preferred treatment for the manager ($r = g$). Payment collectability and product delivery are transaction characteristics represented by evidence $t$ (after proper normalization). Revenue is recognized ($r = g$) if and only if product is delivered and payment is collectible ($t \geq T$). Relative to this standard $T$, one requiring that product be delivered and cash be received implies a higher threshold, while one requiring only product delivery implies a lower threshold. The manager can present favorable evidence of product delivery by either fabricating warehouse records (accounting manipulation) or adopting tactics of channel stuffing (real activity manipulation).

3 Equilibrium analysis

3.1 The benchmark of a simple hypothesis test

Apart from evidence management, the model has been set up deliberately to resemble the test of a simple hypothesis. In this subsection we briefly discuss this statistical benchmark, against which the optimal threshold with evidence management will be contrasted later.

We rephrase the threshold design problem in statistical language. The state $\omega \in \{G, B\}$ is unknown. Set $H_0 : \omega = G$ as the null hypothesis and $H_1 : \omega = B$ as the alternate. Data $t$ is generated according to $f^\omega(t)$ (because of the absence of evidence management). The standard setter’s problem is to find a test procedure that generate a report $r \in \{g, b\}$ for the stakeholder. A test procedure is characterized by a critical region $H$ such that $r = b$ if and only if $t \in H$. MLRP of the evidence distributions implies that the critical region
$H$ is an interval with the right end-point $T$, i.e., $H = [t, T)$. Equivalently, the procedure is characterized by threshold $T$ such that $r = b$ if and only if $t < T$.

For any threshold $T$, report $r$ is associated with both Type I and II errors. Type I is the error of false alarm that reports the good state as bad and Type II the error of undue optimism that reports the bad state as good. They can be written out as

$$
\varepsilon^G(T) \equiv \Pr(b|G; T) = F^G(T), \quad \varepsilon^B(T) \equiv \Pr(g|B; T) = 1 - F^B(T).
$$

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$$

Each recognition error $\varepsilon^\omega$ costs the stakeholder $q_\omega L_\omega$ (ex ante) and the standard setter chooses $T$ to minimize the stakeholder’s total costs. In this setting, the choice of threshold $T$ has been well-studied in statistics and summarized in the following lemma.

**Lemma 1** In the absence of evidence management the optimal threshold $T^{BM}$ is determined by balancing the two recognition errors weighted by their respective decision-making consequences. That is,

$$
-\frac{d \varepsilon^G(T)/dT}{q_\omega L_\omega} |_{t = T^{BM}} = \frac{q_B L_B}{q_G L_G}.
$$

Lemma 1 can be viewed as a Bayesian interpretation of the celebrated Neyman-Pearson Lemma (Neyman and Pearson (1933)). It captures the threshold’s statistical effect on recognition errors. A higher threshold makes it more difficult to be recognized as good, which implies a higher probability of false alarms ($\frac{d \varepsilon^G(T)}{dT} = f^G > 0$) and a lower probability of undue optimism ($\frac{d \varepsilon^B(T)}{dT} = -f^B < 0$). Each error occurs with probability $q_\omega$ and is associated with a decision-making cost $L_\omega$. Thus, the optimal threshold is determined when the marginal benefit of reducing the undue optimism error is equal to the marginal cost of raising the false alarm error.

### 3.2 The manager’s optimal evidence management

Now we analyze the model with evidence management by backward induction. We start with the stakeholder’s decision at date 3 upon receiving report $r$. The stakeholder is not misled by the manager’s evidence management and updates her belief about the state from $\Pr(\omega)$ to $\Pr(\omega|r; \beta^*)$. The difference of her expected payoffs if the stakeholder chooses $d = 1$ versus
\[d = 0\] is

\[\Delta(r) = \sum_{\omega \in \{G,B\}} \Pr(\omega \mid r; \beta^*)(v(\omega, 1) - v(\omega, 0)) = \Pr(G \mid r; \beta^*)(L_G + L_B) - L_B.\]

The stakeholder chooses \(d = 1\) if and only if \(\Pr(G \mid r; \beta^*)\) is sufficiently high. By definition \(\Pr(G \mid g; \beta^*) \geq \Pr(G \mid b; \beta^*)\) for any \(\beta^*\). Assumption 3 implies that \(\Pr(G \mid g; \beta^*) > \frac{L_B}{(L_G + L_B)} > \Pr(G \mid b; \beta^*)\). Thus, the stakeholder’s optimal decision rule is to choose \(d = 1\) if and only if \(r = g\) is observed. That is,

\[d^*(g) = 1 \text{ and } d^*(b) = 0. \tag{4}\]

At date 2 evidence \(t\) is classified to report \(r\) according to threshold \(T\).

At date 1 the manager observes threshold \(T\) and anticipates the stakeholder’s decision rule \(d^*(r)\) at date 3. Then he chooses \(\beta\) to maximize his expected payoff:

\[E_{\omega,r}[u(\omega, d^*(r))] = \sum_{\omega \in \{G,B\}, r \in \{g,b\}} \Pr(\omega) \Pr(r \mid \omega; \beta)u(\omega, d^*(r)).\]

\(\Pr(r \mid \omega; \beta)\) is the state-contingent report distribution when the manager chooses \(\beta\). It can be expressed in terms of the report’s recognition errors. The report’s false alarm error and undue optimism error are

\[\varepsilon^G(T; \beta) \equiv \Pr(b \mid G; \beta) = \tilde{F}^G(T; \beta) = F^G(T), \tag{5}\]

\[\varepsilon^B(T; \beta) \equiv \Pr(g \mid B; \beta) = 1 - \tilde{F}^B(T; \beta) = 1 - F^B(T) + \beta(F^B(T) - F^G(T)). \tag{6}\]

Evidence management increases the manager’s chance of receiving the preferred treatment \(g\) by increasing the undue optimism error \((\frac{\partial \varepsilon^B(T; \beta)}{\partial \beta} = F^B(T) - F^G(T) > 0)\). Using the definitions of \(\varepsilon^G(T; \beta)\) and \(\delta_\omega\), we express the manager’s expected payoff as

\[q_Gu(G, 1) + q_Bu(B, 0) - q_G\varepsilon^G(T)\delta_G + q_B\varepsilon^B(T; \beta)\delta_B. \tag{7}\]

The manager’s expected payoff is \(q_Gu(G, 1) + q_Bu(B, 0)\) when the stakeholder has access to a perfect report. It decreases in the false alarm error but increases in the undue optimism.
error. Balancing this expected payoff against the cost of \( cC(\beta) \), the manager’s optimal evidence management \( \beta^*(T) \) is determined by the following first-order condition (the second-order condition is satisfied by the convexity of \( C(\beta) \)):

\[
q_B[F^B(T) - F^G(T)]\delta_B = cC'(\beta^*(T)).
\] (8)

The left hand side of equation 8 is the marginal benefit of evidence management. It is the product of two components: the marginal probability of receiving \( g \), that is, \( q_B[F^B(T) - F^G(T)] \), and the benefit of receiving \( g \), that is, \( \delta_B \).

**Lemma 2 Evidence management \( \beta^*(T) \)**

1. increases in \( \frac{\delta_B}{C'} \);

2. is not monotonic in \( T \). In particular, \( \frac{\partial \beta^*(T)}{\partial T} > 0 \) for \( T < \hat{T} \), \( \frac{\partial \beta^*(T)}{\partial T} = 0 \) at \( T = \hat{T} \), and \( \frac{\partial \beta^*(T)}{\partial T} < 0 \) for \( T > \hat{T} \). \( \hat{T} \) is the unique solution to \( f^G(T) = f^B(T) \).

The first property of \( \beta^*(T) \) is intuitive. Costly evidence management improves the manager’s chance of receiving \( g \). Thus, the manager engages in more evidence management when the benefit of receiving \( g \) is larger or the cost of evidence management is lower.

The second property might be less intuitive, but it plays an important role later in the characterization of the optimal threshold. It states that evidence management is not monotonic in the threshold. A higher threshold initially increases but eventually reduces evidence management. This result is obtained by differentiating equation 8 with respect to \( T \):

\[
\frac{\partial \beta^*(T)}{\partial T} = \frac{q_B\delta_B}{cC''} (f^B(T) - f^G(T))
\] (9)

Since \( \frac{f^C(T)}{f^B(T)} \) increases strictly in \( T \) for any \( T \in [\ell, \bar{\ell}] \), it can be shown that \( \frac{f^C(T)}{f^B(T)} \) crosses 1 from below once and only once at \( \hat{T} \). This implies that \( \frac{\partial \beta^*(T)}{\partial T} \) is positive when \( T < \hat{T} \), equal to 0 at \( T = \hat{T} \), and negative when \( T > \hat{T} \).

---

5Strict MLRP implies that \( \frac{f^C(T)}{f^B(T)} \) crosses 1 at most once. Suppose \( f^G(T) < f^B(T) \) for any \( T \in [\ell, \bar{\ell}] \). Integrating both sides for \( T \) over \( [\ell, \bar{\ell}] \), we obtain \( \int_{\ell}^{\bar{\ell}} f^G(T) dT < \int_{\ell}^{\bar{\ell}} f^B(T) dT \), which is a contradiction because both sides are equal to 1 by the definition of a probability density function. Thus, \( \frac{f^C(T)}{f^B(T)} \) cannot always stay below 1. Similarly we can show that \( \frac{f^C(T)}{f^B(T)} \) cannot always stay above 1. Therefore, \( \frac{f^C(T)}{f^B(T)} \) crosses 1 from below once and only once at \( \hat{T} \).
To gain intuition about this monotonicity, we examine the firm’s evidence management more closely. Given threshold $T$, the benefit of a unit of evidence management $\beta$ is to replace evidence distribution $f^B(t)$ with $f^G(t)$ in the bad state. Equivalently, the probability of receiving $g$ in the bad state, $1 - F^B(T)$, is replaced by a higher probability of $1 - F^G(T)$. Raising $T$ has two effects on these probabilities. First, a higher $T$ results in a lower probability of receiving $g$ with the manipulated distribution of evidence ($f^G(t)$). That is, it reduces the probability $1 - F^G(T)$ at the rate of $f^G$. This first effect discourages evidence management.

Second, a higher $T$ also results in a lower probability of receiving $g$ with the non-manipulated distribution of evidence ($f^B(t)$). That is, it reduces the probability $1 - F^B(T)$ at the rate of $f^B$. This second effect encourages evidence management as it raises the "opportunity cost" of not engaging in evidence management. The net effect is thus determined by the difference of the rates of these two effects. Raising the threshold reduces evidence management if and only if $f^B(T) < f^G(T)$. MLRP further implies that $f^B(T) - f^G(T)$ is positive first, 0 at $\hat{T}$, and negative afterwards.

This non-monotonicity also seems fairly general. To see it, we can examine two extremes of setting a threshold. On one hand, if the threshold is set sufficiently high, costly evidence management does not help to obtain $g$ and thus does not occur. On the other hand, if the threshold is set sufficiently low, $g$ is obtained without costly evidence management and thus no evidence management occurs either. Thus, whenever evidence management exists, it is not monotonic in $T$.

This kind of non-monotonicity has been observed in other areas. Gibbs (1996) notes that the effect of a promotion standard on an agent’s incentive to work harder shares the similar feature of non-monotonicity. Neither too high nor too low is a standard of promotion motivating. Similarly, it is understood well that the motivational effect of a budget is not monotonic in the level of difficulty to reach it.

### 3.3 The optimal threshold

At date 0 the standard setter chooses threshold $T$ to maximize $W(T)$, anticipating both the manager and the stakeholder’s decision rules. Using the definition of $\omega(T; \beta)$ and $L_\omega, W(T)$
can be written as

\[
W(T) = \sum_{\omega \in \{G,B\}, r \in \{g,b\}} \Pr(\omega) \Pr(r|\omega; \beta^*(T)) v(\omega, d^*(r)) \\
= q_G v(G, 1) + q_B v(B, 0) - q_G L_G \varepsilon^G(T) - q_B L_B \varepsilon^B(T; \beta^*(T))
\]

(10)

\(W^{FB} \equiv q_G v(G, 1) + q_B v(B, 0)\) is the first-best firm value when the stakeholder has access to a perfect report. The actual firm value is short of the first-best because the decision is distorted by both the false alarm error, which costs \(q_G L_G \varepsilon^G(T)\), and the undue optimism error, which costs \(q_B L_B \varepsilon^B(T; \beta^*(T))\). Had the standard setter been able to choose recognition errors directly, she would set both \(\varepsilon^G\) and \(\varepsilon^B\) to be 0, thereby recognizing the transaction’s economic substance “faithfully.” However, the standard setter, unfortunately, cannot choose \(\varepsilon^G\) and \(\varepsilon^B\) independently. Instead, she chooses threshold \(T\) to influence recognition errors indirectly.

Her threshold choice affects recognition errors through two different channels. The first is an \textit{ex ante} strategic effect: threshold \(T\) affects evidence management \(\beta\) that in turn affects the distribution of evidence \(t\) and the recognition errors. The effect of threshold \(T\) on evidence management \(\beta\) is \(\frac{\partial \beta^*(T)}{\partial T}\), as captured in Lemma 2. The effect of evidence management on recognition errors is \(\frac{\partial \varepsilon^B(T; \beta)}{\partial \beta} \big|_{\beta = \beta^*} > 0\). Thus, the threshold’s strategic effect is captured by \(\frac{\partial \varepsilon^B(T; \beta)}{\partial \beta} \frac{\partial \beta^*(T)}{\partial T} \big|_{\beta = \beta^*}\), whose sign is the same as that of \(\frac{\partial \beta^*(T)}{\partial T}\). This strategic effect relies crucially on the fact that the standard setter’s choice of threshold \(T\) not only precedes the manager’s choice of \(\beta\) but also is observed by the manager.

In addition to this strategic effect, the threshold also has an \textit{ex post} statistical effect: for any evidence \(t\) at date 2 (\textit{ex post}), a higher threshold \(T\) reduces the undue optimism error at the expense of increasing the false alarm error. That is, \(\frac{\partial \varepsilon^B(T; \beta^*)}{\partial T} = -(\beta^* f^G + (1 - \beta^*) f^B) < 0\) and \(\frac{\partial \varepsilon^G(T)}{\partial T} = f^G > 0\). This statistical effect is similar to that in the statistical benchmark with the only difference that evidence \(t\) is now understood to be generated from \(\tilde{f}^w(t; \beta^*)\), rather than \(f^w(t)\).

To determine the optimal threshold \(T^*\), we differentiate the firm value \(W(T)\) of equation
The first-order condition could be expressed as

$$-q_G L_G \frac{\partial z^G(T)}{\partial T} - q_B L_B \frac{\partial z^B(T; \beta^*(T))}{\partial T} - q_B L_B \frac{\partial z^B(T; \beta) \partial \beta^*(T)}{\partial \beta} \bigg|_{\beta = \beta^*} = 0.$$  \hspace{1cm} (11)

The optimal threshold $T^*$ is determined by balancing the threshold’s dual effects on recognition errors weighted by their respective costs.\textsuperscript{6} The first two terms in equation 11 capture the threshold’s marginal effect on the firm value via the statistical effect. A higher threshold affects recognition errors by $\frac{\partial z^e(T; \beta^*)}{\partial T}$ and recognition errors reduce the ex ante firm value by $q_\omega L_\omega$. The third term in equation 11 describes the threshold’s marginal effect on the firm value via the strategic effect. The threshold influences the manager’s evidence management decision by $\frac{\partial \beta^*(T)}{\partial T}$, evidence management increases the undue optimism error by $\frac{\partial z^B(T; \beta)}{\partial \beta} \bigg|_{\beta = \beta^*}$, and the undue optimism error reduces the firm value by $q_B L_B$.

The model’s equilibrium is thus characterized by $T^*$ satisfying equation 11, $\beta^*(T)$ determined by equation 8, and the $d^*(r)$ described in equation 4. $W(T^*)$ and Assumption 3 now can be expressed in terms of the model’s parameters.

### 3.4 The properties of the optimal threshold

With the equilibrium fully characterized, we are ready to answer the main research question about the optimal level of $P$ discussed in the Introduction. We first formally define $P$ and relate it to threshold $T$. Consider an accountant who has access to evidence $t$ and understands the manager’s choice $\beta^*$. Her belief of $\omega$ being good is $p(t; \beta^*) \equiv \Pr(G|t; \beta^*)$. Since $p(t; \beta^*)$ is monotonically increasing in $t$ for any given $\beta^*$, the recognition standard $T$ defined in expression 2 can also be expressed by a constant $P$ such that

$$r(t) = g \text{ if and only if } p(t; \beta^*) \geq P.$$  

That is, the preferred treatment is granted if the probability that the state is good, conditional on evidence and logic, exceeds $P$. $T$ and $P$ are thus two representations of thresholds.

\textsuperscript{6}With the general distribution functions, the firm value $W$ is not necessarily concave in $T$. As a result, the first-order condition is necessary but may not be sufficient for $T^*$ to be the maximizer. However, all subsequent results rely only on the first-order condition being necessary.
To avoid confusion, we call $P$ the probability threshold and $T$ the evidence threshold.

The main advantage of working with $P$ instead of $T$ is that $P$ is comparable across different standards. $T$ represents transaction characteristics that are multi-dimensional and vary across different types of transactions. For example, transaction characteristics used in a revenue recognition standard are very different from those employed in a contingency standard. Thus, it is difficult to compare the evidence thresholds across the two standards. In contrast, $P$ is standardized as a probability and thus comparable across different standards. For example, if revenue recognition requires that the probability of revenue being earned exceeds 80% and contingent liability recognition requires that the probability of future loss is 50%, then we might say that the former has a higher probability threshold.

A side benefit of the probability threshold representation is that it makes clear that recognition thresholds exist even in the absence of bright-line cutoffs. Even a purely principle-based recognition standard that does not contain evidence cutoffs still needs to specify a probability threshold. For example, the legal burden of proof is "preponderance of evidence" in most civil cases but "beyond reasonable doubt" in most criminal cases. While neither might be associated with bright-line cutoffs, one probably would agree that they imply two different probability thresholds.

Now we compare the probability thresholds with and without evidence management. They can be calculated by using the Bayes rule, Lemma 1, and the first-order condition of equation 11, respectively.

$$P_{BM} \equiv p(T_{BM}) = \frac{q_G f^G(T_{BM})}{q_G f^G(T_{BM}) + q_B f^B(T_{BM})} = \frac{L_B}{L_B + L_G}.$$

$$P^* \equiv p(T^*; \beta^*) = \frac{q_G f^G(T^*; \beta^*)}{q_G f^G(T^*; \beta^*) + q_B f^B(T^*; \beta^*)} = \frac{L_B}{L_B + L_G + I(T^*)},$$

with

$$I(T^*) = \left. q_B L_B \frac{\partial \beta^B(T; \beta)}{\partial \beta} \frac{\partial \beta^*(T)}{\partial T} \right|_{T=T^*}.$$

The optimal probability threshold in the benchmark $P_{BM}$ is determined by and only by the recognition errors’ relative costs to the stakeholder. The transaction’s details, such
as $f^c(t)$, are irrelevant (of course they still affect the evidence threshold). In contrast, the optimal probability threshold with evidence management $P^*$ has an additional determinant $I(T^*)$ that is a function of the transaction’s details.

**Proposition 1** $P^* > P^{BM}$ for $T^* > \hat{T}$, $P^* = P^{BM}$ for $T^* = \hat{T}$, and $P^* < P^{BM}$ for $T^* < \hat{T}$. $\hat{T}$ is the unique solution to $f^G(T) = f^R(T)$.

Proposition 1 is the paper’s main result. It states that the presence of evidence management does not necessarily result in a higher probability threshold, even though in the model the manager always prefers a good report and evidence management always inflates evidence. Its proof is immediate from inspecting the expression of $I(T^*)$. Since $\frac{q_B L_B \delta_B (T, \beta)}{q_G F^G(T)} |_{T = T^*} > 0$, the sign of $I^*(T)$ is determined solely by $\frac{\partial P^*(T)}{\partial T} |_{T = T^*}$, which is given in Lemma 2.

Consequently, the main intuition for Proposition 1 is derived from the intuition for Lemma 2. Recall that in Lemma 2 raising the threshold has a non-monotonic effect on evidence management. Since the threshold deviates from the statistical benchmark in order to discourage evidence management, the direction of the deviation, or the sign of $P^* - P^{BM}$, depends on the direction of the threshold’s effect on evidence management. When $T^* > \hat{T}$, evidence management is decreasing in the threshold and thus the optimal probability threshold $P^*$ is raised above the statistical benchmark $P^{BM}$. Similarly, when $T^* < \hat{T}$, evidence management is increasing in the threshold and the optimal threshold designed to discourage evidence management is lowered.

The exact location of $T^*$ is determined implicitly by equation 11 and its closed-form solutions are difficult to obtain. To illustrate Proposition 1, we present two examples, one with $P^* > P^{BM}$ and the other $P^* < P^{BM}$. Both use the following set of parameters: $L_G = L_B = 1$, $\delta_B = 1.5$, $C(\beta) = \frac{\delta^2}{2}$, $c = 1$, $F^G(t) = t$ for $t \in [0, 1]$.

**Example 1** $F^R(t) = t^{\frac{1}{10}}$ for $t \in [0, 1]$. $q_G = 0.25$. Then $\hat{T} = 0.08$, $T^* = 0.91$, and $P^* = 60\%$.

**Example 2** $F^R(t) = t^{\frac{4}{10}}$ for $t \in [0, 1]$. $q_G = 0.6$. Then $\hat{T} = 0.18$, $T^* = 0.02$, and $P^* = 33\%$.

In both examples, $P^{BM} = 50\%$ because $L_B = L_G$. Thus, in the absence of evidence management, $r = g$ is recognized as long as the probability of the state being good exceeds
50%, regardless of other aspects of the transaction. In Example 1, even though \( L_B = L_G \), the optimal probability threshold is 60%, higher than 50%. \( r = g \) is not recognized if the probability of the state being good is higher than 50% but lower than 60%. Thus, the response to inflationary evidence management is to raise the probability threshold, that is, to make it more "conservative." For example, the probability threshold of revenue recognition is often higher than 50%. While the excess might be caused by \( L_B > L_G \), Proposition 1 provides an alternative explanation.

In contrast, in Example 2, the optimal probability is 33%, even though \( L_B = L_G \). The reason is that \( T^* < \hat{T} \). Raising the threshold in this region increases evidence management. Thus, the optimal response to inflationary evidence management is to lower the probability threshold, that is, to make it more "aggressive." For example, the probability threshold for recognizing contingent obligations is usually interpreted as 75% (probable) in U.S. GAAP. This means that the probability threshold for the preferred treatment (no recognition of liability) is 25%. While it might be explained as \( L_B < L_G \), Proposition 1 provides an alternative explanation.

\( P^* > P^{BM} \) means that a higher likelihood of the good state is required for granting a preferred treatment and thus might be interpreted as conservatism. Proposition 1 implies that the optimal threshold response to inflationary evidence management is not necessarily conservative. Therefore, it highlights the different ways the instruments in the standard setter’s toolkit work. Gao (2013) shows that as far as the verification requirement is concerned, the optimal response to inflationary evidence management is unambiguously conservative. That is, favorable evidence is subject to more verification. Verification is costly but reduces recognition errors directly. Raising thresholds is free but affects recognition errors indirectly through the statistical and strategic effects. The interaction between the two instruments and their optimal combination are an interesting topic for future research.

Proposition 1 compares the \emph{ex ante} optimal threshold with a statistical benchmark in the absence of evidence management. Now we study another benchmark in which the standard setter chooses an \emph{ex post} efficient threshold, that is, chooses a threshold to maximize the firm value "on the basis of evidence and logic." This is consistent with a typical \emph{ex post} notion of providing maximum information to the stakeholder. Specifically, at date 2, the standard
setter observes evidence $t$ and conjectures correctly that the manager has chosen $\tilde{\beta}$ at date 1. While $\tilde{\beta}$ is endogenously chosen by the manager at date 1, it is a parameter of the evidence generating process at date 2. Conditional on the transaction being type $\omega$, the likelihood of observing $t$ is given by the density $f^\omega(t; \tilde{\beta})$. Thus, a threshold $T$ would result in recognition errors of $\varepsilon^G(T)$ and $\varepsilon^B(T; \tilde{\beta})$ and the firm value of $W^{FB} - q_G L_G \varepsilon^G(T) - q_B L_B \varepsilon^B(T; \tilde{\beta})$. The standard setter who maximizes the firm value at date 2 chooses the \textit{ex post} efficient threshold, denoted as $\tilde{T}$, according to the following first-order condition:

$$q_G L_G \frac{\partial \varepsilon^G(T)}{\partial T} + q_B L_B \frac{\partial \varepsilon^B(T; \tilde{\beta})}{\partial T} = 0. \quad (13)$$

The \textit{ex post} efficient probability threshold $\tilde{P} \equiv p(\tilde{T}; \tilde{\beta})$ can be derived from the Bayes rule and the first-order condition of equation 13.

**Proposition 2**

1. $\tilde{P} = \frac{L_B}{L_B + L_G} = P^{BM}$;

2. When the threshold is set \textit{ex post}, the firm value is lower and evidence management is higher. That is, $W(\tilde{T}) < W(T^*)$ and $\tilde{\beta} > \beta^*$.

The probability threshold implied by $\tilde{T}$ is the same as that in the statistical benchmark without evidence management. Thus, the comparison of $\tilde{P}$ and $P^*$ is straightforward from Proposition 1. Even though the standard setter has rational expectations about evidence management at date 2, she treats evidence management as given when setting the threshold. Thus, the strategic effect is absent and the statistical effect alone determines the optimal threshold. The rational expectations of evidence management modify the evidence generating process from $f^\omega(t)$ to $\tilde{f}^\omega(t; \tilde{\beta})$, but do not affect the conceptual determination of the optimal probability threshold. Therefore, what characterizes the strategic approach to threshold design is the understanding that the threshold affects evidence management, not the rational expectations of evidence management per se.

Part 2 of Proposition 1 summarizes the value of the standard setter’s ability to influence evidence management. Since recognition errors are costly to the stakeholder’s decision-making in the model, the standard setter who chooses the threshold at date 0 considers both the strategic and statistical effects of her choice. For the purpose of the stakeholder’s decision-
making at date 2, $\tilde{T}$ is the \textit{ex post} efficient threshold. For the purpose of curtailing evidence management at date 1, $T = \tilde{t}$ or $T = \tilde{t}$ completely eliminates evidence management. The \textit{ex ante} optimal threshold $T^*$ balances these two effects and thus differs from either benchmark. By moving away from $\tilde{T}$, $T^*$ reduces the \textit{ex post} efficiency of the stakeholder’s decision but at the same time provides better control of the manager’s evidence management, which in turn improves the \textit{ex ante} efficiency of the stakeholder’s decision. It is this commitment to \textit{ex post} inefficient use of information that discourages evidence management, which in turn improves the \textit{ex ante} informativeness of the report and the efficiency of the stakeholder’s decision. In contrast, when the threshold is chosen \textit{ex post}, the threshold’s strategic effect is absent and the standard setter loses one instrument to influence evidence management.

Finally, we conduct comparative statistics.

\textbf{Proposition 3}

1. The \textit{ex ante} firm value ($W^*$) is decreasing in $\frac{\delta_R}{c}$.

2. The optimal evidence threshold ($T^*$) is not monotonic in $\frac{\delta_R}{c}$.

Part 1 of Proposition 3 is intuitive. The manager’s net incentive of evidence management (\textit{i.e.}, $\frac{\delta_R}{c}$) is unambiguously detrimental to the firm value because evidence management reduces the report’s equilibrium informativeness and distorts the stakeholder’s decision. Part 2 of Proposition 3 is also straightforward by now. An increase in $\frac{\delta_R}{c}$ induces more evidence management. Since evidence management decreases in the threshold if and only if $T > \hat{T}$, the optimal evidence threshold increases in $\frac{\delta_R}{c}$ in this region to offset the increase in evidence management. Thus, the non-monotonicity of the strategic effect lies behind the intuition.

\section{Extensions}

This section considers a number of extensions to the baseline model. In the baseline model, a threshold affects recognition errors through both the classification of given evidence (the \textit{ex post} statistical effect) and the influence on evidence management that affects the distribution of evidence (the \textit{ex ante} strategic effect). These dual effects work differently, creating the
qualitative trade-off for the optimal threshold. In this section we check the robustness of the presence of the dual effects to various assumptions. We pay particular attention to the robustness of the non-monotonicity of the strategic effect because it has played an important role in the main results.

4.1 Does disclosure of evidence dominate recognition?

It is necessary to assume the use of recognition in order to study threshold design. We have justified recognition on empirical grounds that binary accounting treatment is prevalent in accounting standards. Recognition always precedes measurement and is the first step to incorporate a transaction into financial statements. Thus, studying threshold design for a given recognition standard appears relevant to the understanding of financial reporting. On the other hand, it is well understood that the discreteness in recognition induces evidence management. Thus, one might be curious about what happens if the evidence is communicated to the stakeholder through disclosure, rather than through recognition as assumed in the baseline model. We show that compared with recognition, full disclosure of evidence in the model lowers the firm value exactly because it induces more evidence management.

**Proposition 4** Compared with the baseline model, the ex ante firm value is lower and evidence management is higher if the accounting standard requires full disclosure of evidence to the stakeholder.

The key proof strategy is to recognize that the stakeholder’s decision rule and the manager’s evidence management in a disclosure equilibrium are payoff equivalent to those in a recognition equilibrium with the threshold being set ex post. Disclosure of evidence $t$ makes the recognition report redundant because the latter is coarser. The stakeholder thus bases the decision on evidence $t$ to maximize the firm value. Since for any evidence management a higher $t$ still indicates a higher likelihood of the good state, the stakeholder’s decision rule takes a cut-off form $T_0$ such that $d^*(t \geq T_0) = 1$ and $d^*(t < T_0) = 0$. $T_0$ is the threshold that maximizes the firm value on the basis of evidence $t$ and the stakeholder’s understanding of the manager’s influence on $t$. Therefore, $T_0 = \tilde{T}$, the ex post efficient threshold defined in equation 13. Full disclosure of evidence leads to the same decision rule for the stakeholder as
in recognition with an \textit{ex post} efficient threshold, which in turn implies that the manager’s evidence management would be the same as $\tilde{\beta}$. Then Proposition 4 directly follows from Proposition 2. The discussion following Proposition 2 suggests that it is the commitment to \textit{ex post} inefficient use of information that discourages evidence management and improves \textit{ex ante} efficiency. Recognition, by coarsening the report, serves as such a commitment. In contrast, disclosure of evidence, by empowering the stakeholder, destroys the commitment. This broad economic force behind Proposition 4, that controlling a manager’s \textit{ex ante} incentive requires the inefficient use of information \textit{ex post}, is a recurring theme in the agency literature (e.g., Arya, Glover, and Sivaramakrishnan (1997), Arya, Glover, and Sunder (1998)).

4.2 Endogenous conflict between the manager and the stakeholder

Facing the manager’s opportunistic evidence management, two intertwined responses can arise. First, the standard setter can change the threshold. Second, the stakeholder can adjust her use of the report. Both responses influence evidence management and they interact with each other. By assuming exogenous $\delta_\omega$, we have effectively isolated the stakeholder’s response and focused exclusively on the threshold response. This differentiates our model from previous studies. For example, if the manager’s benefit from a preferred report ($\delta_\omega$) were fixed, the threshold design would not affect the aggregate evidence management in Dye (2002). In this subsection, we explicitly model the stakeholder’s decision in a capital market setting to provide a "micro-foundation" for $\delta_\omega$. The stakeholder’s response adds an indirect channel through which the threshold affects evidence management, but the direct channel in the baseline model is preserved.

The timeline is the same as before. The manager is interpreted as an entrepreneur with a project that requires investment $K$ at date 2. Without the investment the project is worthless. With the investment it pays out either $Y$ (success) or 0 (failure) at date 3. Its success probability is $\gamma_\omega$, which depends on the firm’s financial condition $\omega$ (the state of nature).

At date 2 after issuing report $r$ the entrepreneur sells the project to investors (the stakeholder) for life cycle reasons at an endogenously determined price $A(r)$. After the purchase investors decide whether to invest $K$ ($d = 1$) or not ($d = 0$). Their payoff related to this decision is $v(\omega, d) = (\gamma_\omega Y - K)d$ and we have $L_G \equiv v(G, 1) - v(G, 0) = \gamma_G Y - K > 0$, and
Denote $\beta^{**}$ as the equilibrium evidence management. Similar to Assumption 3, we assume that the equilibrium report is informative enough to influence the investment decision, that is, 
$$
\sum_{\omega \in \{G, B\}} \Pr(\omega | g; \beta^{**}) \gamma_\omega Y > K > \sum_{\omega \in \{G, B\}} \Pr(\omega | b; \beta^{**}) \gamma_\omega Y.
$$
Thus, the investors’ decision rule is $d^* (g) = 1$ and $d^* (b) = 0$.

$A(r)$ is a transfer between the entrepreneur and investors. The entrepreneur’s payoff is $u(\omega, d^* (r)) = A(r)$. As standard, we assume that investors face a competitive capital market, expect zero net surplus, and price the project accordingly. Then $A(r)$ can be determined as

$$
A(g) = \sum_{\omega \in \{G, B\}} \Pr(\omega | g; \beta^{**}) \gamma_\omega Y - K,
$$
$$
A(b) = 0.
$$

Anticipating $A(r)$, the entrepreneur prefers $r = g$ to $r = b$ and $\delta_B(T; \beta^{**}) \equiv A(g) - A(b) > 0$. Hence, Assumption 2.

The rest setup is the same as in the baseline model. This new setting’s key difference from the baseline model is that $\delta_B(T; \beta^{**})$ is an endogenous function of the threshold and evidence management. To see its impact, we first derive the first-order condition of the entrepreneur’s choice of evidence management at date 1:

$$
q_B(F^B(T) - F^G(T)) \delta_B(T; \beta^{**}) = cC'(\beta^{**}(T)). \tag{14}
$$

Differentiating it with respect to $T$, we have

$$
\underbrace{q_B(F^B - F^G) \delta_B}_{\text{direct channel}} + q_B(F^B - F^G) \left( \frac{\partial \delta_B(T; \beta^{**})}{\partial T} \right) + \frac{\partial \delta_B(T; \beta)}{\partial \beta} \frac{\partial \beta^{**}(T)}{\partial T} \bigg|_{\beta = \beta^{**}} = cC'' \frac{\partial \beta^{**}(T)}{\partial T}. \tag{15}
$$

The comparison between equation 9 and 15 reveals that endogenizing $\delta_B(T; \beta^{**})$ adds an indirect channel for the threshold to affect evidence management. In the baseline model, threshold $T$ affects only the probability of receiving a preferred treatment ($q_B(F^B(T) - F^G(T))$ in equation 14). An endogenous $\delta_B(T; \beta^{**})$ means that threshold $T$ now also affects the benefit of receiving a preferred treatment ($\delta_B(T; \beta^{**})$ in equation 14) when in-
vestors adjust the interpretation and use of the report in their pricing decision. This indirect channel consists of two components. First, $\frac{\delta B(T; \beta)}{\partial T} > 0.7$ Fixing evidence management a higher threshold increases a good report’s informativeness and thus its price. Second, $\frac{\delta B(T; \beta^*)}{\partial \beta} |_{\beta = \beta^*} < 0$. Fixing the threshold higher evidence management reduces a good report’s informativeness and thus its price. The total effect of the indirect channel is thus captured by $\frac{\delta B(T; \beta^*(T))}{\partial T} = \frac{\partial B(T; \beta^*)}{\partial T} + \frac{\partial \delta B(T; \beta^*)}{\partial \beta} \frac{\partial \beta^*(T)}{\partial T}$.

The net effect of the threshold on evidence management in equilibrium, $\frac{\partial \beta^*(T)}{\partial T}$, could be rewritten as

$$\frac{\partial \beta^*(T)}{\partial T} (cC'' - q_B(F^B - F^G) \frac{\partial \delta B(T; \beta)}{\partial \beta} |_{\beta = \beta^*}) = q_B \delta B(f^B(T) - f^G(T)) + \frac{\partial \delta B(T; \beta^*)}{\partial T} q_B(F^B - F^G)$$

While a closed-form solution of $\frac{\partial \beta^*(T)}{\partial T}$ requires specifying the evidence distributions, its sign can be discussed with the general distributions. First, $\frac{\partial \beta^*(T)}{\partial T} \neq 0$ in general. Second, the sign of $\frac{\partial \beta^*(T)}{\partial T}$ can be dominated by $q_B \delta B(f^B(T) - f^G(T))$ because $(cC'' - q_B(F^B - F^G) \frac{\partial \delta B(T; \beta)}{\partial \beta} |_{\beta = \beta^*}) > 0$ and $\frac{\partial \delta B(T; \beta^*)}{\partial T} > 0$. Therefore, the threshold’s strategic effect is still present and its non-monotonicity can be preserved.

Since $A(r)$ is a transfer, its endogenization does not affect the standard setter’s objective function $W(T)$. The new first-order condition for the ex ante optimal threshold $T^{**}$ has the same form as in equation 11 except that $\frac{\partial \beta^*(T)}{\partial T}$ is more complicated with the indirect channel discussed above. As a result, the qualitative characterization of the optimal threshold in the baseline model in Section 3.4 is intact.

### 4.3 Different technologies of evidence management

In the baseline model evidence management improves the distribution of evidence only in the bad state. As a result, evidence management is unambiguously detrimental to the stakeholder’s decision-making. Despite the desirability of reducing evidence management, the threshold’s optimal response is not monotone because of its non-monotone effect on evidence

\[7\] The proof is by straightforward derivatives and utilizes the result that MLRP implies monotone hazard ratio, that is, $\frac{f^B(T)}{1-f^B(T)} > \frac{f^G(T)}{1-f^G(T)}$. 

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management. In this subsection we consider a more general technology of evidence manage-
ment that improves the evidence distributions in both states. As a result, it can be optimal to
courage evidence management. This ambiguity of the desirability of evidence management
adds an orthogonal reason that the threshold’s optimal response to evidence management is
not monotone.

Specifically, we modify $\tilde{f}(t; \beta)$ as $\beta f^{M_\omega}(t) + (1 - \beta) f^\omega(t)$ with $f^{M_\omega}(t)$ strictly increasing
in $t$. The baseline model is a special case of $f^M = f^G$. The manager’s first-order condition
for $\beta$ becomes

$$\sum_{\omega \in \{G, B\}} q_\omega [F^\omega(T) - F^M_\omega(T)] \delta_\omega = c C'(\beta^*(T)).$$

$$\frac{\partial \beta^*(T)}{\partial T} = \sum_{\omega \in \{G, B\}} \frac{q_\omega \delta_\omega}{c C''} (f^\omega(T) - f^M_\omega(T)).$$

Again the non-monotonic strategic effect is preserved because $\frac{\partial \beta^*(T)}{\partial T}$ is not monotonic in $T$. Working through the first-order condition of the optimal threshold, we can characterize
the optimal probability threshold as

$$P^* = \frac{L_B}{L_B + L_G + I(T^*)}$$

with $I(T^*) = \frac{q_B L_B \frac{\partial \varepsilon^B(T; \beta)}{\partial \beta} + q_G L_G \frac{\partial \varepsilon^G(T; \beta)}{\partial \beta}}{q_G f^G} \frac{\partial \beta^*(T)}{\partial T} |_{T = T^*}. (17)$

The optimal probability threshold is the same as that in the baseline model except that
$I(T^*)$ becomes more complicated in comparison with equation 12. The major difference is
that the numerator has a new component $q_G L_G \frac{\partial \varepsilon^G(T; \beta)}{\partial \beta} < 0$. That is, evidence manage-
ment can reduce the false alarm error. Since $q_B L_B \frac{\partial \varepsilon^B(T; \beta)}{\partial \beta} > 0$, the first term in $I(T^*)$,
$q_B L_B \frac{\partial \varepsilon^B(T; \beta)}{\partial \beta} + q_G L_G \frac{\partial \varepsilon^G(T; \beta)}{\partial \beta}$, can go either direction, depending on the relative importance of $\beta$
on the evidence distributions in two states. Since evidence management increases the undue
optimism error and reduces the false alarm error, whether evidence management is detri-
mental to the stakeholder’s decision depends on the relative importance of each effect. As a
result, the sign of $I(T^*)$ is determined not only by $\frac{\partial \beta^*(T)}{\partial T}$ (as in the baseline model) but also
by $q_B L_B \frac{\partial \varepsilon^B(T; \beta)}{\partial \beta} + q_G L_G \frac{\partial \varepsilon^G(T; \beta)}{\partial \beta}$. The latter determinant adds one more reason that the optimal
threshold response is non-monotonic, but it is orthogonal to the reason studied in the baseline model.

We also consider a different technology of evidence management. In the baseline model, the evidence management choice is continuous. We consider another modeling device of evidence management in which the manager either engage in evidence management or not. For example, to improve the chance that a lease contract qualifies for operating lease, the manager has to decide whether to hire a consulting firm to structure the lease contract or not. Specifically, the manager’s private cost of evidence management is $c > 0$, with differentiable density and cumulative distribution functions $h(c)$ and $H(c)$, respectively. We also consider only "bad" evidence management. If the manager engages in evidence management, $t$ is drawn from $f^G(t)$ instead of $f^*(t)$. The manager decides whether to engage in evidence management after observing his cost $c$ but before observing the state.

This new setting preserves the threshold’s strategic effect on recognition errors. For any given $T$, the manager engages in evidence management if and only if $c \leq c^*(T)$, with $c^*(T)$ being determined by

$$q_B[F^B(T) - F^G(T)]\delta_B = c^*(T).$$

Differentiating it with respective to $T$, we have

$$\frac{\partial c^*(T)}{\partial T} = q_B[f^B(T) - f^G(T)]\delta_B$$

Thus, the threshold’s effect on evidence management is qualitatively the same as in the baseline model. As a result, the main results in the baseline model can be replicated with little modification.

4.4 Different timing of evidence management

In the baseline model, the manager makes the choice without observing $\omega$ or $t$. In this subsection we consider the case when the manager chooses evidence management after observing initial evidence $t'$, as in Dye (2002) and Laux and Stocken (2013).

If $t' > T$, evidence management is not necessary and the manager reports $t = t'$. If
$t' < T$, the benefit of inflating $t'$ to $T$ depends on the realization of $t'$. For simplicity, we assume that $\delta_B = \delta_G = \delta$, as in the case of the capital market setting in Subsection 4.2. We also assume that it costs the manager a unit cost of $c$ to add $T - t'$ to the initial evidence $t'$. For any threshold $T$, define $t_0(T)$ as the solution $\delta = (T - t_0(T))c$. The manager’s evidence management decision $\beta$ is a function of $t'$:

$$\beta(t') = \begin{cases} T - t' & \text{for } t' \in (t_0(T), T) \\ 0 & \text{otherwise} \end{cases}$$

One might expect that the threshold’s non-monotonic effect on evidence management will disappear, but this conjecture turns out to be incorrect. Recall that the strategic effect is evaluated from an \textit{ex ante} perspective: how does the choice of threshold $T$ at date 0 affect evidence management at date 1? The expected earning management at date 0 is

$$\beta(T) = E_{t'}[\beta(t')] = \int_{t_0(T)}^{T} (q_G f^G(t) + q_B f^B(t))(T - t)dt$$

$$\frac{\partial \beta(T)}{\partial T} = -(q_G f^G(t_0) + q_B f^B(t_0))(T - t_0) + \int_{t_0(T)}^{T} (q_G f^G(t) + q_B f^B(t))dt \quad (18)$$

A marginal increase of the threshold affects the aggregate amount of evidence management in three ways. First, managers with $t'$ just above $t_0$, who manipulated by an amount of $T - t_0$ before the change, now stop. This population has a mass of $q_G f^G(t_0) + q_B f^B(t_0)$. Hence, the first term in equation 18. Second, all managers with $t'$ between $t_0$ and $T$ has to manipulate more, as captured by the second term. Third, managers with $t'$ just above $T$, who didn’t manipulate before the change, now start to engage in evidence management. This, however, does not affect the total amount of evidence management because each manipulates only an infinitesimal amount. This explains equation 18. In general, $\frac{\partial \beta(T)}{\partial T} \neq 0$ and can be either positive or negative, depending on the shapes of $f^w$ and $F^w$. Therefore, the threshold’s non-monotonic strategic effect is still preserved, and so is the qualitative trade-off of threshold design in the baseline model.
5 Empirical implications

The paper has three major empirical implications. First, the mere difference between $P^*$ and $P^{BM}$ reveals an additional determinant of the optimal threshold. In the absence of evidence management, the optimal threshold $P^{BM} = \frac{L_B}{L_B + L_G}$ is determined solely by costs of recognition errors for the stakeholder. In particular, the transaction’s details are irrelevant. In contrast, since $P^*$ has an additional determinant $I(T^*)$, it varies not only with the stakeholder’s decision problem but also with the transaction-specific features, such as $f^o(t)$ and $c$. $f^o(t)$ is the link between the transaction’s economic substance and evidence. It may be interpreted as the relevance of a transaction characteristic. $c$ captures the link’s vulnerability to the manager’s influence and may be interpreted as the reliability of a transaction characteristic. Since this additional determinant varies across transactions, it seems more suitable for explaining the cross sectional differences in observed thresholds across transactions, industries, jurisdictions, and over time.

For example, the probability threshold $P$ implied in most revenue recognition standards seems to be higher than 50%. One common explanation for the excess is that $L_B > L_G$, that is, early recognition is more costly than delayed recognition. Proposition 1 predicts that even after controlling for $L_B$ and $L_G$, the excess can still exist and it varies with both the relevance and reliability of transaction characteristics used in the standards. For another example, according to the model, one reason for the different probability thresholds of the contingent liabilities under IFRS and the US GAAP can be that preparers’ ability and incentive to structure transactions to influence accounting evidence differ in respective jurisdictions. As standard setters are contemplating on revising revenue recognition standards and converging U.S. GAAP with IFRS, our model could help shed some light on these complex issues.

Second, the *ex ante* optimal threshold, which maximizes the report’s *ex ante* value for decision making (before the evidence is generated), differs from the *ex post* efficient threshold, which maximizes the report’s *ex post* value (after the evidence is generated). Since both approaches maximize the report’s information for the stakeholder’s decision making, standard setters can defend themselves better by making it clear which approach they intend to pursue. If the *ex post* approach is chosen, then standards should be evaluated according to Proposition
2. If the \textit{ex ante} approach is intended, then Proposition 1 should be the basis for the evaluation. In the latter case, Proposition 2 predicts that the \textit{ex ante} optimal threshold is vulnerable to the criticism that it does not make the best use of evidence available \textit{ex post}. Accordingly, some safeguards against such \textit{ex post} pressure, such as the independence of standard setters, are desirable.

Finally, the model reveals the richness of accounting standard setting. The model suggests that one key comparative advantage of standard setters is to document a transaction’s details, which seems to be at the core of accounting standard setting process in practice. Moreover, these details evolve over time and thus standards need to be revised continually.

The tests of all these empirical predictions call for data about the costs of recognition errors ($L_\omega$), the empirical distributions of a transaction’s economic substance and its characteristics ($f^\omega(t)$), and managers’ incentive and ability to influence these distributions ($\delta_\omega$ and $c$).

6 Discussions and conclusions

This paper studies the optimal design of recognition thresholds in accounting standards. When a threshold choice induces managers to influence the very accounting evidence the threshold classifies, the threshold choice has dual effects on recognition errors. On one hand, after the evidence is presented, a higher threshold reduces the undue optimism error at the expense of raising the false alarm error. This is the familiar statistical effect. On the other hand, before the evidence is produced, the threshold influences managers’ evidence management, which in turn affects the distribution of accounting evidence and the \textit{ex post} recognition errors. This is the strategic effect. The statistical and strategic effects differ from each other both quantitatively and qualitatively. As a result, the optimal threshold under the strategic approach, which balances the dual effects, behaves differently from that under the statistical approach. In particular, in the presence of inflatory evidence management the optimal threshold can either be higher or lower than its statistical benchmark. The insights from the statistical approach provide limited and misleading guidance for optimal threshold design.
To focus on recognition threshold design, we have taken the use of recognition standards as given. While this choice might be justified on empirical grounds, the problem of the inherent discontinuity in recognition, the "all-or-nothing" feature, is well-known and various alternatives have been proposed to mitigate it. One alternative is to use a more continuous recognition or measurement approach. Another is to reduce the reliance on bright-line rules and move towards a more principle-based system. Investigating the general rationale for recognition by comparing it with these alternatives is a promising avenue for future research.

To mitigate the concern that the main results are an artifact of imposing suboptimal binary recognition in the model, we have restricted the stakeholder’s decision to be binary. This assumption assures that binary recognition is indeed optimal in the model. In addition, the binary decision assumption also fits the model well with the vast statistical literature on hypothesis testing and, as such, facilitates the model’s contrast with its statistical benchmark. Nonetheless, the binary decision is a restriction on the model. As we understand more about the rationale for binary recognition, we should also extend the threshold design problem to a more general decision making setting.

A related limitation of this paper is that the standard setter in the model serves as a convenient stand-in for a rule designer. The same results can be obtained if the manager and the stakeholder can design the rule and commit to it. Thus, to interpret our model in the context of mandatory financial reporting, we have to assume that reporting regulation serves as a low-cost commitment device, an argument elaborated by Mahoney (1995) and Rock (2001). The economic justifications for mandatory financial reporting are still hotly debated (see Leuz and Wysocki (2007) for a survey of the debate). Apparently, an alternative view about the rationale is likely to have different implications for the optimal rule design.

The two-step representation of the recognition process, as first formalized in Dye (2002), is also useful for studying other issues. The manager can influence the first mapping from the transaction’s economic substance to its accounting evidence, and the standard setter designs the second mapping from accounting evidence to recognition. Recognition can be informative about economic substance but its informativeness is endogenous to the threshold design in the shadow of evidence management. In this representation, the standard setter controls at least three instruments to influence the accounting report: the set of transaction
characteristics admissible to a standard, the verification requirement for the admitted transaction characteristics, and the threshold above which an accounting treatment is accorded. This paper focuses on the design of the last instrument, the evidence threshold. Gao (2013) studies the optimal design of the second instrument (verification requirement) and provides a rationale for conservatism. Future research could examine other instruments or the interactions among the instruments to help understand other prominent institutional features of accounting measurement.

7 Appendix

Proof. of Proposition 2: Part 1 is derived directly from the definition of $\hat{P}$ and the first-order condition of equation 13. To prove Part 2, note that $\hat{T}$ is feasible for the standard setter at date 0 but not chosen. Therefore, we have $\rho \equiv W(T^*; \beta^*) - W(\hat{T}; \hat{\beta}) > 0$. $\beta^* < \hat{\beta}$ is proved by contradiction. Suppose $\beta^* \geq \hat{\beta}$. Since $\frac{\partial W(T; \beta)}{\partial \beta} = -q_B L_B(F^B(T) - F^G(T)) < 0$ for any $T \in (t, \hat{t})$, we have $W(T^*; \beta^*) - W(T^*; \hat{\beta}) \leq 0$. Moreover, by definition $\hat{T}$ maximizes $W(\hat{T}; \hat{\beta})$ for given $\hat{\beta}$. Thus, $W(T^*; \hat{\beta}) - W(\hat{T}; \hat{\beta}) < 0$. Combining the two, we have $\rho = (W(T^*; \beta^*) - W(T^*; \hat{\beta})) + (W(T^*; \hat{\beta}) - W(\hat{T}; \hat{\beta})) = W(T^*; \beta^*) - W(\hat{T}; \hat{\beta}) < 0$. Hence a contradiction. Thus, $\beta^* < \hat{\beta}$. ■

Proof. of Proposition 3: Part 1 is proved by the envelope theorem and Lemma 2. To prove Part 2, we apply the implicit function theorem to equation 11: $\frac{dT^*}{dc} = -\frac{\partial^2 W}{\partial T \partial c}$. Since $T^*$ is the maximizer, $\frac{\partial^2 W}{\partial T^2} < 0$. Thus, $\frac{dT^*}{dc}$ has the same sign as $\frac{\partial^2 W}{\partial T \partial c}$. Differentiating equation 11 with respect to $c$ and simplifying the terms, we have

$$\frac{\partial^2 W}{\partial T \partial c} = -q_B L_B(f^B - f^G) \frac{\partial \beta^*}{\partial c} - q_B L_B(F^B - F^G) \frac{\partial^2 \beta^*}{\partial T \partial c}$$

$$= q_B L_B \left( \frac{F^B - F^G}{c} + \frac{(C')^2 - C'C''}{(C'')^2} \right) \frac{\partial \beta^*}{\partial T}$$

Therefore, the sign for $\frac{dT^*}{dc}$ is the same as $\frac{\partial \beta^*}{\partial T}$. The case for $\delta_B$ can be proved similarly. ■
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