Stress Tests and Information Disclosure*  

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Abstract  

We study an optimal disclosure policy of a regulator who has information about banks’ ability to overcome future liquidity shocks. We focus on the following trade-off: Disclosing some information may be necessary to prevent a market breakdown, but disclosing too much information destroys risk-sharing opportunities (Hirshleifer effect). We find that during normal times, no disclosure is optimal, but during bad times, partial disclosure is optimal. We characterize the optimal form of this partial disclosure. We also relate our results to the debate on the disclosure of stress test results.

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1 Introduction

In the new era of financial regulation following the crisis of 2008, central banks around the world will conduct periodic stress tests for financial institutions to assess their ability to withstand future shocks. A key question that occupies policymakers and bankers is whether the results of the stress tests should be disclosed and, if so, at what level of detail. The debate over this question is summarized in an article in the Wall Street Journal from March 2012. In this article, Fed Governor Daniel Tarullo expresses support for wide disclosure, saying that “the disclosure of stress-test results allows investors and other counterparties to better understand the profiles of each institution.” On the other hand, the Clearing House Association expresses the concern that making the additional information public “could have unanticipated and potentially unwarranted and negative consequences to covered companies and U.S. financial markets.”

A classic concern about disclosure in the economics literature is based on the Hirshleifer effect (Hirshleifer, 1971). According to the Hirshleifer effect, greater disclosure might decrease welfare because it reduces risk-sharing opportunities for economic agents. This is indeed a relevant concern in the context of banks and stress tests. A large literature (e.g., Allen and Gale, 2000) studies risk-sharing arrangements among banks. If banks are exposed to random liquidity shocks, they will create arrangements among themselves or with outside markets to insure against such shocks. If more information about the state of each individual bank and its ability to withstand future shocks is publicly disclosed, then such risk-sharing opportunities will be limited, generating a welfare loss.

While this concern may provide credible content to the “unwarranted and negative consequences” referred to in the above quote from the Clearing House Association, it is hard to deny that greater disclosure that “allows investors and other counterparties

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1See “Lenders Stress over Test Results,” Wall Street Journal, March 5, 2012.
to better understand the profiles of each institution” appears to be crucial at times. In particular, as was clear during the recent financial crisis, when aggregate conditions seem bleak, the lack of disclosure might lead to a breakdown in financial activity. In the context of risk sharing and insurance, if the aggregate state of the financial sector is perceived to be weak, banks would not be able to insure themselves against undesirable outcomes (see, e.g., Leitner, 2005). In this case, some disclosure on certain banks might be necessary to enable some risk sharing and its welfare-improving effects.

In this paper, we study a model to analyze these forces and provide guidance for optimal disclosure policy in light of these forces. In the model, financial institutions suffer a loss if their future capital falls below a certain level. Part of the future capital of the financial institution can be forecasted based on current analysis and will become clear to policymakers conducting stress tests. However, there are also future shocks that cannot be forecasted with such an analysis. Financial institutions can engage in risk-sharing arrangements to guarantee that their capital does not fall below the critical level.

These risk-sharing arrangements work well if the overall state of the financial industry is perceived to be strong. In this case, no disclosure by the regulator is needed. Consistent with the Hirshleifer effect, disclosure can be even harmful because it prevents optimal risk-sharing arrangements from taking place. However, if, on average, banks are perceived to have capital below the critical level, then risk-sharing arrangements that insure them against falling below that level cannot arise without some disclosure. In this case, partial disclosure emerges as the optimal solution.

To study optimal disclosure rules in bad times, we distinguish between two different cases. First, we consider an environment where the information discovered by the regulator in the stress test is not already known to the bank. This is a reasonable assumption if the information involves assessment of bank exposure to aggregate conditions or to the state of other banks, and those are known to the regulator, who
analyzes many banks, and not to the individual banks themselves. In this case, we show that it is optimal to create two scores—a high score and a low score—and to give the high score to a group of banks whose average forecastable capital is equal to the critical level, and a low scores to other banks. This is similar to the Bayesian persuasion solution proposed by Kamenica and Gentzkow (2011).

By providing disclosure that separates some bad banks from the others, the regulator enables risk sharing among the remaining banks. Importantly, for this to work, the regulator must not provide too much information. It is sufficient to use only two scores and classify banks as “good” or “bad.” Providing more detailed information about the “bad” banks does not hurt, but the regulator must not provide more information about “good” banks. In particular, within the group of “good” banks, there are some “bad” banks as well; pooling these banks together enables risk sharing.

Interestingly, the disclosure rule is not necessarily monotone; i.e., it is not always the case that banks below a certain threshold are classified as “bad” and others are classified as “good.” There is a gain and a cost from including a bank in the “good” group. The gain is enabling the bank to participate in the risk sharing, preventing a welfare-decreasing drop in capital. The cost is that placing the bank in the “good” group takes resources, thereby preventing other banks from being in that group. The allocation of banks into the “good” group depends on the gain-to-cost ratio, and this does not always generate a monotone rule; it depends on the distribution of shocks that banks are exposed to. We provide conditions under which the disclosure rule is monotone.

The second environment we consider is one where the information discovered by the regulator in the stress test is known to the bank itself but not to the outside market. In this case, pooling banks into two groups will not generally work. Banks whose forecastable level of capital is significantly above the critical level will refuse to participate in a risk-sharing arrangement with a group whose average forecastable
capital is just at the critical level. Hence, in this case, the optimal disclosure rule has multiple scores. As before, one score is reserved for banks that are revealed to be below the critical capital level, and these banks are shunned from risk-sharing arrangements. Other scores pool together banks below the critical level with a bank above the critical level to enable risk sharing. Different scores are required to accommodate the different reservation utilities of different banks above the critical level of capital.

Interestingly, in this environment, non-monotonicity becomes a general feature of optimal disclosure rules. When considering banks below the critical level of capital, it turns out that the stronger ones will be pooled with a bank whose level of capital is only slightly above the critical level (hence receiving a moderate score), while the weaker ones will be pooled with a bank whose level of capital is significantly above the critical level (hence receiving a high score). As we show in this paper, the increase in cost from pooling with a moderately strong bank to pooling with a very strong bank is not significant for the weakest banks but is significant for the moderately weak banks, and this leads to the non-monotonicity result.

The non-monotonicity result may lead to equilibrium outcomes in which lower types end up with higher expected payoffs. This may be plausible if the bank and the regulator learn the bank’s type at the same time during the stress test, but may not be plausible if the bank learns its type before the regulator and can freely dispose assets (e.g., Innes, 1990). To explore the latter case, we solve for optimal disclosure rules when we add a constraint that higher types end up with higher equilibrium payoffs. Interestingly, the non-monotonicity result may continue to hold in this case, but to satisfy the constraint on equilibrium payoffs, the probability that low types participate in risk sharing is reduced. Another new insight is that banks that are above the critical level may be pooled into the same score and hence sell at prices that are above their reservation utilities. This increases the expected payoff for high types and relaxes the constraint on equilibrium payoffs for low types.
In summary, our paper generates the following results about optimal disclosure rules. First, no disclosure is optimal during good times, but partial disclosure is optimal during bad times. Second, partial disclosure takes the form of different scores pooling together banks of different levels of strength. The number of scores increases as we move from a case in which banks do not already have the information revealed in the stress test to the case in which they do possess this information. Third, non-monotonicity appears to be a pervasive feature of optimal disclosure rules, such that a given score pools together strong banks with weak banks. This type of non-monotonicity may continue to hold even when we impose monotonicity on equilibrium expected payoffs.

1.1 Related literature

Our paper is related to the literature on Bayesian persuasion, going back to Kamenica and Gentzkow (2011). The solution for the first case in which the bank does not know its type is similar to the solution in Kamenica and Gentzkow (2011), but since we put more structure on the planner’s objective function, we obtain more results. In particular, we show that disclosure should be based on the gain-to-cost ratio and provide conditions under which a simple cutoff rule is optimal. The second case in which the bank knows its type is completely new and provides new results.\(^2\)

The literature on disclosure of regulatory information is reviewed in a recent paper by Goldstein and Sapra (2014), which highlights the disadvantages of disclosure. Morris and Shin (2002) show that disclosure might be bad if economic agents share strategic complementarities and wish to act like each other even though it is not socially optimal. Providing a public signal then makes them place a too large weight on

\(^2\)In a different context, Gick and Pausch (2014) study a persuasion game in which investors with heterogeneous priors can take one of two actions and the regulator’s objective is to get as close as possible to an outcome in which some predetermined fraction of investors take the first action. They show that in general, it is optimal for the regulator to choose a signal that is not too informative because full information induces investors to herd on the same action.
it because it provides information not only about fundamentals but also about what others know about the fundamentals. However, Angeletos and Pavan (2007) show that this conclusion may not hold when agents share strategic substitutes or when coordination is socially desirable. Leitner (2012) shows that disclosing information may reduce the regulator’s ability to obtain information about contracts that banks enter with one another. In his setting, it is optimal to reveal partial information: the regulator should set a position limit for each bank and reveal only whether a bank has reached its limit. The idea that disclosing information may reduce the regulator’s ability to collect information from banks also appears in Prescott (2008). Bond and Goldstein (2012) show that disclosure of information by the government to the market might harm the government’s ability to learn from the market. Hence, the government may want to disclose information only on variables on which it cannot learn from the market. Increased disclosure might also be harmful due to the adverse effect it might have on the ex-ante incentives of bank managers, as in the traditional corporate-finance literature emphasizing the tension between ex-post and ex-ante optimal actions (e.g., Burkart, Gromb, and Panunzi, 1997). Morrison and White (2013) and Shapiro and Skeie (2013) study how the regulator’s disclosure policy is affected by reputational concerns. Our paper analyzes a different tradeoff involving risk-sharing opportunities, which are at the heart of financial activity.

In a recent paper, Bouvard et al (2013) study how disclosure affects the possibility of bank runs when there are two types of banks and the regulator has private information about banks’ types as well as the proportion of banks of each type. They show that during normal times disclosing information is undesirable because it can lead to bank runs, but during crises, disclosing information is desirable because it can prevent some runs. This result relates to one of our results but is based on completely different considerations. In addition, most of our results on the design of optimal disclosure rules are absent in their setting because they assume that there are only two
types of banks.

In a related paper, Lizzeri (1999) studies the optimal disclosure policy of an intermediary who is hired by a firm to certify the quality of its products.\(^3\) Lizzeri (1999) shows that a monopolist intermediary may choose to restrict the flow of information and reveal only the minimum information that is required for an efficient exchange. Disclosing less information allows the intermediary to extract more rents from firms that are being rated. Instead, in our setting, providing less information allows for better risk sharing.

There is also an extensive literature that studies information disclosure by firms, particularly whether the regulator should mandate firms to disclose information.\(^4\) Our paper contributes to this literature by illustrating a case in which the regulator would like to restrict information flow from firms. A strong firm ignores the fact that revealing information destroys risk-sharing opportunities for weak firms, but the regulator takes this negative externality into account.

In a different context, Marin and Rahi (2000) provide a theory of market incompleteness, which is based on the tradeoff between adverse selection and the Hirshleifer effect. Adverse selection favors an increase in the number of securities because it reduces information asymmetries among agents. The Hirshleifer effect favors a reduction in the number of securities. Our paper does not talk about security design but instead discusses how the regulator should pool banks into groups to enable risk sharing. Because the payoff function in our setting exhibits some convexity (a bank suffers a loss if its capital falls below a certain level), two groups may be necessary even when banks do not have private information. When banks have private information, more groups are necessary to accommodate the different reservation utilities

\(^3\) See also Kartasheva and Yilmaz (2012), who extend Lizzeri’s framework by adding different outside options for firms as well as information asymmetries among potential buyers. In their setting the first-best outcome is full disclosure, while in our setting, the first-best outcome typically involves pooling and, hence, only partial disclosure.

of banks above the critical level.

Finally, the idea that risk-sharing arrangements may break down when aggregate conditions are bleak relates to Leitner (2005). He shows that in this case, it is optimal for banks to remain unlinked rather than form a financial network. In one interpretation of our model, we show how the disclosure policy affects the financial networks that banks form.

2 A model

2.1 The bank

In our model, risk sharing takes a simple form. A bank has an asset that yields a random cashflow. The bank can replace the random cashflow with a deterministic cashflow by selling the asset in a competitive market. The sale price, and hence the bank’s ability to share risk, depends on the regulator’s disclosure policy. The regulator does not inject money in our model.

There are three dates \( t = 0, 1, 2 \). A bank has an asset that yields a random cash flow at date 1 and no cash flows afterward. This cash flow is the sum of two random variables \( \tilde{\theta} \) and \( \tilde{\varepsilon} \), where \( \tilde{\theta} \) is referred to as the bank’s type and \( \tilde{\varepsilon} \) is the bank’s idiosyncratic risk, which is independent of its type. At date 0, the bank can sell the asset in a perfectly competitive market for an amount \( x \), which will be derived endogenously. The amount of cash available for the bank at date 1, which we denote by \( z \), is therefore \( z = \tilde{\theta} + \tilde{\varepsilon} \) if the bank keeps the asset, and \( z = x \) if the bank sells the asset. Everyone is risk neutral, and the risk-free rate is normalized to be zero percent; therefore, \( x \) equals the expected value of the asset \( \tilde{\theta} + \tilde{\varepsilon} \), conditional on the information available to the market.

The bank’s date-2 payoff is:

\[
R(z) = \begin{cases} 
  z & \text{if } z < 1 \\
  z + r & \text{if } z \geq 1, 
\end{cases}
\]  

(1)
where $r > 0$. This payoff function is a reduced form to capture the general idea that banks suffer a loss when their cash holdings fall below some threshold.\(^5\) The payoff function can also represent a project that yields a positive net present value $r$ but requires a minimum level of investment. For various reasons (e.g., projects cash flows are nonverifiable), the bank cannot finance the project if it does not have sufficient cash in hand. For convenience, we stick to the project interpretation, but the reader can think of other interpretations.\(^6\)

The bank acts to maximize its expected payoff at $t = 2$: $E[R(z)]$. As will be clear later, this provides incentives for banks to sell their assets in the financial market for an amount of at least one dollar. This is an insurance to guarantee that the bank can later make the investment. More generally, selling the asset can be thought of as engaging in a risk-sharing arrangement.\(^7\)

The random variables $\tilde{\theta}$ and $\tilde{\varepsilon}$ are drawn at date 0, and we denote their realizations by $\theta$ and $\varepsilon$, respectively. The bank’s type $\tilde{\theta}$ is drawn from a finite set $\Theta \subset \mathbb{R}$ according to a probability distribution function $p(\theta) = \Pr(\tilde{\theta} = \theta)$. The idiosyncratic risk $\tilde{\varepsilon}$ is drawn from a cumulative distribution function $F$ that satisfies $E(\tilde{\varepsilon}) = 0$; for simplicity, we assume that $F$ is continuous. The probability structure (i.e., the functions $p$ and $F$) is common knowledge.

The planner observes $\theta$. The market observes neither $\theta$ nor $\varepsilon$. As for the bank, we focus on two cases:

1. The bank observes neither $\theta$ nor $\varepsilon$.

\(^5\)A similar discontinuity in the payoff function appears in Leitner (2005) and in Elliott et al. (forthcoming).

\(^6\)Other payoff functions that exhibit discontinuity may lead to similar results. For example, we can assume that if $z < 1$, the bank obtains $az$ for some $a \in [0, 1)$; and if $a < 1$, we can set $r = 0$. For example, $a = r = 0$ may capture the idea that when the asset value falls below some threshold, there is a bank run and the bank is left with nothing.

\(^7\)We rule out partial insurance in which a bank with type $\theta < 1$ sells its asset for a price 1, which is paid with probability $\theta$ (i.e., the bank transfers the asset with probability 1 but receives payment with probability that is less than 1). This can be motivated by assuming that banks enter risk-sharing arrangements by forming links as in Leitner (2005). In his model, the bank’s investment can succeed only if all the banks to which it is linked invest as well; hence, helping just a fraction of the banks in the network does not help.
The bank observes $\theta$ but not $\epsilon$.

The first case captures the idea that the government may have some information advantage relative to banks. This is a plausible assumption when asset values depend on future government actions or when asset values depend on interactions among banks, and the government’s ability to collect information from multiple banks allows it to come up with better estimates. The second case captures the idea that the government and banks share the same information, which is unobservable to other market participants. For example, the bank may know its ability to withstand future liquidity shocks, and the government can find out this information by conducting stress tests.

Denote the lowest type by $\theta_{\text{min}}$ and the highest type by $\theta_{\text{max}}$. We assume that $\theta_{\text{max}} > 1$, so if information on $\theta$ were publicly available, at least some types could sell their assets for more than one dollar and invest in their projects. We also assume that:

**Assumption 1:** $F(1 - \theta_{\text{min}}) < 1$ and $F(1 - \theta_{\text{max}}) > 0$.

This implies that for any type realization there is a positive probability that the asset cash flow will be more than 1; but there is also a positive probability that the asset cash flow will be less than 1.

### 2.2 Disclosure rules

The planner’s problem is to choose a disclosure rule, as defined below, to maximize total surplus, taking as given the effect of disclosure on the bank’s ability to sell its asset for at least one dollar. Since the market breaks even on average, maximizing total surplus is the same as maximizing the bank’s expected payoff.

Formally, a *disclosure rule* is a set of “scores” $S$ and a function that maps each type to a distribution over scores. Since $\Theta$ is assumed to be finite, we also assume that $S$ is finite. Denote by $g(s|\theta)$ the probability that the planner assigns a score
$s \in S$ when he observes type $\theta$; that is, $g(s|\theta) = \Pr(\tilde{s} = s|\tilde{\theta} = \theta)$. (For every $\theta \in \Theta$, $\sum_{s \in S} g(s|\theta) = 1$.) For example, full disclosure is obtained when for every type $\theta$, the planner assigns some score $s_\theta \in S$ with probability 1, such that $s_\theta \neq s_{\theta'}$ if $\theta \neq \theta'$. No disclosure is obtained when the planner assigns the same distribution over scores to all types; e.g., each type obtains the same score.

For use below, denote $\mu(s) = E[\tilde{\theta} + \tilde{\varepsilon}|\tilde{s} = s]$, which is the expected value of the bank’s asset conditional on the bank obtaining score $s$. Since $\tilde{\varepsilon}$ is independent of $\tilde{\theta}$, and since $E(\tilde{\varepsilon}) = 0$, we obtain that

$$
\mu(s) = E[\tilde{\theta}|\tilde{s} = s] = \sum_{\theta \in \Theta} \theta \Pr(\tilde{\theta} = \theta|\tilde{s} = s) = \frac{\sum_{\theta \in \Theta} \theta p(\theta) g(s|\theta)}{\sum_{\theta \in \Theta} p(\theta) g(s|\theta)},
$$

where the last equality follows from Bayes’ rule.

### 2.3 Sequence of events

We assume that the planner can commit to assigning scores according to the disclosure rule chosen. Hence, the sequence of events is as follows:

$t = 0$:

(a) The planner announces its disclosure rule.

(b) The bank’s type $\theta$ is realized and observed by the planner.

(c) The planner assigns the bank a score $s$, according to the disclosure rule, and publicly announces the score.

(d) The market offers to purchase the asset at a price $x(s)$.

(e) The bank either keeps the asset or sells it for a price $x(s)$.

$t = 1$:

The bank invests if its available cash $z$ is above 1.

$t = 2$:

The bank obtains $R(z)$.

The planner’s disclosure rule and assigned scores specify a game between the bank and the market. We focus on perfect Bayesian equilibria of this game. Specifically,
the bank chooses whether to sell or keep the asset to maximize its expected profits, conditional on its information, and the market chooses a price \( x(s) \) that equals the expected value of the asset conditional on public information, taking as given the bank’s equilibrium strategy. We assume that if the bank is indifferent between selling and not selling, it sells. The planner chooses a disclosure rule that maximizes the bank’s expected payoff, taking as given the equilibrium strategies of the market and of the bank.

Finally, note that there is a big difference between the bank and the planner even in the second case in which the bank and the planner share the same information about \( \theta \). The bank maximizes its ex-post payoff after \( \theta \) is realized. The planner maximizes the bank’s ex-ante payoff before \( \theta \) is realized. If there are many banks, one can think of \( p(\theta) \) as the fraction of banks with a realization of \( \theta \). In this case, maximizing the bank’s ex-ante payoff is the same as maximizing the sum of banks’ ex-post payoffs. Hence, the bank and the planner have different objective functions ex post: the bank cares only about its own payoff, while the planner cares about the sum of payoffs of all banks.

3 Bank does not observe its type

We start with the case in which the bank observes only the score \( s \). We solve the game backward. One observation that simplifies the analysis is that the bank’s decision of whether to sell the asset depends on \( s \) but not on \( \theta \) or \( \varepsilon \). Hence, selling does not convey any additional information to the market. Consequently, the market sets a price \( x(s) = \mu(s) \). It then follows from the payoff structure in (1) that:

\textbf{Lemma 1} In equilibrium, the bank sells the asset if and only if it obtains a score \( s \) such that \( \mu(s) \geq 1 \).
The proof of Lemma 1 and all other proofs are in the appendix. The idea behind Lemma 1 is simple. If \( \mu(s) > 1 \), selling guarantees that the bank will have sufficient funds to invest in its positive NPV project; hence, the bank acts like a risk averse agent and is happy to replace the asset’s random cash flow with its expected value. If instead, \( \mu(s) < 1 \), the bank prefers to keep the asset because if the bank sells the asset, the bank will surely have insufficient funds to invest, but if the bank keeps the asset, there is a positive probability that the asset’s cash flow will turn out to be high and the bank will have sufficient funds. In this case the bank acts like a risk-loving agent.

The expected payoff for a bank of type \( \theta \), given disclosure rule \((S, g)\), is then

\[
 u(\theta) \equiv \sum_{s: \mu(s) < 1} [\theta + r \Pr(\bar{\varepsilon} \geq 1 - \theta)]g(s|\theta) + \sum_{s: \mu(s) \geq 1} [\mu(s) + r]g(s|\theta).
\]  

The first term represents the cases in which the bank keeps the asset, and the second term represents the cases in which the bank sells the asset. The planner’s problem is to choose a disclosure rule \((S, g)\) to maximize the bank’s ex-ante expected payoff \(\sum_{\theta \in \Theta} p(\theta)u(\theta)\).

For use below, we refer to a score with \( \mu(s) \geq 1 \) as a “high” score and a score with \( \mu(s) < 1 \) as a “low” score. Denote the probability that a bank of type \( \theta \) obtains a high score by \( h(\theta) \); that is, \( h(\theta) = \sum_{s: \mu(s) \geq 1} g(s|\theta) \). This is the probability that the bank sells its asset or, more broadly, engages in risk sharing.

**Lemma 2** The planner’s problem reduces to finding a function \( h : \Theta \rightarrow [0, 1] \) to maximize

\[
\sum_{\theta \in \Theta} p(\theta) \Pr(\bar{\varepsilon} < 1 - \theta)h(\theta),
\]

subject to the constraint

\[
\sum_{\theta \in \Theta} p(\theta)(\theta - 1)h(\theta) \geq 0.
\]
The term $\Pr(\tilde{\varepsilon} < 1 - \theta)$ in the objective function (4) represents the gain from giving a high score to type $\theta$. The gain is that type $\theta$ can invest and receive $r$ even if it has a low realization of cash flow. The constraint (5) can be thought of as the planner’s resource constraint. The planner would like to give high scores to everyone but faces the constraint that the average cash flow conditional on obtaining a high score must be at least 1. This constraint reduces to constraint (5).\footnote{Formally, for every high score $s$, $\mu(s) \geq 1$, which from equation (2) reduces to $\sum_{\theta \in \Theta} p(\theta)(\theta - 1)g(s|\theta) \geq 0$. Summing over all high scores, we obtain constraint (5).} Essentially, by giving a high score, the planner implements a cross subsidy from types with $\theta > 1$ to types with $\theta < 1$, so a high type sells its asset for less than what the asset is truly worth, and a low type sells its asset for more than what the asset is worth.

The solution to the planner’s problem depends on whether there are sufficient resources or insufficient resources. If $E(\tilde{\varepsilon}) \geq 1$, the planner can give a high score to every type without violating the resource constraint. Hence, it is optimal that every type obtains a high score with probability 1. If instead $E(\tilde{\varepsilon}) < 1$, the resource constraint is binding, so some types must obtain a low score. In this case, the optimal disclosure rule depends on the “gain-to-cost ratio” from giving a high score to type $\theta$. The gain is the term $\Pr(\tilde{\varepsilon} < 1 - \theta)$ in the objective function. The cost is that type $\theta$ requires resources in the amount $1 - \theta$. So the gain-to-cost ratio is

$$G(\theta) \equiv \frac{\Pr(\tilde{\varepsilon} < 1 - \theta)}{1 - \theta}. \quad (6)$$

For types with $\theta \geq 1$, it is optimal to assign a high score with probability 1 because there is no cost; these types provide resources. For types with $\theta < 1$, it follows from the linearity of the problem that it is optimal to assign the high score with probability 1 to as many types as possible subject to the resource constraint and with preference to types with a high gain-to-cost ratio. Types with a high gain-to-cost ratio obtain a high score, and types with a low gain-to-cost ratio obtain a low score. The high score pools all the types that are at or above 1 with some type that are below 1, such that
the average cash flows of banks receiving the high score equals 1.

**Proposition 1** When a bank does not observe its type, the optimal disclosure rule is such that

(i) If $E(\tilde{\theta}) \geq 1$, then $h(\theta) = 1$ for every $\theta \in \Theta$.

(ii) If $E(\tilde{\theta}) < 1$, then

$$h(\theta) = \begin{cases} 1 & \text{if } \theta \geq 1 \text{ or if } \theta < 1 \text{ and } G(\theta) > G^* \\ 0 & \text{if } \theta < 1 \text{ and } G(\theta) < G^*, \end{cases}$$

where $G^*$ is the lowest $G \in \{G(\theta)\}_{\theta \in \Theta}$ that satisfies $\sum_{\theta \geq 1} p(\theta)(\theta - 1) + \sum_{\theta < 1; G(\theta) > G^*} p(\theta)(\theta - 1) \geq 0$.

When different types have different gain-to-cost ratios, the optimal disclosure rule uniquely determines the probability that each type obtains a high score, but there is more than one way to implement it. For example, the first part in Proposition 1 says that if there are sufficient resources, every bank must obtain a high score with probability 1. This can be implemented by giving all banks the same score; i.e., no disclosure. This can also be implemented by assigning more than one score such that the average cash flows for banks receiving each score is at least 1. In the special case $\theta_{\min} \geq 1$, the planner can assign a different score to each type; i.e., full disclosure.

In contrast, if there are insufficient resources (second part), the planner must assign at least two scores, a high score and a low score; two scores are sufficient. In this case, full disclosure is suboptimal because under full disclosure, only types above 1 sell their assets, while under the optimal disclosure rule, some types that are below 1 also sell their assets.

**Corollary 1** 1. Full disclosure achieves the optimal outcome if and only if $\theta_{\min} \geq 1$.

\[\text{If there is only one type for which } G(\theta) = G^*, \text{ then for this type, } h(\theta) = \alpha, \text{ where } \alpha \text{ is uniquely determined from the resource constraint: } \sum_{\theta \geq 1} p(\theta)(\theta - 1) + \sum_{\theta < 1; G(\theta) > G^*} p(\theta)(\theta - 1) + \alpha \sum_{\theta; G(\theta) = G^*} p(\theta)(\theta - 1) = 0. \text{ If there is more than one type for which } G(\theta) = G^*, \text{ assigning } h(\theta) = \alpha \text{ to each of these types is optimal, but any } \{h(\theta)\}_{\theta; G(\theta) = G^*} \text{ that satisfy } \alpha \sum_{\theta; G(\theta) = G^*} p(\theta)(\theta - 1) = \sum_{\theta; G(\theta) = G^*} p(\theta)h(\theta)(\theta - 1) \text{ is also optimal.} \]
2. No disclosure achieves the optimal outcome if and only if $E(\tilde{\theta}) \geq 1$.

In general, the banks that obtain a low score in the second part of Proposition 1 are not necessarily the lowest types. There are two forces that push the gain-to-cost ratio $G(\theta)$ in different directions. On the one hand, the benefit from insurance (and hence from obtaining a high score) is higher for lower types because lower types are more likely to end up with low realizations of cash flow. But on the other hand, the cost of giving a high score to low types is also higher because low types require more resources. The function $G(\theta)$, and hence the optimal disclosure rule, depends on the distribution of the idiosyncratic risk $\tilde{\varepsilon}$.

For example, if $G(\theta)$ is increasing when $\theta < 1$, the optimal disclosure rule involves a cutoff, such that types above the cutoff obtain a high score and types below the cutoff obtain a low score. A sufficient condition for this to happen is that the probability distribution of the idiosyncratic risk satisfies the following condition:

**Condition 1** $F(\varepsilon)/\varepsilon$ is decreasing when $\varepsilon > 0$.

**Corollary 2** If $E(\tilde{\theta}) < 1$, and if Condition (1) is satisfied, the optimal disclosure rule involves a cutoff such that types below the cutoff obtain a low score (and hence do not engage in risk sharing) and types above the cutoff obtain a high score (and hence engage in risk sharing).

Condition (1) is satisfied by any cumulative distribution function that is concave on the positive region. Examples include a normal distribution with mean zero and a uniform distribution with mean zero.

However, if $F(\varepsilon)/\varepsilon$ is increasing when $\varepsilon > 1 - \theta_{k+1}$ ( $\theta_{k+1}$ denotes the highest type below 1), then $G(\theta)$ is decreasing when $\theta \leq 1 - \theta_{k+1}$. In this case, the optimal disclosure rule is nonmonotone in type. It includes a cutoff such that types below the cutoff and types above 1 obtain a high score, while types in the middle obtain a low score.
Finally, we assumed above that all types of banks have the same \( r \), that is, the same investment opportunities. The results extend easily to the case in which \( r \) depends on the bank’s type according to some function \( r(\theta) \). In this case, the gain-to-cost ratio becomes \( r(\theta)G(\theta) \). Everything else being equal, the gain of giving a high score is higher if the bank’s continuation value is higher. Hence, if \( r'(\theta) > 0 \), the optimal rule may involve a cutoff even if Condition (1) does not hold.

4 Bank observes its type

So far, we assumed that the bank does not observe its type. We showed that it is possible to implement the optimal disclosure rule with two scores, such that the planner pools everyone who sells under the same score. In this section, we show that this conclusion may no longer be true when the bank observes its type. The difference is that now each type has a “reservation price,” i.e., a minimum price at which it is willing to sell. When different types have different reservation prices, the planner may need to assign more than two scores to distinguish among them. We also discuss how the planner should assign these multiple scores to low types who are pooled with high types.

We first derive banks’ reservation prices. Define

\[
\rho(\theta) = \begin{cases} 
\max\{1, \theta - r \Pr(\tilde{\varepsilon} < 1 - \theta)\} & \text{if } \theta \geq 1 \\
\min\{1, \theta + r \Pr(\tilde{\varepsilon} \geq 1 - \theta)\} & \text{if } \theta < 1.
\end{cases}
\]  

Then,

**Lemma 3** A bank of type \( \theta \) will sell its asset if and only if the price is at least \( \rho(\theta) \).

\footnote{An example of a probability distribution function that satisfies the condition above is a truncated Cauchy distribution (See Nadarajah and Kotz, 2006). In particular, there exists \( A > 0 \), such that if \( X \) is a random variable drawn from a truncated Cauchy distribution on \([-A, 0]\), the random variable \( X - E(X) \) satisfies the condition above. This distribution puts higher weights on higher values, has low probability for negative values, and has a fat left tail to ensure that the mean is zero.}
We refer to $\rho(\theta)$ as type $\theta$’s reservation price. As illustrated in Figure 1, the reservation price is increasing in $\theta$. For high types, $\theta > 1$, the reservation price is lower than the true value $\theta$ because these types are willing to pay a premium $r \Pr(\bar{\varepsilon} < 1 - \theta)$ to guarantee that they will have the minimum amount necessary for investment. But the price must be at least one for this type of insurance to work. Low types, $\theta < 1$, should also have at least one dollar if they want to insure themselves, but the very low types may be willing to sell their assets for even less than one dollar. Such a sale goes against insurance, so the very low types will be willing to do so only if the price is strictly higher than the true value.

If $\rho(\theta_{\text{max}}) = 1$, so the highest reservation price is one, the optimal disclosure rule from Section 3 remains optimal. The case $\rho(\theta_{\text{max}}) = 1$ happens when $\theta_{\text{max}} - r \Pr(\bar{\varepsilon} < 1 - \theta_{\text{max}}) \leq 1$; i.e., when $r$ is sufficiently high, so the cost of not obtaining insurance is very high, or when $\theta_{\text{max}}$ is sufficiently low, so the cost of selling at a price of 1 rather than the true value $\theta_{\text{max}}$ is not too high.

**Proposition 2** If $\theta_{\text{max}} - r \Pr(\bar{\varepsilon} < 1 - \theta_{\text{max}}) \leq 1$, i.e., $r$ is sufficiently high or $\theta_{\text{max}}$ is sufficiently low, Proposition 1 continues to hold even if banks observe their types.

The rest of this section focuses on the more interesting case $\rho(\theta_{\text{max}}) > 1$. We first establish that:

**Lemma 4** Under an optimal disclosure rule:

1. Every type $\theta \geq 1$ sells its asset with probability 1.
2. Whenever type $\theta \geq 1$ receives score $s$, the price is $x(s) = \mu(s)$.
3. If the highest type that obtains score $s$ is less than 1, then every type keeps its asset upon obtaining score $s$.

The idea behind the first part in Lemma 4 is that if a type $\theta \geq 1$ did not sell its asset, the planner could strictly increase the payoff of that type, without affecting
the payoffs of other types, by fully revealing $\theta$’s type. Then the market would offer to buy the asset of type $\theta$ at a price $\theta$, and type $\theta$ would accept the offer.

The second part in Lemma 4 follows from the first part and the observation that the reservation price is increasing in $\theta$. These imply that every type sells its asset upon obtaining score $s$, and hence selling does not convey any additional information to the market.

The third part in Lemma 4 reflects the fact that if there is no type above 1 that obtains score $s$, the price $x(s)$ must be less than 1. But then banks will sell only if the price is strictly above their true value. However, this cannot be an equilibrium outcome, since the market would lose money. Note that this result holds under any disclosure rule, not only an optimal one.

For use below, denote the types in $\Theta$ by $\theta_{\text{max}} = \theta_1 > \theta_2 > \ldots > \theta_m = \theta_{\text{min}}$ and suppose that $\theta_k \geq 1 > \theta_{k+1}$, so there are exactly $k$ types at or above 1. Denote $\rho_i = \rho(\theta_i)$.

Denote by $S_i$ the set of scores that type $\theta_i$ obtains with a positive probability but higher types do not obtain; that is, $S_i = \{s \in S : g(s|\theta_i) > 0 \text{ and } g(s|\theta') = 0 \text{ for every } \theta' > \theta\}$. From Lemma 4, we know that type $\theta$ sells its asset upon obtaining score $s$ if and only if $s \in \cup_{i=1}^k S_i$; that is, the highest type that obtains score $s$ must be at least 1. Hence, the expected payoff for type $\theta$ is

$$V(\theta) \equiv \sum_{s \notin \cup_{i=1}^k S_i} [\theta + r \Pr(\tilde{\varepsilon} \geq 1 - \theta)]g(s|\theta) + \sum_{s \in \cup_{i=1}^k S_i} [\mu(s) + r]g(s|\theta).$$

This is similar to equation (3), but the summations are over the events $s \in \cup_{i=1}^k S_i$ and $s \notin \cup_{i=1}^k S_i$ instead of $\mu(s) < 1$ and $\mu(s) \geq 1$.

The social gain from type $\theta$ selling its asset is as in Section 3: type $\theta$ can continue its project even if it has a low realization of cash flow. Hence, the planner’s objective function is similar to that in Lemma 2, with $h(\theta)$ replaced by $h(\theta) = \sum_{s:s \in \cup_{i=1}^k S_i} g(s|\theta)$. Define $h_i(\theta) = \sum_{s \notin S_i} g(s|\theta)$. Then $h(\theta) = \sum_{i=1}^k h_i(\theta)$.
However, now there may be more than one resource constraint. In particular, from Lemma 3 and Lemma 4, we know that for each \( i \in \{1, \ldots, k\} \) and \( s \in S_i \), we must have
\[
x(s) = \mu(s) \geq \rho_i. \tag{10}
\]
That is, if the highest type that obtains score \( s \) is type \( \theta_i \geq 1 \), the expected cash flow conditional on obtaining score \( s \) must be at least as high as type \( \theta_i \)’s reservation price.

From equation (2), equation (10) reduces to
\[
\sum_{\theta \in \Theta} p(\theta)(\theta - \rho_i)g(s|\theta) \geq 0.
\]
Summing over all \( s \in S_i \), we obtain
\[
\sum_{\theta \in \Theta} p(\theta)(\theta - \rho_i)h_i(\theta) \geq 0. \tag{11}
\]
Equation (11) is a generalization of the resource constraint (5). Now the cost of giving a “high” score to type \( \theta \) depends on the reservation price of the highest type that obtains that score because banks that obtain the same score sell at the same price.

The planner’s problem reduces to choosing the probabilities \( \{h_i(\theta)\}_{i \in \{1, \ldots, k\}} \) to maximize the social gain from risk sharing, \( \sum_{\theta \in \Theta} p(\theta)\Pr(\bar{\varepsilon} < 1 - \theta) \sum_{i=1}^{k} h_i(\theta) \), subject to the \( k \) resource constraints, namely, constraint (11) must hold for every \( i \in \{1, \ldots, k\} \). Lemma A-3 in the appendix provides more details.

As in Corollary 1, full disclosure is optimal only if there are no types below 1. No disclosure is optimal only if there are sufficient resources, but the condition for no disclosure changes to \( E(\bar{\theta}) \geq \rho_1 \), so that equation (10) holds for the highest type.

The rest of this section focuses on the case in which resources are scarce, so the optimal disclosure rule is such that there is at least one type that keeps its asset with a positive probability. A sufficient condition for this to happen is that \( E(\bar{\theta}) < 1 \). In this case, all resource constraints are binding. In particular, if the highest type that obtains score \( s \) is \( \theta_i \geq 1 \), the price must equal \( \rho_i \). This means that all lower types that obtain score \( s \) also sell for a price \( \rho_i \). An implication of this is that if types \( \theta_i > \theta_j \geq 1 \) have different reservation prices (which is the case when \( \rho_i > 1 \)), the planner must assign them different scores. Formally,
Proposition 3 Suppose $E(\tilde{\theta}) < 1$. Under an optimal disclosure rule, types that are above 1 and that have different reservation prices must obtain different scores.

Intuitively, if types $\theta_i > \theta_j \geq 1$ have different reservation prices but the same score, the sale price depends on the reservation price of the highest type. This means that the lowest type sells the asset for more than its reservation price and, therefore, ends up with more resources than it requires. But this is a waste of resources without any gain. The planner can do better by assigning the lower type its own score, so that this type ends up with less resources. This frees up resources that can be used to subsidize types with $\theta < 1$. This, in turn, increases the probability that these low types invest in their projects.

It follows that when $E(\tilde{\theta}) < 1$, and $\rho_1 > \rho_2 > \ldots > \rho_k$, the planner must assign at least $k + 1$ scores, $s_0, s_1, \ldots, s_k$, such that for each $i \in \{1, \ldots, k\}$, score $s_i$ pools together type $\theta_i$ with a type (or types) that are below 1, and score $s_0$ pools together only types that are below 1. A bank sells its asset if and only if $s \neq s_0$. When a bank obtains score $s_i \neq s_0$, the bank sells the asset at a price $\rho_i$. Since $\rho_1 > \rho_2 > \ldots > \rho_k$, it is natural to think of score $s_1$ as the highest, score $s_2$ as the second highest, etc. We can assume, without loss of generality, that scores $s_0, s_1, \ldots, s_k$ are the only scores.\footnote{Lemma A-2 in the appendix provides more details.}

Next we discuss how the planner should assign scores to types that are below 1; that is, how the planner should pool types that are below 1 with types that are above 1. Suppose first that there is only one type above 1, type $\theta_1$. The analysis is similar to the the one in Section 3, but now the gains-to-cost ratio depends on $\rho_1$:

$$G_1(\theta) \equiv \frac{\Pr(\tilde{\varepsilon} < 1 - \theta)}{\rho_1 - \theta}. \quad (12)$$

In particular, the gain of pooling type $\theta < 1$ with type $\theta_1 > 1$ is the same as in Section 3, but the cost is higher, since type $\theta$ ends up with $\rho_1 > 1$ rather that 1. This reflects the fact that when a low type is pooled with a high type, the market price reflects the reservation price of the highest type.
Suppose now that there are two types that are above 1, \( \theta_1 > \theta_2 > 1 \). The gain from pooling type \( \theta < 1 \) with either type \( \theta_1 \) or type \( \theta_2 \) is the same. However, the cost is different: it is less costly to pool type \( \theta \) with type \( \theta_2 \) because then type \( \theta \) ends up with less resources. The “net” benefit of pooling type \( \theta \) with type \( \theta_2 \) rather than with type \( \theta_1 \) is

\[
\frac{G_2(\theta)}{G_1(\theta)} = \frac{\Pr(\bar{\varepsilon} < 1 - \theta)}{\rho_2 - \theta} \frac{\rho_1 - \theta}{\Pr(\bar{\varepsilon} < 1 - \theta)} = \frac{\rho_1 - \theta}{\rho_2 - \theta} > 1. \tag{13}
\]

Since the net benefit is higher when \( \theta \) is higher, the planner would prefer to pool type \( \theta_2 \) with higher types (among those with \( \theta < 1 \)) and type \( \theta_1 \) with lower types. Hence, if, for example, \( \theta' < \theta'' < 1 \), we may obtain an outcome in which type \( \theta' \) is pooled with type \( \theta_1 \) and sells its asset for price \( \rho_1 \), and type \( \theta'' \) is pooled with type \( \theta_2 \) and sells it asset for a price \( \rho_2 \). In this case, the lower types sells for a higher price; that is, the lower type obtains a higher score.

The intuition above extends to the case in which there are more than two types above 1. Formally,

**Proposition 4** Suppose \( E(\bar{\theta}) < 1 \) and \( \theta' < \theta'' < 1 \). Under an optimal disclosure rule, if there is a positive probability that type \( \theta' \) obtains score \( s' \neq s_0 \) and type \( \theta'' \) obtains score \( s'' \neq s_0 \), then the prices must satisfy \( x(s'') \leq x(s') \). In other words, among the types \( \theta < 1 \) that sell their assets, lower types obtain higher scores.

Propositions 3 and 4 imply that when banks observe their types, the sale price is increasing in type when \( \theta > 1 \) but decreasing in type when \( \theta < 1 \). Hence, non-monotonicity is a general feature of optimal disclosure rules. (In contrast, in Section 3, all types that sell their assets sell for the same price, and only the probability of selling the asset could be non-monotone.) The next example illustrates this.

**Example 1** Suppose that there are eight types: \( \theta_1 > \theta_2 > 1 > \theta_3 > \ldots > \theta_8 \). Suppose that \( \rho_1 > \rho_2 \geq 1 \) and \( E(\bar{\theta}) < 1 \). Then we need at least three scores:
Suppose the gains-to-cost functions that are associated with score $s_1$ and score $s_2$ are both increasing in $\theta$; that is, the functions $G_1(\theta) = \frac{\Pr(\tilde{\eta} < 1 - \theta)}{\rho_1 - \theta}$ and $G_2(\theta) = \frac{\Pr(\tilde{\eta} < 1 - \theta)}{\rho_2 - \theta}$ are both increasing in $\theta$ (see Figure 2). Suppose

$$p_2(\theta_2 - \rho_2) = p_3(\rho_2 - \theta_3) + \frac{1}{3}p_4(\rho_2 - \theta_4) \quad (14)$$

$$p_1(\theta_1 - \rho_1) = \frac{2}{3}p_4(\rho_1 - \theta_4) + \frac{1}{5}p_5(\rho_1 - \theta_5) \quad (15)$$

As will become clear, equation (14) is the resource constraint that is associated with score $s_2$, and equation (15) is the resource constraint that is associated with score $s_1$.

The optimal disclosure rule is as follows. (Each element in the table is the probability of assigning score $s$ to type $\theta$.)

<table>
<thead>
<tr>
<th>$s_0$</th>
<th>$s_1$</th>
<th>$s_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_8$</td>
<td>$\theta_7$</td>
<td>$\theta_6$</td>
</tr>
<tr>
<td>$\frac{1}{5}$</td>
<td>$\frac{2}{3}$</td>
<td>$\frac{4}{5}$</td>
</tr>
</tbody>
</table>

To see why, note that since $G_1(\theta)$ and $G_2(\theta)$ are both increasing in $\theta$, score $s_0$ is given to low types. (Note that since $\rho_1 > \rho_2$, $G_1(\theta)$ is below $G_2(\theta)$ for every $\theta < 1$.)

Regarding scores $s_1$ and $s_2$, we know from Proposition 3 that with probability 1, type $\theta_1$ obtains score $s_1$, and type $\theta_2$ obtains score $s_2$. As for the other types, which are below 1, we know from Proposition 4 that score $s_2$ is given to higher types compared with score $s_1$. It then follows from equation (14) that score $s_2$ is first given to type $\theta_3$. Since there are remaining resource even if type $\theta_3$ obtains score $\theta_3$ with probability 1, score $s_2$ is also given to type $\theta_4$, but only with probability $\frac{1}{3}$. This exhausts all resources that type $\theta_2$ contributes. Similarly, score $s_1$ is given to the next highest types until all resources are exhausted. Hence, type $\theta_4$ obtains score $s_1$ with probability $\frac{2}{3}$ (so that it sells its asset with probability 1), and type $\theta_5$ obtains score $s_1$ with probability $\frac{1}{5}$, so that the resource constraint (15) is satisfied with equality. All remaining types obtain score $s_0$.

Note that while the sale price in Example 1 is non-monotone in type, the probability of selling the asset is monotone. In particular, as in Corollary 2, there exists a
cutoff such that types above the cutoff sell their asset, and types below the cutoff do not sell. This follows since we assumed in the example that the gains-to-cost function that is associated with each score \( s \neq s_0 \) is increasing in \( \theta \). A sufficient condition for this to happen is that condition 1 holds and \( \rho_1 \) is sufficiently low.\(^{12}\) However, if \( \rho_1 \) is sufficiently high, condition 1 implies that the gains-to-cost function \( G_1(\theta) \) is decreasing in \( \theta \).\(^{13}\) In this case, there does not exist a cutoff such that types above the cutoff sell and types below the cutoff do not sell. Hence, we obtain two forms of non-monotonicity: First, the probability of selling the price does not increase in type. Second, the sale price does not increase in type. The next example illustrates this.

**Example 2** Consider Example 1 but assume that \( \rho_1 \) is sufficiently high, so that \( G_1(\theta) \) is decreasing in \( \theta \). In addition, instead of equation (15), assume that

\[
p_1(\theta_1 - \rho_1) = p_8(\rho_1 - \theta_8) + \frac{1}{10}p_7(\rho_1 - \theta_7),
\]

which will be the resource constraint that is associated with score \( s_1 \). In this case, the optimal disclosure rule is

\[
\begin{array}{cccccccc}
\theta_8 & \theta_7 & \theta_6 & \theta_5 & \theta_4 & \theta_3 & \theta_2 & \theta_1 \\
1 & \frac{1}{10} & 1 & 1 & \frac{1}{2} & \frac{1}{3} & 1 & 1 \\
\end{array}
\]

In particular, as before, score \( \theta_2 \) is assigned to type \( \theta_3 \) and type \( \theta_4 \), such that the resource constraint (14) is binding. However, since the gains-to-cost function that is associated with score \( s_1 \) is decreasing in type, score \( s_1 \) is given to the lowest type. Hence, type \( \theta_8 \) obtains score \( s_1 \) with probability 1, and type \( \theta_7 \) obtains score \( s_1 \) with probability \( \frac{1}{10} \). Then the resource constraint (16) is satisfied with equality. The remaining score \( s_0 \), is given to all remaining types (those in the middle). Hence, the probability of selling the asset \((1 - s_0)\) is non-monotone. \(\blacksquare\)

\(^{12}\)To see that, note that \( G_i(\theta) \) increases when \( \theta < 1 \) if and only if \( F(\varepsilon)/(\varepsilon + \rho_i - 1) \) is decreasing when \( \varepsilon > 0 \), or equivalently, if for every \( \varepsilon > 0 \), \( \frac{F(\varepsilon)}{F(\varepsilon)} > \varepsilon + \rho_i - 1 \). By continuity, if \( \rho_i \) is sufficiently small (\( \rho_i \downarrow 1 \)), condition 1 implies \( \frac{F(\varepsilon)}{F(\varepsilon)} > \varepsilon + \rho_i - 1 \).

\(^{13}\)In particular, \( \frac{F(\varepsilon)}{F(\varepsilon)} < \varepsilon + \rho_i - 1 \) for every \( \varepsilon > 0 \), so \( G_i(\theta) \) is decreasing when \( \theta < 1 \).
5 Non-monotonicity and free disposal (to be completed)

As we saw in Proposition 4, under the optimal disclosure rule, low types sell their assets for higher prices. This may lead to equilibrium outcomes in which low types end up with higher expected payoff than high types. For example, in Example 1, type \( \theta_4 \) ends up with a higher expected payoff than type \( \theta_3 \). This may be plausible if the bank and the regulator learn \( \theta \) at the same time and the bank cannot affect \( \theta \). However, the equilibrium above may not work if the bank learns its \( \theta \) before the regulator and can “freely dispose” assets. In the latter case, a high type will have strong incentives to increase its equilibrium payoff by destroying assets. An interesting question is whether Proposition 4 continues to hold in this case. We explore this issue in this section.

Specifically, we solve the planner’s problem from Section 4 with the additional constraint that the bank’s equilibrium payoff is weakly increasing in type. We refer to this constraint as the monotonicity constraint and to a solution to this problem as an optimal monotone rule. Optimal monotone rules continue to satisfy Lemma 4.\(^{14}\) Hence, the expected payoff for a bank of type \( \theta \) is as in equation (9). The monotonicity constraint is that for every two types \( \theta' < \theta \),

\[
V(\theta') \leq V(\theta). 
\]  

Interestingly, Proposition 4 may continue to hold even under optimal monotone rules, but to satisfy the monotonicity constraint the probability that a low type sells its asset is reduced. In contrast, Proposition 3 may no longer hold. In particular, it might be optimal that a higher type sells its asset at a price above its reservation price. This increases the payoff for the higher type and relaxes the monotonicity constraint for lower types. We illustrate this through an example. The example also

\(^{14}\)All proofs in this section will be added in a future draft.
that when we do not impose monotonicity, the optimal disclosure rule is such that

\[ \text{Example 3} \]

Suppose that there are two types above 1, \( \theta_1 > \theta_2 \), with reservation prices \( \rho_1 > \rho_2 \geq 1 \), and suppose that the gain-to-cost ratios are increasing. Suppose that when we do not impose monotonicity, the optimal disclosure rule is such that types \( \theta_1 \) and \( \theta_4 \) obtain score \( s_1 \) with probability 1 (and sell at price \( \rho_1 \)) and types \( \theta_2 \) and \( \theta_3 \) obtain score \( s_2 \) with probability 1 (and sell at price \( \rho_2 \)). The other types obtain score \( s_0 \). In this case the monotonicity constraint is violated because \( V(\theta_4) > V(\theta_3) \).

Suppose now that we add the monotonicity constraint (17). We show in the appendix that it continues to be optimal to assign three scores \( s_0, s_1, \) and \( s_2 \), such that \( x(s_1) = \rho_1 \) and \( x(s_2) = \rho_2 \), and that the optimal disclosure rule takes the form

\[
\begin{array}{c|cccc}
\theta_5 & \theta_4 & \theta_3 & \theta_2 & \theta_1 \\
\hline
s_1 & \gamma & \beta & \alpha & \alpha & 1 \\
\hline
s_2 & 1-\alpha & 1-\alpha & & & \\
\hline
s_0 & & & & & \\
\end{array}
\]

Hence, we need to find the optimal \( \alpha, \beta, \) and \( \gamma \).

Under the rule above, the resource constraint for score \( s_2 \) continues to be binding. The probability \( \beta \) can be found from the monotonicity constraint \( V(\theta_3) = V(\theta_4) \). The probability \( \gamma \) can be found from the resource constraint for score \( s_1 \). Hence, the problem boils down to choosing \( \alpha \).

If \( \alpha = 0 \), the monotonicity constraint \( V(\theta_3) = V(\theta_4) \) implies that \( \beta < 1 \), and the resource constraint for score \( s_1 \) implies that \( \gamma > 0 \). When we increase \( \alpha \), we increase the payoff for type \( \theta_3 \), and this allows to increase \( \beta \). However, when \( \alpha \) increases, there is a waste of resources, and the probability \( \gamma \) is reduced. The optimal choice of \( \alpha \) depends on the tradeoff above. For example, if the probabilities \( p(\theta_2) \) and \( p(\theta_3) \) are sufficiently low, wasting resources is second order, and it is optimal to choose \( \alpha > 0 \). Otherwise, it is optimal to choose \( \alpha = 0 \). 

---

\[ ^{15} \text{This rule is optimal if} \ p(\theta_3)(\rho_2 - \theta_3) = p(\theta_2)(\theta_2 - \rho_2) \text{ and } p(\theta_4)(\rho_1 - \theta_4) = p(\theta_1)(\theta_1 - \rho_1). \]

\[ ^{16} \text{Specifically, } p(\theta_3)(\rho_2 - \theta_3) = p(\theta_2)(\theta_2 - \rho_2) \text{ from footnote 15 implies that } (1 - \alpha)p(\theta_3)(\rho_2 - \theta_3) = (1 - \alpha)p(\theta_2)(\theta_2 - \rho_2). \]

\[ ^{17} \text{For example, if there are 5 types } (\theta_5, \theta_4, \theta_3, \theta_2, \theta_1) = (0.7, 0.8, 0.9, 1.1, 1.3), \ r = 0.8, \text{ and } \varepsilon \text{ is uniform on } [-0.4, 0.4], \text{ then choosing } \alpha = 0, \beta = 0.8, \text{ and } \gamma = 0.16 \text{ is optimal when the probabilities } p(\theta) \text{ are given by } p(\theta_5, \theta_4, \theta_3, \theta_2, \theta_1) = (0.4, 0.2, 0.2, 0.1, 0.1). \text{ If instead } p(\theta_5, \theta_4, \theta_3, \theta_2, \theta_1) = (0.4, 0.001, 0.001, 0.1, 0.498), \text{ it is optimal to choose } \alpha = 0.95, \beta = 0.99, \text{ and } \gamma = 0. \]
6 Conclusion

Our paper provides a model of an optimal disclosure policy of a regulator, who has information about banks (e.g., the regulator has conducted stress tests). The regulator’s disclosure policy affects whether banks can take corrective actions, particularly whether banks can engage in risk-sharing arrangements to protect themselves against the possibility that their future capital falls below some critical level. We show that during normal times, no disclosure is necessary, but during bad times, partial disclosure is needed. Partial disclosure takes the form of different scores pooling together banks of different levels of strength. Two scores are sufficient if banks do not have the information that the regulator has. In this case, the optimal disclosure rule may take a simple form, such that banks whose forecasted capital is below some threshold obtain the low score and banks whose forecasted capital is above the threshold obtain the high score; we provide conditions for this to happen. More than two scores may be needed if a bank shares the same information that the regulator has about the bank. In this case, the optimal disclosure rule is non-monotone: among the strong banks, the stronger banks obtain higher scores, but among the weak banks that are pooled with strong banks, the weaker banks obtain higher scores. This type of non-monotonicity continues to hold even if we impose monotonicity on equilibrium payoffs.

Note that the regulator does not lie in our model and that a high score does not necessarily mean that the bank is strong. It only means that, on average, the bank’s cash flow is above the critical level. When the planner gives a score, the planner can also announce the average cash flow for bank receiving the score. In addition, one can think of scores more broadly than just grades. Scores separate banks into groups, and assigning scores is isomorphic to recommending banks which groups to form. For example, one can think of scores as suggesting mergers among banks or joint liability arrangements as in Leitner (2005). We solved for the optimal design of groups under the constraint that each bank prefers to join the recommended group rather than stay
in autarky, and under the assumption that idiosyncratic risk is fully diversified within a group. This might be the case if there is a continuum of banks of each type, or more realistically, if the regulator provides insurance against idiosyncratic risk within a group. We do abstract, however, from other issues of group formation, such as whether a bank receiving one score will attempt to form a link with a bank receiving a different score.

Finally, while our model focuses on the optimal disclosure policy by a regulator, we believe that it can be used as a benchmark to think of credit rating agencies. For example, our model suggests that low types receiving high scores may be a feature of a socially optimal outcome. An interesting question is how the optimal disclosure rule looks like when the regulator faces competition from credit rating agencies, or whether it is optimal to implement risk sharing when the regulator and credit rating agencies have a different objectives. This is left for future research.

References


Appendix

Proof of Lemma 1. From the text, the equilibrium price is $x(s) = \mu(s)$. If the bank sells the asset at price $\mu(s)$, its (expected) final payoff is $R(\mu(s))$. If the bank keeps the asset, its (expected) final payoff, conditional on its information, is $E[R(\tilde{\theta} + \tilde{\epsilon}|\tilde{s} = s)] = \mu(s) + r \Pr(\tilde{\theta} + \tilde{\epsilon} \geq 1|\tilde{s} = s)$. Hence, if $\mu(s) \geq 1$, it is optimal to sell, since $R(\mu(s)) = \mu(s) + r > E[R(\tilde{\theta} + \tilde{\epsilon}|\tilde{s} = s)]$. If $\mu(s) < 1$, it is optimal to keep the asset, since $R(\mu(s)) = \mu(s) < E[R(\tilde{\theta} + \tilde{\epsilon}|\tilde{s} = s)]$. The strict inequality follows from Assumption 1. Q.E.D.

Proof of Lemma 2. The planner’s problem is to find a disclosure rule $(S, g)$ to maximize $\sum_{\theta \in \Theta} p(\theta)u(\theta)$. Since equation (3) reduces to

$$u(\theta) = \sum_{s: \mu(s) < 1} [\theta + r \Pr(\tilde{\epsilon} \geq 1 - \theta)]g(s|\theta) + \sum_{s: \mu(s) \geq 1} [\mu(s) + r]g(s|\theta),$$

it follows that:

$$\sum_{\theta \in \Theta} p(\theta)u(\theta) = \sum_{\theta \in \Theta} p(\theta) \sum_{s: \mu(s) < 1} \theta g(s|\theta) + \sum_{\theta \in \Theta} p(\theta) \sum_{s: \mu(s) < 1} r \Pr(\tilde{\epsilon} \geq 1 - \theta) g(s|\theta)$$

$$+ \sum_{\theta \in \Theta} p(\theta) \sum_{s: \mu(s) \geq 1} \mu(s) g(s|\theta) + \sum_{\theta \in \Theta} p(\theta) \sum_{s: \mu(s) \geq 1} r g(s|\theta).$$

The sum of the first and third terms in the right-hand-side of the equation above reduces to $E(\tilde{\theta})$, as follows:

$$\sum_{\theta \in \Theta} p(\theta) \sum_{s: \mu(s) < 1} \theta g(s|\theta) + \sum_{\theta \in \Theta} p(\theta) \sum_{s: \mu(s) \geq 1} \mu(s) g(s|\theta)$$

$$= \sum_{\theta \in \Theta} \theta p(\theta) \sum_{s: \mu(s) < 1} g(s|\theta) + \sum_{\theta \in \Theta} \mu(s) \sum_{s: \mu(s) \geq 1} p(\theta) g(s|\theta)$$

$$= \sum_{\theta \in \Theta} \theta p(\theta) \sum_{s: \mu(s) < 1} g(s|\theta) + \sum_{\theta \in \Theta} \sum_{s: \mu(s) \geq 1} \theta p(\theta) g(s|\theta)$$

$$= \sum_{\theta \in \Theta} \theta p(\theta) \sum_{s: \mu(s) < 1} g(s|\theta) + \sum_{\theta \in \Theta} \theta p(\theta) \sum_{s: \mu(s) \geq 1} g(s|\theta) = E(\tilde{\theta}),$$
where the third line follows from equation (2). Hence,

\[
\sum_{\theta \in \Theta} p(\theta)u(\theta) = E(\tilde{\theta}) + \sum_{\theta \in \Theta} p(\theta)r \Pr(\tilde{\epsilon} \geq 1 - \theta) \sum_{s: \mu(s) < 1} g(s|\theta) + r \sum_{\theta \in \Theta} p(\theta) \sum_{s: \mu(s) \geq 1} g(s|\theta)
\]

\[
= E(\tilde{\theta}) + \sum_{\theta \in \Theta} p(\theta)r \Pr(\tilde{\epsilon} \geq 1 - \theta)[1 - h(\theta)] + r \sum_{\theta \in \Theta} p(\theta)h(\theta)
\]

\[
= E(\tilde{\theta}) + \sum_{\theta \in \Theta} p(\theta)r \Pr(\tilde{\epsilon} \geq 1 - \theta) + r \sum_{\theta \in \Theta} p(\theta)[1 - \Pr(\tilde{\epsilon} \geq 1 - \theta)]h(\theta)
\]

Hence,

\[
\sum_{\theta \in \Theta} p(\theta)u(\theta) = E(\tilde{\theta}) + r \sum_{\theta \in \Theta} p(\theta) \Pr(\tilde{\epsilon} \geq 1 - \theta) + r \sum_{\theta \in \Theta} p(\theta) \Pr(\tilde{\epsilon} < 1 - \theta)h(\theta) \quad (1)
\]

The first two terms in the right-hand side of (1) are exogenous and are not affected by the planner’s disclosure rule. Only the third term is endogenous and affected by the planner’s disclosure rule. Hence, maximizing \( \sum_{\theta \in \Theta} p(\theta)u(\theta) \) is equivalent to maximizing (4).

From Lemma A-1 below, we can focus, without loss of generality, on disclosure rules with only two scores, \( s_0 \) and \( s_1 \), such that \( \mu(s_0) < 1 \) and \( \mu(s_1) \geq 1 \). From Lemma 1, we know that \( h(\theta) = g(s_1|\theta) \). Hence, the relevant constraint is \( \mu(s_1) \geq 1 \). From equation (2), the constraint \( \mu(s_1) \geq 1 \) reduces to \( \sum_{\theta \in \Theta} p(\theta)(\theta - 1)g(s_1|\theta) \geq 0 \), which is equivalent to constraint (5). Q.E.D.

**Lemma A-1** Assume that the bank does not observe its type. For every disclosure rule \( (S, g) \), we can construct a disclosure rule that induces the same probability that a bank of type \( \theta \) sells its asset (i.e., \( h(\theta) \)) but that uses only two scores, \( s_0, s_1 \), such that \( \mu(s_0) < 1 \) and \( \mu(s_1) \geq 1 \).

**Proof of Lemma A-1.** For a given disclosure rule \( (S, g) \), define a disclosure rule \( (\tilde{S}, \tilde{g}) \), such that \( \tilde{S} = \{s_0, s_1\} \) and such that for every \( \theta \in \Theta, \tilde{g}(s_0|\theta) = \sum_{s: \mu(s) < 1} g(s|\theta) \) and \( \tilde{g}(s|\theta) = \sum_{s: \mu(s) \geq 1} g(s|\theta) \). From Lemma 1, we need to show that \( \mu_{\tilde{g}}(s_1) \geq 1 \) and \( \mu_{\tilde{g}}(s_0) < 0 \), where the subscript \( \tilde{g} \) indicates that the expected values are given
under the optimal disclosure rule, there exists a resource constraint remains unchanged. In particular, since type the original by showing that the alternate rule increases the value of the objective function. In addition, since the first and fourth equalities follow from equation (2) and the second equality follows from the definition of $\tilde{g}$. Similarly, we can show that $\mu_{\tilde{g}}(s_0) < 1$. Q.E.D.

**Proof of Proposition 1.**

Part (A): Assigning $h(\theta) = 1$ for every $\theta \in \Theta$ achieves the maximal attainable value for the objective function and satisfies the planner’s resource constraints. Any other disclosure rule reduces the value of the objective function, by Assumption 1.

Part (B): First, by Assumption 1, it is clearly (uniquely) optimal to set $h(\theta) = 1$ for every $\theta \geq 1$. In addition, if $h(b_j) > 0$ for some $j$, it is optimal to set $h(b_i) = 1$ for every $i < j$. To see why, suppose, by contradiction, that under an optimal disclosure rule there exists $i < j$, such that $h(b_j) > 0$ but $h(b_i) < 1$. Consider a small $\Delta > 0$, let $\Delta' = \frac{P(b_i)}{P(b_j)} \frac{1-b_j}{1-b_i} \Delta$, and consider an alternate disclosure rule in which we increase $h(b_i)$ by $\Delta$ and reduce $h(b_j)$ by $\Delta'$. We obtain a contradiction to the optimality of the original by showing that the alternate rule increases the value of the objective function without violating the resource constraint. In particular, since type $b_i$ has a higher gains-to-cost ratio than type $b_j$, it follows that $\Delta P(b_i) \Pr(\bar{\epsilon} < 1 - b_i) > \Delta P(b_i) \frac{1-b_i}{1-b_j} P(b_j) \Pr(\bar{\epsilon} < 1 - b_j)$, and so the alternate rule increases the value of the objective function. In addition, since $\Delta P(b_i) (b_i - 1) = \Delta \frac{P(b_i)}{P(b_j)} \frac{1-b_i}{1-b_j} P(b_j) (b_j - 1)$, the resource constraint remains unchanged.

Since $\theta_{\text{max}} > 1$, the resource constraint is slack if $h(\theta) = 0$ for every $\theta < 1$. Hence, under the optimal disclosure rule, there exists $i$, such that $h(b_j) > 0$. Denote the
lowest such \( j \) by \( j^* \). Then \( h(b_i) = 0 \) when \( i > j^* \), and it follows from above that \( h(b_i) = 1 \) when \( i < j^* \). Finally, note that if \( j^* \neq l^* \), it is possible to increase the objective function without violating the constraint. Q.E.D.

**Proof of Corollary 1.**

Part 1: Under full disclosure, type \( \theta \) is offered a price \( \theta \), and hence, type \( \theta \) sells its asset if and only if \( \theta \geq 1 \) (Lemma 1). Hence, under full disclosure, \( h(\theta) = 1 \) if and only if \( \theta \geq 1 \). If \( \theta_{\text{min}} \geq 1 \), then \( E(\tilde{\theta}) \geq 1 \) and full disclosure is optimal by the first part of Proposition 1. If \( \theta_{\text{min}} < 1 \), then either \( E(\tilde{\theta}) \geq 1 \), and full disclosure is suboptimal by the first part of Proposition 1, or else \( E(\tilde{\theta}) < 1 \) and full disclosure is suboptimal by the second part of Proposition 1. In particular, under full disclosure, \( h(\theta) = 0 \), for every \( \theta < 1 \), while under the optimal disclosure rule, there must exist \( \theta' > 0 \), such that \( h(\theta') > 0 \). The last statement follows since \( \theta_{\text{max}} > 1 \).

Part 2: Under no disclosure, every bank is offered a price \( E(\tilde{\theta}) \). Hence, it follows from Lemma 1 that under no disclosure, the bank will sell its asset if and only if \( E(\tilde{\theta}) \geq 1 \). Hence, if \( E(\tilde{\theta}) \geq 1 \), we know from the first part of Proposition 1 that no disclosure is optimal. If \( E(\tilde{\theta}) < 1 \), we know from the second part of Proposition 1 that no disclosure is suboptimal because under the optimal disclosure rule, at least some banks sell (since \( \theta_{\text{max}} > 1 \)). Q.E.D.

**Proof of Corollary 2.** From Proposition 1, it is sufficient to show that if condition 1 holds, \( G(\theta) = \frac{F(1-\theta)}{1-\theta} \) is increasing in \( \theta \) whenever \( \theta < 1 \). Denote \( \varepsilon = 1 - \theta \). Then we need to show that \( \frac{F(\varepsilon)}{\varepsilon} \) is decreasing in \( \varepsilon \) whenever \( \varepsilon > 0 \). This follows from condition 1. Q.E.D.

**Proof of Lemma 3.** Suppose a bank is offered a price \( x \), and the bank knows that it is type \( \theta \). If the bank sells the asset, it obtains \( R(x) \). If the bank keeps the asset, it obtains \( E[R(\theta + \tilde{\varepsilon})] \). Hence, the bank sells if and only if

\[
R(x) \geq E[R(\theta + \tilde{\varepsilon})].
\]  
(2)
Observe that $E[R(\theta + \tilde{\xi})] = \theta + r \Pr(\tilde{\xi} \geq 1 - \theta)$, and $R(x) = \begin{cases} x + r & \text{if } x \geq 1 \\ x & \text{if } x < 1 \end{cases}$. Hence, if $\theta \geq 1$, then $E[R(\theta + \tilde{\xi})] \geq 1$, and so equation (2) can hold only if $x \geq 1$. In this case, equation (2) reduces to $x + r \geq \theta + r \Pr(\tilde{\xi} \geq 1 - \theta)$, which reduces to $x \geq \theta - r \Pr(\tilde{\xi} < 1 - \theta)$. If instead $\theta < 1$, then whenever $x \geq 1$, we clearly have $E[R(\theta + \tilde{\xi})] < x + r$, so equation (2) holds; and if $x < 1$, equation (2) reduces to $x \geq \theta + r \Pr(\tilde{\xi} \geq 1 - \theta)$. Q.E.D.

**Proof of Proposition 2.** First observe that since $\theta_{\max} > 1$, the condition $
abla \theta_{\max} - r \Pr(\tilde{\xi} < 1 - \theta_{\max}) \leq 1$ is equivalent to $\rho(\theta_{\max}) = 1$. Since $\rho(\theta)$ is increasing in $\theta$, every type will agree to sell a price 1.

Consider any disclosure rule $(g, S)$. If $\mu(s) \geq 1$, the market price will be $x(s) = \mu(s)$, and every type will sell. If $\mu(s) < 1$, the price must be below 1, since otherwise everyone will sell, and the market will lose money. But then only types below 1 may sell, and the proof of Part 3 in Lemma 4 implies that in equilibrium, no type sells upon receiving score $s$. Hence, Lemma 1 continues to hold, and the bank’s ex-ante expected payoff given disclosure rule $(g, S)$ is the same as in Section 3. Hence, Proposition 1 continues to hold. Q.E.D.

**Proof of Lemma 4**

Part 1. The proof is by contradiction. Consider an optimal disclosure rule $(S, g)$ and suppose there exists a type $\theta' \geq 1$ and a score $s' \in S$, such that $g(s'|\theta') > 0$ and such that type $\theta'$ does not sell its asset upon obtaining score $s'$.

Consider an alternate disclosure rule $(\tilde{S}, \tilde{g})$, in which we add a score $\tilde{s} \notin S$ that type $\theta'$ obtains instead of score $s'$. Specifically, $\tilde{S} = S \cup \{\tilde{s}\}$ and $\tilde{g}(s|\theta) = \begin{cases} g(s|\theta) & \text{if } \theta \neq \theta' \text{ and } s \neq s' \\ 0 & \text{if } \theta = \theta' \text{ and } s = s' \end{cases}$. Under the alternate rule, the only type that obtains score $\tilde{s}$ is $\theta'$. Hence, $x(s') = \theta'$. Since $\rho(\theta') \leq \theta'$, type $\theta'$ sells its asset upon obtaining score $\tilde{s}$. Hence, the alternate rule increases the probability that type $\theta$ invests in its project, while keeping the probabilities that each of the other types invests un-
changed. Hence, the alternate rule increases the bank’s ex ante expected payoff. But this contradicts the optimality of the original disclosure rule \((S, g)\).

Part 2. Consider an optimal disclosure rule \((S, g)\) and suppose there exist a type \(\theta \geq 1\) and a score \(s \in S\), such that \(g(s|\theta) \geq 0\). From part 1, we know that type \(\theta\) sells the asset upon obtaining score \(s\). Hence, \(\rho(\theta) \leq x(s)\). From part 1, we also know that every type \(\theta' > \theta\) such that \(g(s|\theta') > 0\) sells. Finally, every type \(\theta' < \theta\) such that \(g(s|\theta') > 0\) sells, since \(\rho(\theta') < \rho(\theta) \leq x(s)\). Hence, every type that obtains score \(s\) sells the asset upon obtaining the score. Consequently, selling does not convey any additional information to the market, and the market sets a price \(x(s) = \mu(s)\), which is based only on the information that is contained in the score.

Part 3. The proof is by contradiction. (Note that it applies to the equilibrium that is induced by any disclosure rule, not necessarily the optimal.) Suppose that the highest type that obtains score \(s\) is less than 1 (that is, \(g(s|\theta) = 0\) for every \(\theta \geq 1\)), and suppose that the equilibrium that is induced by disclosure rule \(g\) is such that some types sell upon obtaining score \(s\). Denote the highest type that sells by \(\theta'\). (\(\theta' < 1\).) The sale price must satisfy \(x(s) \leq \theta'\), so that the market expected profits are non-negative. Since \(\theta' < \rho(\theta') \leq 1\), we obtain that \(x(s) < \rho(\theta')\). But this contradicts the fact that type \(\theta'\) sells. Q.E.D.

**Lemma A-2** Assume that the bank observes its type. For every disclosure rule \((S, g)\) that is optimal, we can construct a disclosure rule that induces the same probability that a bank of type \(\theta\) sells its asset (and hence, is also optimal) but that uses at most \(k+1\) scores, \(s_0, s_1, s_2, \ldots, s_k\) such that when \(s_i \neq s_0\), the highest type that obtains score \(s_i\) is type \(\theta_i\).

**Proof of Lemma A-2** Suppose \((S, g)\) is an optimal disclosure rule. For every \(i \in \{1, \ldots, k\}\), define \(S_i = \{s : \mu(s) \in [\rho_i, \rho_{i-1})\}\), where \(\rho_0 = \infty\). Let \((\tilde{S}, \tilde{g})\) be a
disclosure rule with $k+1$ scores $\tilde{S} = \{s_0, s_1, s_2, \ldots, s_k\}$, such that for every $\theta \in \Theta,$

$$\tilde{g}(s_i|\theta) = \begin{cases} \sum_{s \in S_i} g(s|\theta) & \text{if } i \in \{1, 2, \ldots, k\} \\ \sum_{s \notin \bigcup_{i=1}^k S_i} g(s|\theta) & \text{if } i = 0 \end{cases}$$

Under disclosure rule $(S, g)$, type $\theta_i \geq 1$ sells the asset upon obtaining score $s$ if and only if $\mu(s) \geq \rho_i$. This happens with probability $\sum_{j=1}^i \sum_{s \in S_j} g(s|\theta)$. Type $\theta < 1$ sells if and only if $\mu(s) \geq \rho_k$, which happens with probability $\sum_{j=1}^k \sum_{s \in S_j} g(s|\theta)$. Following similar steps as in the proof of Lemma A-1, we obtain that (i) $\mu_{\tilde{g}}(s_0) < \rho_k$, and (ii) for every $i \in \{1, 2, \ldots, k\}$, $\mu_{\tilde{g}}(s_i) \in [\rho_i, \rho_{i-1})$. Hence, the probability that type $\theta$ sells the asset under disclosure rule $(\tilde{S}, \tilde{g})$ is the same as under disclosure rule $(S, g)$.

Q.E.D.

**Lemma A-3** Suppose banks know their types. For $i \in \{1, \ldots, k\}$, denote $h_i(\theta) = \sum_{s \in S_i} g(s|\theta)$. The planner’s problem reduces to finding a set of functions $\{h_i : \Theta \rightarrow [0, 1]\}_{i=1, \ldots, k}$ to maximize

$$\sum_{\theta \in \Theta} p(\theta) \Pr(\tilde{\varepsilon} < 1 - \theta) \sum_{i=1}^k h_i(\theta),$$

such that the following constraints hold:

(i) For every type $\theta \in \Theta$,

$$\sum_{i=1}^k h_i(\theta) \leq 1. \quad (4)$$

(ii) For every $i \in \{1, \ldots, k\}$,

$$\sum_{\theta \in \Theta} p(\theta)(\theta - \rho_i)h_i(\theta) \geq 0. \quad (5)$$

(iii) For every $i \in \{1, \ldots, k\}$, $h_i(\theta) = 0$ if $\theta > \theta_i$.

**Proof of Lemma A-3.** Maximizing the bank’s ex-ante expected payoff $\sum_{\theta \in \Theta} p(\theta) u(\theta|g)$ is equivalent to maximizing (3). (The proof is an extension of the proof of Lemma 2.)
More details to be added.) The first constraint says that the probability that a bank obtains a score $s \in \bigcup_{i=1}^{k} S_i$ is at most 1. The second constraint follows by summing the resource constraints for each $s \in S_i$. The third constraint follows from the definition of $S_i$. Q.E.D.

**Lemma A-4** If $E(\bar{\theta}) < 1$, there must be a type $\theta' < 1$ that keeps its asset (i.e., obtains score $s_0$) with a positive probability.

**Proof of Lemma A-4.** The proof is by contradiction. Consider the planner’s problem in Lemma A-3. Suppose that no type obtains score $s_0$ with a positive probability; that is, $\sum_{i=1}^{k} h_i(\theta) = 1$ for every type $\theta \in \Theta$. Then since $\rho_i \geq 1$ for every $k \geq 1$, it follows that $\sum_{i=1}^{k} \sum_{\theta \in \Theta} p(\theta)(\theta - \rho_i)h_i(\theta) \leq \sum_{i=1}^{k} \sum_{\theta \in \Theta} p(\theta)(\theta - 1)h_i(\theta) = \sum_{\theta \in \Theta} p(\theta)(\theta - 1) \sum_{i=1}^{k} h_i(\theta) = E(\bar{\theta}) - 1 < 0$. However, summing up all $k$ resource constraints, we obtain $\sum_{i=1}^{k} \sum_{\theta \in \Theta} p(\theta)(\theta - \rho_i)h_i(\theta) \geq 0$. Hence, a contradiction. Q.E.D.

**Lemma A-5** If $E(\bar{\theta}) < 1$, then under an optimal disclosure rule, all resource constraints are binding.

**Proof of Lemma A-5.** The proof is by contradiction. Suppose $(S, g)$ is an optimal disclosure rule and suppose there exists a score $s$, such that the highest type that obtains score $s$ is $\theta_i$ and such that the resource constraint that is associated with score $s$ is not binding; that is, $\sum_{\theta \in \Theta} p(\theta)(\theta - \rho_i)g(s|\theta) > 0$. Since $E(\bar{\theta}) < 1$, we know from Lemma A-4 that there exists type $\theta' < 1$ that obtains score $s_0$ with a positive probability. Consider an alternate disclosure rule in which the planner reduces the probability that type $\theta'$ obtains score $s_0$ by a small $\Delta$ and increases the probability that type $\theta'$ obtains score $s$ by $\Delta$. The alternate rule increases the value of the objective function without violating any of the constraints. But this contradicts the optimality of the original disclosure rule. Q.E.D.
Proof of Proposition 3. Consider the planner’s problem in Lemma A-3. We can assume, without loss of generality, that \( \rho_1 > \rho_2 > \ldots > \rho_k \). We want to show that if \( E(\bar{\theta}) < 1 \), then \( h_i(\theta_i) = 1 \) for every \( i \in \{1, \ldots, k\} \). The proof is by contradiction. Suppose there exists \( i \in \{1, \ldots, k\} \), such that \( h_i(\theta_i) < 1 \). From Lemma 4, we know that \( \theta_i \) sells its asset with probability 1. Hence, there must be \( j < i \), such that \( h_j(\theta_i) > 0 \). We obtain a contradiction by showing that there is an alternate solution that increases the value of the objective function in Lemma A-3 without violating the constraints.

Case 1: \( \rho_j \geq \theta_i \). Consider alternating the original solution as follows: Reduce \( h_j(\theta_i) \) by a small amount \( \Delta \) and increase \( h_i(\theta_i) \) by the same amount. Since \( \rho_i \leq \theta_i \), increasing \( h_i(\theta_i) \) weakly relaxes the resource constraint \( i \), and since \( \rho_j \geq \theta_i \), reducing \( h_j(\theta_i) \) weakly relaxes the resource constraint \( j \). In addition, at least one of these two constraints is strictly relaxed: if \( \theta_i = 1 \), then \( \rho_j > \theta_i \), and constraint \( j \) is strictly relaxed; otherwise \( \rho_i < \theta_i \), and constraint \( i \) is strictly relaxed. Finally, the value of the objective function and all other constraints remain unchanged. But this contradicts Lemma A-5.

Case 2: \( \rho_j < \theta_i \). In this case, \( \theta_i \) adds resources to the resource constraint \( j \), and reducing \( h_j(\theta_i) \) tightens the constraint. Since the resource constraint \( j \) is binding (Lemma A-5), there must be a type \( \theta'' < \rho_j \), such that \( h_j(\theta'') > 0 \); this type takes resources from constraint \( j \). Fix a small \( \Delta > 0 \) and let \( \Delta' = \frac{\rho(\theta_i)(\theta_i - \rho_j)}{\rho(\theta'')(\rho_j - \theta'')} \Delta \); observe that \( \Delta' > 0 \). Consider an alternate solution in which for type \( \theta_i \), we reduce \( h_j(\theta_i) \) by \( \Delta \) but increase \( h_i(\theta_i) \) by \( \Delta \), and for type \( \theta'' \), we reduce \( h_j(\theta'') \) by \( \Delta' \) but increase \( h_i(\theta'') \) by \( \Delta' \). Under the alternate rule, the probability that each type sells its asset remains unchanged, so the objective function remains unchanged. The resource constraint \( j \) remains unchanged since \(-p(\theta_i)(\theta_i - \rho_j)\Delta - p(\theta'')(\theta'' - \rho_j)\Delta' = 0 \). In contrast, since
\( \rho_j > \rho_i \) (as \( j < i \)), the resource constraint \( i \) is loosened:

\[
 p(\theta_i)(\theta_i - \rho_i) \Delta + p(\theta'')(\theta'' - \rho_i) \Delta' = p(\theta_i)(\theta_i - \rho_i) \Delta + p(\theta'')(\theta'' - \rho_i) \frac{p(\theta_i)(\theta_i - \rho_j)}{p(\theta'')(\theta_j - \theta'')} \Delta \\
> p(\theta_i)(\theta_i - \rho_i) \Delta + p(\theta'')(\theta'' - \rho_j) \frac{p(\theta_i)(\theta_i - \rho_j)}{p(\theta'')(\theta_j - \theta'')} \Delta \\
= p(\theta_i)(\theta_i - \rho_i) \Delta - p(\theta_i)(\theta_i - \rho_j) \Delta \\
= p(\theta_i)(\rho_j - \rho_i) \Delta > 0.
\]

All other constraints remain unchanged. But this contradicts Lemma A-5. Q.E.D.

**Proof of Proposition 4.** Consider the planner’s problem in Lemma A-3. The proof is by contradiction. Suppose \( (h_i)_{i=1,\ldots,k} \) is an optimal solution, such that \( h_i(\theta) > 0 \) for some type \( \theta < 1 \), and suppose, by contradiction, that there exists \( \theta' < \theta \) and \( j > i \), such that \( h_j(\theta') > 0 \). Assume, without loss of generality, that \( \rho_j < \rho_i \).

Fix a small \( \Delta > 0 \), and let \( \Delta' = \frac{p(\theta(\theta - \rho_i))}{p(\theta')(\theta' - \rho_i)} \Delta \); observe that \( \Delta' > 0 \). Consider alternating the original solution as follows: For type \( \theta \), reduce \( h_i(\theta) \) by \( \Delta \) and increase \( h_j(\theta) \) by \( \Delta \). For type \( \theta' \), reduce \( h_j(\theta') \) by \( \Delta' \) and increase \( h_i(\theta') \) by \( \Delta' \). Under the alternate rule, the probability that each type sells its asset remains unchanged, so the objective function remains unchanged. The resource constraint \( i \) remains unchanged since \( -\Delta p(\theta)(\theta - \rho_i) + \Delta' p(\theta')(\theta' - \rho_i) = 0 \). The resource constraint \( j \) is loosened since

\[
\Delta p(\theta)(\theta - \rho_j) - \Delta' p(\theta')(\theta' - \rho_j) = \Delta p(\theta)(\theta - \rho_j) - \Delta' p(\theta')(\theta' - \rho_j) - \frac{p(\theta)(\theta - \rho_i)}{p(\theta')(\theta' - \rho_i)} (\theta' - \rho_j) \\
= \Delta p(\theta)[(\theta - \rho_j) - \frac{p(\theta)(\theta - \rho_i)}{p(\theta')(\theta' - \rho_i)} (\theta' - \rho_j)] \\
= \Delta p(\theta) \frac{\rho_j - \rho_i}{\rho_i - \rho_j} (\theta' - \theta) \\
\Delta p(\theta) \frac{\rho_i - \rho_j}{(\theta' - \rho_i)} > 0,
\]

where the inequality follows since \( \rho_i > \rho_j \geq 1 > \theta > \theta' \). All other constraints remain unchanged. So the alternate solution gives the same value for the objective but relaxes one of the resource constraints. But this contradicts Lemma A-5. Q.E.D.
Figure 1: The figure illustrates the reservation price $\rho(\theta)$ as a function of $\theta$.

Figure 2: The figure illustrates the gain-to-cost functions that are associated with the highest score $s_1$ and the second highest score $s_2$. 