Does Audit Transparency Improve Audit Quality and Investment Efficiency?*

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Abstract

We examine the effects of disclosing information about the precision of audit opinion (i.e., audit transparency) on auditor quality and investment efficiency in a setting where audit quality is affected by auditor’s effort, motivated by auditor’s liability in the event of audit failure. We show that while higher audit transparency enhances the information decision usefulness of financial reports for investors, it can also adversely affect auditor’s incentives, and consequently lowers expected audit quality and investment efficiency. We show that the underlying quality of financial reporting is an important determinant for this tradeoff, and the case for audit transparency is weaker when the underlying financial reporting quality is high. Our analysis also suggests that it is useful to evaluate audit liability rules in conjunction with audit disclosure rules.

1 Introduction

We analytically evaluate and compare audit quality and investment efficiency between audit regimes with different disclosure rules regarding audit opinions. We define an audit opinion

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as high quality (and more useful to investors) when it can more accurately reveal managerial misreporting. Accordingly, we define an audit regime as more transparent when it requires disclosures of information to help investors assess the level of audit quality.

The paper is primarily motivated by the on-going debate on whether more information should be disclosed by auditors in addition to their summary evaluation on the fairness of their clients’ financial reports. This debate dates back to those proposals in the 1990s to require disclosure of the materiality threshold used by auditors in reaching their audit opinions (see Patterson and Smith (2003)). It is also directly related to the current initiatives by Public Company Accounting Oversight Board (PCAOB) and other regulatory bodies around the world that call for more disclosure in audit opinions to help investors assess their accuracy/reliability. While the content of the additional disclosure requirement at debate depends on the specific initiatives/proposals, the general idea is that more information should assist investors to evaluate the usefulness of audit opinions. Proponents argue that more information not only assists investors’ investment decision, it can also provide stronger incentives for auditors to exert more effort in order to improve audit quality. Opponents, however, argue that the additional information may induce undue reliance by investors in making investment decisions, while at the same time may increase audit costs and auditor’s liability.

The paper also aims to contribute to the theoretical literature on how audit regulations affect audit quality and the usefulness of financial reporting. While most prior studies focus on the effects of various auditor liability rules (e.g., Dye (1993), Narayanan (1994), Hillegeist (1999), etc.), we examine the effects of auditor disclosure rules. Both liability and disclosure rules play important yet distinct roles in affecting the overall information environment. Audit liability rules affect the information environment indirectly by affecting

1For example, one proposed disclosure item is critical audit matters that "posed the most difficulty to the auditor in obtaining sufficient appropriate audit evidence or forming an opinion on the financial statements." The proposal is outlined in PCAOB Release No. 2013-005, August 13, 2013. Similar initiatives are also being evaluated outside the US by International Auditing and Assurance Standards Board (IAASB), the Financial Reporting Council (FRC) of United Kingdom, and the European Commission (EC).
auditors’ incentives, whereas the effects of disclosure rules can be both direct (more disclosure provides investors with more information) and indirect (by affecting auditor’s incentives). In addition, auditor liability rules are part of the legal systems, whereas auditor disclosure rules are often set by audit standard setters (e.g., PCAOB). As such, studying the effect of auditor disclosure rules can also help us better understand how financial reporting quality, as embodied in the financial reporting standards (e.g., U.S. GAAP and IAS), and auditing quality, as embodied in auditing standards, jointly affect the decision usefulness of financial information for investors, and how effort to improve one set of standards (GAAP or GAAS) depends on the quality of the other. Answer to these questions are important not only for standard setters, to the extent that auditing standards differ across jurisdictions, they can also shed light on empirical findings related to the cross-country differences in the effects of adopting the same set of accounting reporting standards (e.g., IFRS).\(^2\)

We address these issues in a setting where investors decide whether to invest in a firm based on both their private information signal about the firm’s future prospect and an audited financial report. Our model explicitly recognizes that the usefulness of an audited financial report depends on both the quality of the underlying financial reporting in the firm’s true fundamentals (i.e., financial reporting risk), and the quality of an audit as captured by the likelihood that the firm’s auditor discovers managerial misreporting of true accounting signals (i.e., detecting risks).\(^3\) We allow audit quality to depend on auditor’s effort but the audit technology is imperfect in that higher effort only increases audit quality in expectation. We assume the auditor’s effort is motivated by the auditor’s liability in the event of an audit failure, which occurs when the auditor’s opinion fails to catch managerial misreporting and

\(^2\) See, for example, Armstrong et al. (2010), Li (2010), and Daske, et al. (2013).

\(^3\) Two other concepts of risk/uncertainty are often studied in the literature: the firm/client’s business risk (which refers to the likelihood that the client/firm’s fundamental is good or bad), and the auditor’s reporting risk (which refers to the likelihood that the auditor’s reported opinion differs from his underlying evidence). In our setting, financial reporting risk is the likelihood that accounting report captures the true fundamental, conditional on the realization of the fundamental. Our model also has features similar to those in Hillegeist (1999) such that auditors will always honestly report their evidence and therefore there is no reporting risk in our model. Lu (2005) and Lu and Sapra (2009) are examples of studies on reporting risks.
investors’ investment project fails. Thus, a key trade-off in our setting is between potential inefficient uses of information by investors (over-weighing the audit opinion because they collect the auditor’s liability as damage compensation for failed investment) and effort motivation for auditors (because investors’ reliance on the audit opinion is a necessary condition for the auditor to exert effort).

We study and compare two disclosure regimes regarding audit quality: a Disclosure regime where required disclosure by the auditor is sufficient to assist investors to assess the audit quality for both qualified and unqualified opinions; and a No Disclosure regime where investors can obtain information about audit quality only for qualified opinions.

Our first main result is with regard to audit quality. We find that the equilibrium effort level is higher under the Disclosure regime than under the No Disclosure regime only when the quality of the underlying financial report is relatively low. The intuition comes from the fact that investors’ reliance on the auditor’s opinion is a necessary condition for audit failure, and therefore acts as an incentive mechanism for motivating the auditor’s effort. The effectiveness of this mechanism depends on whether investors’ reliance is primarily driven by the audit opinion’s informativeness value (i.e., investors try to glean the firm’s future prospect from available information) or by its insurance value (i.e., investors attempt to use information to predict the audit failure where damage compensation is received). Information about audit quality allows investors to fine-tune their reliance upon the auditor’s opinion (versus their private signal). When the underlying reporting quality is low, investors primarily rely on the audit opinion for its insurance value, more (less) so when the realized audit quality is low (high). As a result, investment (and hence audit failure) is more sensitive toward the auditor’s effort, enhancing the auditor’s incentives to exert effort. On the other hand, when the underlying reporting quality is high, investors rely on the audit opinion primarily for its informativeness value, and therefore are less likely to invest when the realized audit quality is low. Since investment is a necessary condition for audit failure, this implies that from the auditor’s perspective, lower audit quality can reduce expected liability, muting the auditor’s incentives to exert effort. In contrast, investors cannot fine-tune their decisions based on observed audit quality under the No Disclosure regime, which results in higher equilibrium
Our second main result is with respect to investment efficiency. Here we show that the effect of audit transparency on investment efficiency also depends on the underlying accounting quality, and on whether investors rely on the audit opinion for its informativeness value or its insurance value. When investors are primarily concerned with the opinion’s informativeness value, the fine-tuning allowed by the Disclosure regime improves investment efficiency. However, this improvement comes at the expense of audit quality, as discussed above. On the other hand, when investors are primarily concerned with the opinion’s insurance value, the fine-tuning allowed by the Disclosure regime improves auditor effort (and expected audit quality). However, it exacerbates investors’ over-weighting of the auditor’s opinion, which can in turn lower investment efficiency. The resulting comparison is a complex tradeoff of these forces. Numerical examples suggest that on the net, investment efficiency is lower under the Disclosure regime than under the No Disclosure regime when the underlying reporting quality is high.

To assess the robustness of our results, we also analyze a simplified setting by endogenizing the optimal liability rule under each regime. We find that while an interior optimal liability rule can motivate effort and restore investment efficiency under the No Disclosure regime, a similar rule cannot be found under the Disclosure regime for all levels of underlying reporting quality. Specifically, when the underlying reporting quality is high, the optimal auditor’s liability may be muted in order to minimize the distortion caused by investors’ over-weighting of the audit opinion due to its insurance value.

Our study belongs to the broad literature on understanding how audit rules and regulations affect market participants’ behaviors (e.g., Dye (1993), Narayanan (1994), Hillegeist (1999), etc.), and more specifically, the literature evaluating their effects on audit quality and investment efficiency (e.g., Schwartz (1997), Pae and Yoo (2001), Deng, Melumad, and Shibano (2011)). While most prior studies focus on the effect of audit liability rules, we contribute to the literature by examining the effect of audit disclosure rules (i.e., audit trans-

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4See also Newman, et al. (2005) and Deng, Melumad, and Shibano (2011) for reviews of related literature.
Our analysis shows that these two types of regulations have different impacts on audit quality and investment efficiency and their effects cannot be offset.

Modeling wise, our focus on disclosure rules also implies that our setting differs in several key aspects from existing studies. First, while most prior studies focus on strategic interactions between managers and auditors, we focus on the consequences of strategic interactions between auditors and investors. In our setting, investors base their investment decisions on both the audited accounting report and their private information, and auditors perform only the attestation role (i.e., their job is to verify whether managers truthfully report the underlying accounting signals, regardless of how informative the accounting signals are). Second, whereas prior studies assume that information about auditor quality is known (or is inferrable in equilibrium), we allow the possibility that different disclosure requirements for auditors affect investors’ assessment of audit quality. While these distinctions are necessary ingredients to examine the effect of audit transparency on audit quality and investment efficiency, they also provide an opportunity to shed light on the interaction between the accounting reporting regime and audit reporting regime, and their joint effect on audit quality and investment efficiency. An important insight from our analysis is that the effect of improving financial reporting quality depends crucially on the auditing rules, both those related to audit liability and those related to audit disclosure.

The rest of the paper proceeds as follows. Section 2 sets up the basic model. Section 3 provides the main analyses. Section 4 evaluates alternative liability rules and Section 5 concludes.

Dye and Sridhar (2007) analyzes how a firm’s existing owners’ disclosure precision choice is changed when this choice becomes publicly observable in an overlapping generation model. While the driving force in their paper is the optimal risk sharing between different generations of owners, we look at how audit disclosure regulation interacts with the underlying financial reporting quality in affecting auditor effort provision and investment efficiency.
2 Model Setup

Consider a representative firm that has access to an investment project. The project requires an up-front investment $K$ and yields a random payoff depending on the underlying state of nature. If undertaken, the project generates a terminal cash flow of either $RK$ (with $R > 1$) when the state of nature is good (denoted by $G$) or 0 when the state is bad (denoted by $B$). The common prior for a good state is $\mu = \Pr (G) \in (0, 1)$. Without loss of generality, we assume the \textit{ex ante} net present value of the project (before any information signal becomes available) is zero, implying that $R = \frac{1}{\mu}$.

There are three types of risk-neutral players: a group of potential investors, a manager, and an auditor. Investors decide whether to invest $K$ to fund the project whose payoff directly accrues to them. The manager receives an incremental private benefit $\lambda > 0$ only when investors decide to take the project. We normalize investors’ payoff to zero in the case when the project is rejected.

The state of nature is initially unknown to everyone. It can be partially revealed by a noisy signal from the firm’s accounting system, denoted as $R \in \{R_G, R_B\}$ with the probability structure of

$$p(R_G|G) = p(R_B|B) = q \in \left(\frac{1}{2}, 1\right).$$

The higher $q$ is, the more accurately the accounting signal captures the underlying state. Therefore, $q$ is a measure for the quality of the accounting system, which is exogenously determined by the prevailing financial reporting rules and standards (e.g., GAAP).

\footnote{Each investor would invest his share of the total investment. Since investors have identical preferences and information structure, it is without loss of generality that we treat them as a collective group who decides whether to invest the total amount.}

\footnote{An alternative interpretation for our setting is that investors decide whether to purchase a firm’s stocks, either through an equity issuance by the firm or from the secondary market. Investors rely on the firm’s accounting and auditing reports to assess the firm’s prospect. The firm (either manager or existing shareholders) prefers that investors choose to invest, either because the manager enjoys empire building, or because the existing shareholders prefer a higher share price or better liquidity in case they need to liquidate their holdings.}
The manager privately observes the accounting signal $R$, after which he proposes a report $\hat{R} \in \{\hat{R}_G, \hat{R}_B\}$ to the auditor and investors. Report $\hat{R}_G$ ($\hat{R}_B$) claims that the privately observed accounting signal is $R_G$ ($R_B$). We assume that the manager’s private benefit $\lambda$ from undertaking the project is sufficiently large, such that he strictly prefers a favorable report $\hat{R}_G$ to a unfavorable one $\hat{R}_B$. While the assumption is a simplification, it is needed to allow a role for the auditor. If it is public knowledge that managers always truthfully reveal $R$, auditors are not needed in the first place.

After observing the manager’s report $\hat{R}$, the auditor spends resources and exerts effort, denoted by $e \in [0, 1]$, to collect audit evidence $\Omega \in \{\Omega_g, \Omega_b\}$ to verify the accounting signal. The auditing technology is imperfect and correctly reveals the underlying accounting signal only with probability $\gamma$:

$$p(\Omega_g|R_G) = p(\Omega_b|R_B) = \gamma.$$  

$\gamma$ reflects the notion of audit quality: the higher $\gamma$ is, the more likely audit evidence reveals the underlying accounting signal, the more likely the auditor can detect manager’s misreporting. Without loss of generality, we assume that there are two levels of audit quality $\gamma \in \{\gamma_h, \gamma_l\}$ with $1 \geq \gamma_h > \gamma_l \geq \frac{1}{2}$ and that higher auditor’s effort can stochastically improve the audit quality in that $\Pr(\gamma = \gamma_h) = e$ and $\Pr(\gamma = \gamma_l) = 1 - e$. The auditor privately observes $e$ and $\gamma$. She also bears the cost of effort, given by $C(e)$, with $C' \geq 0$, $C'' > 0$, $C'(0) = 0$ and $C'(1) = \infty$.

After observing evidence $\Omega$, the auditor issues an audit opinion, denoted by $AO \in \{U, Q\}$ where $U$ stands for an unqualified opinion and $Q$ for a qualified opinion. We assume that the auditor can issue a qualified opinion only when her evidence supports it (i.e., $\Omega = \Omega_b$). This is consistent with the practice that a qualified opinion usually is accompanied with detailed.

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8Our results are qualitatively unchanged if we allow stochastic misreporting by the manager. Stochastic misreporting can be introduced in two ways. First, we can allow the manager to choose $t \in [0, \bar{t}]$ such that $\Pr(\hat{R}_G|R_b) = t$, where $\bar{t} < 1$ is an exogenous upper bound on the manager’s misreporting. It is easy to see that in this case the manager will optimally choose $t$. Second, we can alternatively do away with the exogenous bound $\bar{t}$ and instead introduce an increasing convex cost $C(t)$ to manager’s misreporting. Our results are robust to both modeling alternatives.
discussions and hence is likely to be based on evidence collected.\footnote{As will become clear next, if auditors are allowed to issue qualified opinions upon observing $\Omega_g$, they will always do so to avoid audit failure and potential liability. This will lead to a trivial equilibrium where auditors exert no effort, always issue qualified opinions, and investors never rely on auditor’s opinions. Anticipating this, the firm/investors would not pay for the auditor’s service to begin with. Alternatively, Lu and Sapra (2008) assume an exogenous cost from qualified opinions. The nature of audit evidence in their model differs from ours. In their model, the auditor either knows for sure whether manager lied, or is left uncertain. In the latter case, auditor needs to decide whether to issue qualified or unqualified opinion. In our model, auditor never knows for sure whether manager lied and their opinion can only be based on their audit evidence.}

Investors observe both the manager’s report and the auditor’s opinion. In addition, investors collectively have access to a noisy signal of their own $S \in \{S_g, S_b\}$ that is informative of the underlying state with

$$p(S_g | G) = p(S_b | B) = p \in \left[ \frac{1}{2}, 1 \right].$$

$p$ reflects the quality of investors’ signal and is itself a random variable, uniformly distributed on $[\frac{1}{2}, 1]$. $p$ and $S$ are realized and privately observed by investors after the auditor chooses her effort $e$ and issues her opinion. Investors then decide whether to invest in the project based on information available to them.

The auditor gets a non-contingent fee $F$ from the firm at the beginning of their relationship. We assume a competitive audit market such that the audit fee is set to equal the auditor’s cost of effort and expected liability in the event of an audit failure.\footnote{Once set, $F$ doesn’t affect any subsequent behaviors. Since the focus of our analysis is not on the audit fee, we treat it as a known parameter.} An audit failure occurs when investors choose to invest and the state turns out to be $B$; and at the same time, the accounting signal correctly captures the state (i.e., $R = R_B$) but the auditor fails to detect managerial misreporting by issuing an unqualified opinion.

We assume that in the event of an audit failure, the auditor’s liability is $\alpha K$ which accrues to investors as damage compensation. $\alpha \in (0, 1)$ is a known parameter that reflects the severity of the auditor’s liability. For expositional ease, in our main setup we will treat $\alpha$ as exogenous and doesn’t allow it to vary with either the auditing regulatory regime (to
be discussed below) or the quality of the underlying accounting system \( q \). We will extend our model to endogenize \( \alpha \) in section 4.

Alternatively, one can model the auditor’s liability as a function of realized \( \gamma \)’s. For this arrangement to be implementable, the court not only needs to be able to verify the \textit{ex post} realization of \( \gamma \)’s but also has to know the exact space of all possible \( \gamma \)’s. Therefore, making the auditor liability depend only on investors’ investment amount \( K \) as our model formulates is equivalent to assuming an informationally constrained court. Our results are not qualitatively affected if the court sets \( \alpha \) as a function of a noisy signal (of the realized \( \gamma \)) that it can observe \textit{ex post}.

We study two auditing regulatory regimes, a No Disclosure regime \((ND)\) and a Disclosure regime \((D)\), that differ the amount of information available to investors in assessing audit quality \((\gamma)\). Specifically, in the No Disclosure regime, auditors are required to provide additional information that can help investors assess audit quality only when they issue qualified opinions.\textsuperscript{11} The No Disclosure regime corresponds to the existing regulatory requirement that the auditor provides a pass/fail assessment in her opinion and is required or expected to provide further information only in the case of qualified opinions. In contrast, in the Disclosure regime, auditors need to provide such information for \textit{both} qualified and unqualified opinions. This regime corresponds to the PCAOB’s proposed regulation that auditors need to discuss "critical audit matters" to provide more information to investors regarding how confident they are with respect to their opinions. In our model, information on auditors’ confidence regarding their opinions corresponds to the information about whether audit quality \((\gamma)\) is high or low.

The timeline of the model is summarized below:

- Date 1. The auditor is hired and paid with a non-contingent fee \( F \). The firm installs its accounting information system (the quality of which is \( q \)). Nature chooses the state \( G \) or \( B \).

\textsuperscript{11}Our results are unaffected under the alternative (presumably less empirically descriptive) assumption that investors do not observe audit quality in all cases (including with a qualified opinion) in the No Disclosure regime.
- Date 2. \( R \in \{R_G, R_B\} \) is generated by the accounting system. The manager privately observes \( R \) and reports \( \hat{R}_G \) to the auditor and investors.

- Date 3. The auditor determines her effort \( e \) and issues her opinion based on collected evidence.

  - In the No Disclosure regime, \( \gamma \) is disclosed only if a qualified opinion is issued.
  - In the Disclosure regime, \( \gamma \) is disclosed for both qualified and unqualified opinions.

- Date 4. Investors observe their private information (\( p \) and \( S \)) and make investment decisions.

- Date 5. The state of nature is revealed. Project payoff is realized and distributed. Auditor’s liability is assessed.

Figure 1 illustrates the information structure modeled in the paper. Figure 1A shows the auditor’s audit evidence \( \Omega \), while Figure 1B corresponds to investors’ signal \( S \).

Figure 1a: Auditor’s Signal

Figure 1b: Investor’s Signal

Fig 1 Graphical Illustration of Auditor’s and Investor’s Signal
We next define the equilibrium concept for our model.

**Definition 1** Let \( i \in \{D, ND\} \) denote the auditing regime. An equilibrium in regime \( i \) consists of the auditor’s effort choice \( e_i^* \) and opinion issued \( AO_i^* (\Omega) \in \{U, Q\} \), investors’ conjecture of auditor effort \( \hat{e}_i \), investors’ information set \( \Phi_i \) and investment decision function \( I_i^* (\Phi_i) \) such that

1. \( \{e_i^*, AO_i^* (\Omega)\} = \arg \min_{\{e, AO\in\{U,Q\}\}} \mathbb{E} \left[ \alpha K (B, R_B, \hat{R}_G, AO = U) + C(e) | I_i^* (\Phi_i) \right] \), where \( 1 (\ast) \) is an indicator function taking a value of 1 if and only if event \( \ast \) occurs;

2. \( I_i^* (\Phi_i) \) generates an optimal investment decision that maximizes investors’ payoff based on investors’ information set \( \Phi_i \). Specifically, investors’ information set in each auditing regime is given by

\[
\Phi_D = \left( \hat{R}_G, p, S, \hat{e}, AO, \gamma \right) \quad \text{and} \\
\Phi_{ND} = \left( \hat{R}_G, p, S, \hat{e}, AO, \gamma \right) \text{ if } AO = Q, \quad \hat{\gamma} \text{ if } AO = U
\]

where \( \hat{\gamma} = \hat{e} \gamma_h + (1 - \hat{e}) \gamma_l \).

That is, with the No Disclosure regime investors form a conjecture \( \hat{\gamma} \) about \( \gamma \) for an unqualified opinion based on their conjecture of auditor’s effort \( \hat{e} \), i.e., \( \hat{\gamma} = \hat{e} \gamma_h + (1 - \hat{e}) \gamma_l \); and

3. In equilibrium, investors’ conjecture is confirmed, i.e., \( \hat{e} = e \).

We evaluate the social welfare consequences of different regimes using a measure of *ex ante* investment efficiency, defined below.

**Definition 2** Investment Efficiency is denoted by \( IE \) and equals

\[
IE = -\mu \Pr (\text{Project Rejected}|G) (R - 1) K - (1 - \mu) \Pr (\text{Project Taken}|B) K.
\]

In other words, investment efficiency reflects the expected loss from investment decisions. It decreases with the expected Type I (i.e., a profitable project not taken) and Type II (i.e., an unprofitable project taken) loss from the project. The loss conditional on the Type I and Type II error is \( (R - 1) K \) and \( K \), respectively.
3 Main Results

3.1 Auditor’s opinion decision

We start with the auditor’s optimal opinion decision. Under the assumption that the auditor cannot issue a qualified opinion when her evidence is \( \Omega_g \), we only need to examine whether the auditor has incentive to issue unqualified opinion when her audit evidence is \( \Omega_b \). Since her effort is sunk at this stage, the auditor is only concerned about her expected liability. Therefore, she will optimally issue a qualified opinion, as otherwise she would be exposed to possible liability. Note that this will be the case in both disclosure regimes. This observation is stated in the following lemma; the proof is straightforward and hence omitted.

Lemma 1 Under both disclosure regimes, the auditor issues a qualified opinion if and only if her evidence is \( \Omega_b \).

3.2 Investors’ investment decision

We next examine investors’ investment decision. Investors will take the project if their expected payoff is larger than the upfront cost of \( K \). The expected payoff consists of two components: the project’s terminal cash flow (i.e., \( RK \)) when the underlying state is good, and the damage compensation (i.e., \( \alpha K \)) when an audit failure occurs. It follows that investors will take the project under ND if and only if

\[
RK \Pr (G|\hat{R}_G, p, S, AO, \hat{\gamma}) + \alpha K \Pr (B, R_B|\hat{R}_G, p, S, AO = U, \hat{\gamma}) 1 (AO = U) \geq K.
\]

Similarly, investors will take the project under D if and only if

\[
RK \Pr (G|\hat{R}_G, p, S, AO, \gamma) + \alpha K \Pr (B, R_B|\hat{R}_G, p, S, AO = U, \gamma) 1 (AO = U) \geq K.
\]

Holding everything else constant, investors’ payoff is higher when they obtain damage compensation than when they do not, leading to the possibility of suboptimal uses of information by investors as far as investment efficiency is concerned. To facilitate later discussion, we first analyze investors’ investment decision in a benchmark case where \( \alpha = 0 \) and summarize the results in Lemma 2 below.
Lemma 2 Let
\[ \tilde{\gamma} = \begin{cases} \hat{\gamma}, \text{with an unqualified opinion under ND;} \\ \gamma, \text{with a qualified opinion under ND;} \\ \gamma, \text{under } N = D. \end{cases} \]
and define
\[ p^*(\gamma) = \tilde{\gamma}q + (1 - \tilde{\gamma})(1 - q). \tag{4} \]

When \( \alpha = 0 \), investors’ optimal investment decision is given by

<table>
<thead>
<tr>
<th>Scenario where ( \Phi = )</th>
<th>Investment Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( (\hat{R} = \hat{R}_G, AO = Q, S = S_b) )</td>
<td>Not invest</td>
</tr>
<tr>
<td>2. ( (\hat{R} = \hat{R}_G, AO = U, S = S_g) )</td>
<td>Invest</td>
</tr>
<tr>
<td>3. ( (\hat{R} = \hat{R}_G, AO = Q, S = S_g) )</td>
<td>Invest iff ( p \geq p^*(\tilde{\gamma}) )</td>
</tr>
<tr>
<td>4. ( (\hat{R} = \hat{R}_G, AO = U, S = S_b) )</td>
<td>Invest iff ( p \leq p^*(\tilde{\gamma}) )</td>
</tr>
</tbody>
</table>

Lemma 2 establishes the First Best solution for investment efficiency, because investors’ payoff comes only from the project’s terminal cash flow and therefore their objective is simply to maximize investment efficiency. In Scenarios (1) and (2) where the auditor’s opinion and investors’ signal are consistent with each other, it is optimal for investors to follow what these signals suggest: no investment in Scenario (1) when both signals suggest the state is bad and invest in Scenario (2) when both signals suggest the state is good. In these scenarios, information about audit quality is irrelevant.

In Scenarios (3) and (4) where investors’ signal conflicts with the auditor’s opinion, investors optimally follow the signal that is more informative about the underlying state. Specifically, investors compare \( p \) (the precision/informativeness of their own signal \( S \) with respect to the state) with the informativeness of the auditor’s opinion with respect to the state, which is defined as the likelihood of observing audit evidence \( \Omega_j \in \{\Omega_q, \Omega_g\} \) conditional on the state of \( j \in \{G, B\} \):

\[
\Pr (\Omega_j|j) = \Pr (\Omega_j|R_j) \Pr (R_j|j) + \Pr (\Omega_j|R_{-j}) \Pr (R_{-j}|j) \\
= \gamma q + (1 - \gamma)(1 - q) = p^*(\gamma).
\]
When $\alpha > 0$, investors rely on auditor’s opinion not only for its informative value effect in predicting the project’s terminal cash flow, but also for its insurance value effect (i.e., obtaining damage compensation from the auditor when an audit failure occurs). Since an auditor failure can possibly happen only if the auditor issues an unqualified opinion and the project is taken, this insurance effect biases investors’ investment decision away from the First Best, when the auditor issues an unqualified opinion. Proposition 1 below summarizes investors’ optimal investment rule with $\alpha > 0$.

**Proposition 1** Let $\tilde{\gamma}$ be as defined in Lemma 2. When $\alpha > 0$, investors’ optimal investment decision is given by

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<td>Invest</td>
</tr>
<tr>
<td>3. $\left( \hat{R} = \hat{R}_G, AO = Q, S = S_g \right)$</td>
<td>Invest iff $p \geq p^*(\tilde{\gamma})$</td>
</tr>
<tr>
<td>4. $\left( \hat{R} = \hat{R}_G, AO = U, S = S_b \right)$</td>
<td>Invest iff $p \leq \bar{p}(\tilde{\gamma})$</td>
</tr>
</tbody>
</table>

where

$$\bar{p}(\tilde{\gamma}) \equiv p^*(\tilde{\gamma})\rho(\tilde{\gamma}, \alpha, q)$$

with $\rho(\tilde{\gamma}, \alpha, q) \equiv \frac{1}{1 - \alpha q (1 - \tilde{\gamma})} > 1$. (5)

As expected, here investors deviate from the First Best investment rule by over-weighing the auditor’s unqualified opinion and under-weighing a conflicting signal $S$. Specifically, the investment threshold $\bar{p}(\tilde{\gamma}) = p^*(\tilde{\gamma})\rho(\tilde{\gamma}, \alpha, q)$ is larger than the First Best threshold $p^*(\tilde{\gamma})$ in scenario 4 which is the only situation where the auditor is potentially liable. Thus, $\rho(\tilde{\gamma}, \alpha, q)$ reflects the auditor opinion’s insurance value. Holding $\tilde{\gamma}$ constant, the larger the auditor’s liability (higher $\alpha$), the larger $\rho(\tilde{\gamma}, \alpha, q)$ is and the higher the insurance value of the auditor’s opinion.

Proposition 1 applies to both the Disclosure regime and the No Disclosure regime. The main difference between the two regimes is the specific value of $\tilde{\gamma}$ at which the investment
threshold \( \bar{p}(\hat{\gamma}) \) is evaluated at. In the Disclosure regime, \( \bar{p}(\hat{\gamma}) \) depends on the actual \( \gamma \) observed; whereas in the No Disclosure regime, \( \bar{p}(\hat{\gamma}) \) is evaluated at investors’ conjectured audit quality \( \hat{\gamma} \) as defined in (3) if and only if the auditor issues an unqualified opinion.

Note that although the presence of audit liability may lead to inefficient uses of information by investors, it does not necessarily imply that the overall investment efficiency is lower than without any audit liability. This is because without audit liability, the auditor clearly will not exert any effort in the model, reducing the overall investment efficiency.

### 3.3 Effects of audit transparency

#### 3.3.1 No disclosure regime

We first determine the auditor’s effort choice in the No Disclosure regime. The auditor chooses effort \( e \) to minimize both the cost of effort and the expected liability, given investors’ investment decision. Conditional on \( \gamma \) and investors’ conjectures \( \hat{e} \), the auditor’s probability assessment of an audit failure is:

\[
\Pr(\text{audit failure} \mid \gamma, \hat{\gamma}) = l(q, \gamma, \mu) \bar{p}^2(\hat{\gamma})
\]

where \( l(q, \gamma, \mu) \equiv (1 - \mu)(1 - \gamma)q \)

and \( \hat{\gamma} = \hat{e} \gamma_h + (1 - \hat{e}) \gamma_l. \) (9)

From the auditor’s perspective, the likelihood of an audit failure depends on two events: *auditor vulnerability* and *investors reliance*. *Auditor vulnerability* refers to the event where the auditor issues an unqualified opinion in the bad state \( (B) \) and the underlying accounting signal is accurate \( (R = R_B) \). The probability of this event is given by \( l(q, \gamma, \mu) \), which is decreasing in \( \gamma \), suggesting a benefit for the auditor to exert effort to reduce her vulnerability.

The event of *investor reliance* happens when investors rely on the auditor’s unqualified opinion and take the project in the bad state. Such an event can happen in two situations: in scenario 2 when investors also receive an erroneous signal \( S_g \), the *ex ante* probability of which is \( \int_{1/2}^{1} 2(1 - p)dp = \frac{1}{4} \); and in scenario 4 when investors ignore \( S_b \), the *ex ante* probability of which is \( \int_{1/2}^{p(\hat{\gamma})} 2pdp = \bar{p}^2(\hat{\gamma}) - \frac{1}{4} \). The sum of the two probabilities is simply
Given $\Pr (\gamma = \gamma_h) = e$, the auditor’s total expected cost for a given effort level $e$ is

$$[e \Pr (\text{audit failure} | \gamma_h, \hat{x}) + (1 - e) \Pr (\text{audit failure} | \gamma_l, \hat{x})] \alpha K + C(e).$$  \hspace{1cm} (10)

The first term reflects the expected liability and the second term the cost of effort. The auditor’s equilibrium effort choice is solved by choosing $e$ to minimize (10) and is summarized in Proposition 2 below.

**Proposition 2** Under the No Disclosure regime,

(a) given investors’ conjecture $\hat{e}$, the auditor’s optimal effort choice is determined by

$$\alpha K [l(q, \gamma_l, \mu) - l(q, \gamma_h, \mu)] p^2(\hat{x}) = C'(e).$$  \hspace{1cm} (11)

Imposing the rational expectation equilibrium condition, the auditor’s equilibrium effort $e^*_{ND}$ is characterized by

$$\alpha K [l(q, \gamma_l, \mu) - l(q, \gamma_h, \mu)] p^2(e^*_{ND} \gamma_h + (1 - e^*_{ND}) \gamma_l) = C'(e^*_{ND})$$  \hspace{1cm} (12)

and strictly lies between 0 and 1;

(b) there exists at least one stable equilibrium under the No Disclosure regime;

(c) $\frac{de^*_{ND}}{dq} > 0$ for any stable equilibrium;

(d) there exists a $\gamma_0 > 0$ such that $\forall \gamma_h < \gamma_0$, the investment efficiency strictly decreases with $q$.

(11) shows the marginal benefit and cost of the auditor’s effort. Holding investors’ conjecture constant at $\hat{e}$, a higher effort improves the accuracy of audit evidence in the bad state and reduces the auditor’s vulnerability, as reflected by $[l(q, \gamma_l, \mu) - l(q, \gamma_h, \mu)]$ on the the left-hand side (LHS) of (11). A higher effort is also costlier to the auditor as shown in the right-hand side (RHS) of (11). The equilibrium condition is given by replacing investors’
conjecture $\hat{e}$ in (11) with the auditor’s actual effort. This ensures that investors’ conjecture is rational in equilibrium.

The equilibrium uniqueness is not guaranteed as both sides of (12) can be increasing in the auditor’s effort. Multiple equilibria can occur because investors’ conjecture $\hat{e}$ can be self-fulfilling. Under certain parameter values, the higher the effort investors conjecture, the more likely they rely on the auditor’s opinion (i.e., $\bar{p}(\hat{\gamma})$ increases in $\hat{\gamma}$). This in turn increases the auditor’s expected liability and can provide more incentives for effort. With multiple equilibria comes the issue of equilibrium selection. We note that any equilibrium with \[ \frac{\partial [LHS \ of \ (12)]}{\partial e} \bigg|_{e=e^*_{ND}} > C''(e^*_{ND}) \] is unstable in that a small deviation in investors’ conjecture $\hat{e}$ will not converge back to that equilibrium (Stokey, Lucas, and Prescott (1989)). Proposition 2(b) shows that under the assumption of $C''(1) = \frac{\partial [LHS \ of \ (12)]}{\partial e} \bigg|_{e=e^*_{ND}} \] there must exist at least a stable equilibrium where $\frac{\partial [LHS \ of \ (12)]}{\partial e} \bigg|_{e=e^*_{ND}} < C''(e^*_{ND})$.

Proposition 2(c) can be proved by noticing that a larger $q$ unambiguously increases the marginal benefit of the auditor’s effort: both terms on the LHS of (11), $l(q, \gamma_l, \mu) - l(q, \gamma_h, \mu)$ and $\bar{p}^2(\hat{\gamma})$, are strictly increasing in $q$, while the RHS is unaffected. The intuition comes from the fact that the auditor’s incentives to exert effort is motivated by the threat of audit failure. The odds of an audit failure can be reduced either when the auditor exerts more effort to reduce her vulnerability, and/or when investors rely less on the auditor’s opinion (i.e., less investors’ facilitation). Both forces can be affected by $q$. First, $l(q, \gamma_l, \mu) - l(q, \gamma_h, \mu)$ increases with $q$. The intuition is the familiar informativeness principle in agency theory (Holmstrom (1979)) in that a higher $q$ reduces the noise in vulnerability as a performance measure for the auditor’s effort. To see this, take the extreme case where $q = 1/2$. Then, even if the underlying state is bad, there is still a high chance that the underlying accounting system generates $R_G$, reducing the likelihood of an audit failure and thus disincentivizing the auditor to exert effort. A larger $q$ reduces this noise in audit failure as a performance measure and therefore promotes a higher effort.

Second, a larger $q$ also increases the chances that the auditor’s vulnerability is "relied upon" by investors. This happens because ceteris paribus, a larger $q$ makes the auditor’s opinion more useful to predict the underlying state and therefore induces investors to rely
more on the auditor’s opinion than their own signal: $\bar{p}^2(\hat{\gamma})$ increases with $q$. More reliance means that when the auditor fails to catch managerial misreporting, her mistake is more likely to lead to a full-blown auditor failure, thus providing more incentives for the auditor exert effort.

As shown in Proposition 2(d), although a larger $q$ induces a higher auditor effort, increasing $q$ can potentially reduce investment efficiency. Intuitively, increasing $q$ strengthens the insurance effect of the auditor’s opinion by making investors increasingly confident that the auditor has committed an audit failure when the auditor issues an unqualified opinion and the opinion contradicts investors’ signal $S$. To see this, in the extreme case of $q = 1/2$, the auditor’s signal becomes independent of $S$ and thus is not useful in predicting whether the auditor has made a mistake or not. The larger $q$ is, the more correlated $S$ and $\Omega$ are, and the more certain investors are that the auditor has committed an audit failure when their signal conflicts with the auditor’s opinion. An increased likelihood of an audit failure enhances the insurance effect and induces investors to ignore their own signal more often with a larger $\bar{p}$. Proposition 2(d) shows that this unintended consequence of increasing $q$ becomes dominant when $\gamma_h$ is sufficiently small. Intuitively, when the auditing technology is poor ($\gamma_h$ is low), the auditor’s opinion is not that informative of the underlying state, thus making ignoring a conflicting $S$ less costly and the insurance value dominant.

### 3.3.2 Disclosure regime

The auditor’s effort choice under the Disclosure regime is characterized by Proposition 3 and determined similarly as that in the No Disclosure regime. The main difference is that $\hat{\gamma}$ in the assessed probability of an audit failure (as in (7)) is replaced by the actual realization of $\gamma$.

**Proposition 3** In the Disclosure regime, the auditor’s optimal effort choice $e_D^*$ is uniquely determined by

$$\max \left\{ \alpha K[l(q, \gamma_l, \mu)\bar{p}^2(\gamma_l) - l(q, \gamma_h, \mu)\bar{p}^2(\gamma_h)], 0 \right\} = C'(e_D^*),$$

(13)

where $l(q, \gamma, \mu)$ and $\bar{p}(\gamma)$ are as defined in Proposition 2.
Similar to (12) in Proposition 2, the left-hand side of (13) expresses the marginal benefit of the auditor’s effort. However, there are two differences here. First, (12) admits multiple self-fulfilling equilibria whereas (13) pins down an unique equilibrium. Multiple equilibria do not arise in the Disclosure regime because investors directly observe $\gamma$ and no longer need to base the investment decision on their conjecture.

Second, (12) guarantees an interior solution, while a corner solution of $e^*_D = 0$ is possible under (13). This is because unlike in the No Disclosure regime, the marginal benefit of effort are not necessarily always positive. To see this, let’s denote the auditor’s probability assessment of an audit failure on $\gamma$ under the Disclosure regime conditional as $\Pr(\text{audit failure}|\gamma)$.

It is easy to obtain
\[
\frac{\partial \Pr(\text{audit failure}|\gamma)}{\partial \gamma} = \frac{\partial l(q, \gamma, \mu)}{\partial \gamma} \bar{p}^2(\gamma) + 2l(q, \gamma, \mu) \bar{p}(\gamma) \frac{\partial \bar{p}(\gamma)}{\partial \gamma}.
\]

(14) shows that a larger $\gamma$ has two effects on $\Pr(\text{audit failure}|\gamma)$. The first is to reduce the auditor vulnerability, as captured by $\frac{\partial l(q, \gamma, \mu)}{\partial \gamma} < 0$. This effect is also present in the No Disclosure regime and it’s the primary force to motivate the auditor’s effort. The second effect, captured by $\frac{\partial \bar{p}(\gamma)}{\partial \gamma}$, is to enable investors to adjust their investment decision as a function of the realized $\gamma$, i.e., $\frac{\partial \bar{p}(\gamma)}{\partial \gamma} \neq 0$. This effect is absent in the No Disclosure regime where the decision is based on investors’ conjecture $\hat{\gamma}$ but not the realized $\gamma$. A larger $\gamma$ increases the auditor opinion’s information value ($\frac{\partial \bar{p}(\gamma)}{\partial \gamma} > 0$) but decreases its insurance value ($\frac{\partial \bar{p}(\gamma, a, q)}{\partial \gamma} < 0$). When the information value effect overwhelms the insurance value effect, a larger $\gamma$ increases the chance of an audit failure. Since more auditor effort increases $\gamma$ in expectation, this would dampen the auditor’s incentives to exert effort and lead to a possible corner solution of no effort.

The effect of audit transparency (whether to disclose information about $\gamma$) on the auditor’s effort is established in Proposition 4 below.

**Proposition 4** Define $q^*$ as the unique solution to $2q - 1 - \alpha q^2 = 0$ for $q \in \left[\frac{1}{2}, 1\right]$.

\[
e^*_D \geq e^*_\text{ND} \text{ if and only if } q \leq q^*.
\]
Proposition 4 shows that more audit transparency increases the auditor’s effort (i.e., higher audit quality in expectation) only when the underlying accounting quality is relatively poor; and the Proposition is crucially linked to the sign of $\frac{\partial p(q)}{\partial \gamma}$. Note that (14) implies $\frac{\partial p(q)}{\partial \gamma} < 0$ is a sufficient condition to ensure a strictly positive equilibrium auditor’s effort. As the proof for Proposition 4 shows, $q < q^*$ is a sufficient and necessary condition for $\frac{\partial p(q)}{\partial \gamma} < 0$.

The intuition for Proposition 4 can be illustrated in terms of the informativeness principle of optimal performance measures in agency theory (Holmstrom (1979)). Since an audit failure acts as the performance measure for the auditor’s effort, its usefulness is enhanced when it becomes more sensitive to the auditor’s effort. That is, when the auditor anticipates a lower effort is more likely to lead to an audit failure, she will have more incentives to work harder. Recall that an audit failure depends on both the auditor’s vulnerability ($l(q, \gamma, \mu)$) and investor reliance ($\hat{p}^2(\gamma)$). While the auditor’s vulnerability ($l(q, \gamma, \mu)$) is largely exogenous and depends only on the underlying accounting quality ($q$) and the underlying state ($\mu$), how sensitive investors’ reliance ($\hat{p}^2(\gamma)$) is toward the auditor’s effort is endogenously determined by the motives behind their use of information.

On the one hand, when $q$ is relatively large and thus the accounting signal is quite informative regarding the state, investors use the auditor’s opinion primarily for its information value to make correct investment decisions (invest when $G$ is more likely and not invest when $B$ is more likely). Specifically, they will rely more on the auditor’s opinion when $\gamma_h$ is observed than when $\gamma_l$ is realized, simply because a more precise opinion helps better capturing the underlying state. Thus, a higher effort by the auditor in fact may lead to more reliance by investors and a higher chance of an audit failure. Anticipating this, the auditor’s incentives to exert effort are muted.

On the other hand, when $q$ is relatively small and the accounting signal is not very informative to predict the state, investors primarily use the auditor’s opinion for its insurance value. A smaller $\gamma$ in this case further enhances the insurance value of auditor’s opinion. This is because when $\gamma$ is low, investors are more certain that the auditor is vulnerable. Consequently, $\gamma_l$ induces more reliance by investors on the auditor’s opinion than $\gamma_h$, increasing the odds of a lower auditor’s effort to render an audit failure. Anticipating this, the
It is worth noting that when investors rely on the auditor’s opinion for its insurance value, they do so at the expense of investment efficiency (i.e., sometime they purposely disregard their own informative signal and follow the auditor’s opinion precisely when the auditor’s opinion is of low precision). The silver lining of the insurance effect, however, is to provide extra incentive to motivate auditor effort, although this effect is only present in the disclosure regime.

Since $e_{ND}^*$ is always increasing in $q$ (as shown in Proposition 2(c)), the finding in Proposition 4 that $e_D^* > e_{ND}^*$ only for $q < q^*$ suggest that the marginal effect of $q$ on the equilibrium effort in the disclosure regime can actually be negative. This is indeed confirmed by the next proposition.

**Proposition 5** Under the Disclosure regime, when $\alpha$, $q$ and $\gamma_h$ are sufficiently small,

(a) the equilibrium auditor’s effort strictly decreases with $q$;

(b) the investment efficiency strictly decreases with $q$.

The effect of $q$ on the auditor’s effort can be analyzed by examining how $q$ affects the marginal benefit of effort (i.e., the LHS of (14)). The LHS contains two terms: the first term is similar to that in (11) and therefore is always decreasing in $q$. This force is the same as that under the No disclosure regime and provides the auditor more incentive to work, as shown in Proposition 2(c).

However, $q$’s effect on the second term, $l(q, \gamma, \mu)\bar{p}(\gamma)\frac{\partial \pi(\gamma)}{\partial \gamma}$, is more subtle. Specifically, it depends on the sign of $\frac{\partial \pi(\gamma)}{\partial \gamma}$ and of $\frac{\partial^2 \pi(\gamma)}{\partial \gamma \partial q}$:

$$\frac{\partial l(q, \gamma, \mu)\bar{p}(\gamma)\frac{\partial \pi(\gamma)}{\partial \gamma}}{\partial q} = \left[\frac{\partial l(q, \gamma, \mu)}{\partial q}\bar{p}(\gamma) + \frac{\partial \bar{p}(\gamma)}{\partial q}l(q, \gamma, \mu)\right]\frac{\partial \pi(\gamma)}{\partial \gamma} + l(q, \gamma, \mu)\frac{\partial^2 \pi(\gamma)}{\partial \gamma \partial q} > 0$$

It can be shown that $\frac{\partial^2 \pi(\gamma)}{\partial \gamma \partial q} > 0$ as long as $\alpha$ is not too large, thus making it possible that increasing $q$ lowers the marginal benefit of the auditor’s effort and reduces the equilibrium audit quality.
To see the intuition behind Proposition 5(a), consider the extreme case where $q = 1/2$ and $\alpha$ is close to zero. Here the auditor’s opinion is irrelevant for assessing the project’s underlying state of the world (i.e., the auditor’s opinion has no information value); and investors do not care much of the opinion’s insurance value. As such, $\overline{p}(\gamma_h) \approx \overline{p}(\gamma_l) \approx 1/2$. This in turn implies that auditor’s effort does not very much affect the probability that his vulnerability is acted upon by investors. When $q$ increases, investors’ reliance on the auditor’s opinion is more sensitive to $\gamma \left( \frac{\partial^2 p^*(\gamma)}{\partial \gamma \partial q} > 0 \right)$. However, since this reliance is purely for the information value of the auditor’s opinion, it has the perverse effect on the auditor’s effort. In contrast, under the No Disclosure regime, the link between the auditor’s actual effort and investors’ equilibrium reliance is weaker because it is based on the conjectured effort (and audit quality), which gives rise to a positive relationship between the equilibrium auditor’s effort and $q$, as shown in Proposition 2(c).

Finally, under the Disclosure regime, increasing $q$ on the margin has two effects. The first effect is similar to what is shown in Proposition 2(d), where $q$ increases the insurance value of the auditor’s opinion and can lead to a reduction in investment efficiency. Furthermore, under the Disclosure regime, a higher $q$ has an additional impact on investment efficiency via its adverse impact on the auditor’s effort provision. As demonstrated in Proposition 5(d), the combined effects are that a marginal increase in $q$ may reduce the equilibrium efficiency.

### 3.4 Effects of auditor transparency on investment efficiency

A main argument for improving audit transparency (i.e., forcing auditors to disclose $\gamma$) is that it can help improve the decision usefulness of audited accounting reports. In our setting, investors use the auditor’s opinion for the investment decision. Therefore, we evaluate the decision usefulness of auditor transparency by comparing the equilibrium investment efficiency across the two regimes.

Three forces are at play here. First, disclosing $\gamma$ allows investors to tailor their investment decision to the realized precision of the auditor’s opinion. We term this effect as Blackwell effect, manifested by $\overline{p}(\gamma)$ being a function of $\gamma$. This effect tilts our efficiency comparison in
the Disclosure regime’s favor. That is, Ceteris Paribus, the flexibility to adjust the investment decision as a function of $\gamma$ should improve the ex ante investment efficiency under the Disclosure regime relative to the No Disclosure regime.

Second, there is an insurance effect. Because investors receive damages when an audit failure occurs, their investment decision deviates from the First Best. This effect is manifested by $\rho (\tilde{\gamma}, \alpha, q)$ in (5). While this insurance effect is present under both regimes, it is easy to verify that $\rho (\tilde{\gamma}, \alpha, q)$ is a convex function in $\tilde{\gamma}$, implying that the deviation from the First Best is weaker under the No Disclosure regime than under the Disclosure regime. Intuitively, not knowing $\gamma$ under the No Disclosure regime hampers investors’ ability to take full advantage of the insurance, thus alleviating the inefficient use of information by investors and resulting in more efficient investment. Thus, this insurance effect works in favor of the No Disclosure regime.

Finally, we have an effort effect. Specifically, Proposition 4 shows that the equilibrium effort can be either higher or lower under the Disclosure regime than under the No Disclosure regime depending on the magnitude of $q$. The efficiency comparison of the two regimes hence is determined by a fairly complex tradeoff among these three forces, which unfortunately does not easily lend itself to a complete analytical solution. To fix idea, Claim 1 below sheds light on a partial tradeoff between the Blackwell and insurance effect.

Claim 1 Holding the auditor’s effort constant at the same level for the two regimes, $IE_D > IE_{ND}$ if and only if $q > q^*$ where $q^*$ is defined in Proposition 4.

Claim 1 shows that, in a hypothetical situation void of a differential effort effect between the two regimes, Blackwell effect dominates insurance effect if and only if $q > q^*$. The intuition is that when the underlying accounting quality is low (i.e., $q$ is small), the auditor’s opinion cannot provide much information for the project’s terminal case flow and thus investors simply use the opinion for insurance purposes. When $q = q^*$, these two effects exactly cancel each other out, making $IE_D = IE_{ND}$.

When the effort effect is present, the picture becomes more complicated. As Proposition 4 shows, the auditor’s effort is higher under the No Disclose regime if and only if $q > q^*$, thus
countervailing the directional prediction outlined in Claim 1. Next, we present three sets of numerical examples to illustrate the tradeoff between these forces. In all examples, the auditor’s effort function is represented by $C(e) = \frac{1}{3} ce^3$ and $K(1-\mu) = 1$. These examples differ in the level of liability. In each example, we plot the equilibrium effort level and investment efficiency as a function of $q$. For investment efficiency, we plot both the effect around $q^*$ as well as globally.

In Figure 2, $\alpha$ is relatively large ($\alpha = 0.8$). Figure 2a shows that the auditor’s effort under the Disclosure regime is higher if and only if $q < q^* = 0.69$. Figure 2b shows that around $q^*$ the efficiency comparison follows Claim 1’s prediction. That is, when $q$ is slightly below $q^*$, the investment efficiency is higher in the No Disclosure Regime and the opposite holds when $q$ is slightly above $q^*$. However, as shown in Figure 2c, when $q$ is much larger than $q^*$, the effort difference between the two regimes becomes the dominant force, resulting in a higher investment efficiency under the No Disclosure regime.

Fig 2a Effort level in the two regimes when $\gamma_h = 0.70$, $\gamma_l = 0.50$ and $\alpha = 0.8$. 

Fig 2b Investment efficiency in the two regimes when $\gamma_h = 0.70$, $\gamma_l = 0.50$ and $\alpha = 0.8$. 

Fig 2c Investment efficiency in the two regimes when $\gamma_h = 0.70$, $\gamma_l = 0.50$ and $\alpha = 0.8$. 
Fig 2b Investment efficiency around $q^*$ in the two regimes when $\gamma_h = 0.70$, $\gamma_l = 0.50$ and $\alpha = 0.8$.

Figure 2c Investment efficiency with respect to $q$ over the whole range in the two regimes when $\gamma_h = 0.70$, $\gamma_l = 0.50$ and $\alpha = 0.8$.

Figure 3 and 4 illustrate cases where $\alpha$ is moderately big ($\alpha = 0.5$) and $\alpha$ is relatively small ($\alpha = 0.1$), respectively. They are qualitatively similar to Figure 2: the efficiency comparison is consistent with Claim 1 around $q^*$; but the No Disclosure regime becomes dominant in terms of investment efficiency when $q$ is sufficiently big.

Fig 3a Effort level in the two regimes when $\gamma_h = 0.70$, $\gamma_l = 0.50$ and $\alpha = 0.5$. 

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Fig 3b Investment efficiency when \( q \) is around \( q^* \) in the two regimes when \( \gamma_h = 0.70 \), 
\( \gamma_l = 0.50 \) and \( \alpha = 0.5 \).

Fig 3c Investment efficiency with respect to \( q \) over the whole range in the two regimes when 
\( \gamma_h = 0.70 \), \( \gamma_l = 0.50 \) and \( \alpha = 0.5 \).

Fig 4a Effort level in the two regimes when \( \gamma_h = 0.70 \), \( \gamma_l = 0.50 \) and \( \alpha = 0.1 \).
4 Endogenizing Liability Parameter $\alpha$

Our analyses up to this point have treated the auditor’s liability parameter $\alpha$ as exogenous. While an exogenous $\alpha$ simplifies the math, it raises the issue whether our results are robust to an endogenously chosen liability, which we address in this section. In order to maintain the key forces identified in our setting without losing tractability, we make three changes to the main setup. First, the auditor’s effort $e$ is assumed to be binary, with $e \in \{e_h, e_l\}$ and $1 > e_h > e_l > 0$. The incremental cost of effort when $e_h$ is taken is $C$. Second, investors’ signal precision $p$ is also binary, with $p \in \{p_h, p_l\}$ and $1 > p_h > p_l > \frac{1}{2}$. Without of a loss of generality, we set $\gamma_l = \frac{1}{2}$ and assume that when investors are indifferent between following...
their private signal and following the auditor’s opinion they will rely on the latter.

To focus on interesting cases, we restrict our attention to situations where investors’ information is not too precise (either when it is $p_l$ or when it is $p_h$) such that they will never rely on the auditor’s report regardless of $q$ or $\gamma$.\footnote{Specifically, these assumptions entail $p_h > \gamma_h > p_l > \frac{1}{2}$; $q > \hat{q}$ where $\hat{q}$ is such that $p^* (\gamma_h, \hat{q}) = p_l$; and $p_h \in \Theta = \left( \max \left\{ 3 \gamma_h - 2 p_l, \frac{4 \gamma_h - p_l - 2}{2 \gamma_h - 1} \right\}, \frac{\gamma_h}{1 - \frac{2 - \frac{1}{2}}{2 \gamma_h - 1}} \right)$. These parameter restrictions do not result in an empty set. For example, when $p_l = 0.65$, $\gamma_h = 0.68$, $p_h = 0.75 \in \Theta = (0.74, 0.83)$, $\hat{q} = 0.8$ and $q^{**} = 0.89$.}

In other words, these assumption assumes that there is value added from audited financial report. We also emphasize investment efficiency in that we assume the personal cost to auditor $C$ is sufficiently small, relative to the investment amount ($K$), and that the auditor effort is sufficiently productive (that $e_h - e_l$ is sufficiently big). Under these assumptions, we allow $\alpha$ to be chosen to maximize investment efficiency given the disclosure environment. Thus, $\alpha$ can be different in the two regimes and can be a function of $q$. The following proposition characterizes and compares the equilibrium solution under the two regimes.

**Proposition 6** Assume the value of auditor’s effort is sufficiently high (relative to its cost) and that the informativeness of audited report is sufficiently high (relative to investors’ private information).

(a) Under the No Disclosure regime, setting $\alpha^{ND} = \frac{C}{(1-\mu)(e_h-e_l)(\gamma_h-\frac{1}{2})(1-\frac{1}{2}p_h)qI}$ induces the auditor to exert effort $e_h$ and maximizes the expected investment efficiency. Under $\alpha^{ND}$, investment efficiency strictly increases with $q$.

(b) Under the Disclosure regime, there exists a $q^{**}$ such that, for $\forall q \leq q^{**}$, $\alpha^{D1} = 2 - \frac{1}{p_l}$ induces the auditor to exerts effort $e_h$ and maximizes investment efficiency. Under $\alpha^{D1}$, investment efficiency increases in $q$. For $\forall q > q^{**}$, $\alpha = 0$ maximizes investment efficiency but can only induces $e_l$. Under $\alpha = 0$, investment efficiency increases in $q$. There is a discontinuous drop in investment efficiency at $q^{**}$.

(c) Investment efficiency is strictly higher under the No Disclosure regime than that under the Disclosure regime if and only if $q \in (q^{**}, 1]$. 

\textit{Specified formulas and parameter restrictions are given in the text.}
Proposition 6 shows that our results are robust to endogenizing the liability parameter $\alpha$. This may come as a surprise as one suspects that any reduced incentives for the auditor to exert effort can be made up for by ramping up liability. However, as Proposition 6 shows that increasing $\alpha$ and thus restoring the auditor’s effort incentive are optimal if and only if $q$ is relatively small. The intuition is as follows. Though increasing $\alpha$ could increase effort provision, it comes with a cost in the form of increased insurance effect that leads to more inefficient use of information by investors. Such cost becomes high when $q$ is big; and in this case the optimal solution is to forego motivating high effort by the auditor. This result suggests that auditor liability rules and auditor disclosure rules are not perfect substitutes for each other as far as maximizing investment efficiency is concerned.

The simplified binary setting in this section also enables us to precisely compare the investment efficiency under the two regimes. Specifically, Proposition 6 shows that when $q$ is relatively big, $e_h$ is not motivated, which renders the investment efficiency lower under the Disclosure regime than under the No Disclosure regime. This implies that, somewhat surprisingly, the higher underlying financial reporting quality (more informative accounting signal), the more opaque the information environment should be for the auditor’s opinion from an optimal investment efficiency perspective.

5 Conclusions

To be finished.

References


51(3), 495-547.


6 Appendix

Proof of Proposition 1 When \( S \) is consistent with the auditor’s opinion (scenarios 1 and 2), it is obvious that investors optimally invest when \( S = S_G \) and the auditor unqualifies; and that they do not invest when \( S = S_B \) and the auditor qualifies, the proof of which is hence omitted. When \( S = S_G \) and the auditor qualifies (scenario 3), investors’ expected payoff from taking the project net of the initial investment is

\[
\Pr(G \mid \hat{R} = \hat{R}_G, AO = Q, S = S_g, \gamma) \cdot RK - K = \frac{\mu p [q (1 - \gamma) + (1 - q) \gamma]}{\mu p [q (1 - \gamma) + (1 - q) \gamma] + (1 - \mu)(1 - p)(1 - q)(1 - \gamma) + q\gamma} \cdot \frac{K}{\mu} - K \geq 0,
\]

if and only if \( p \geq q\gamma + (1 - q)(1 - \gamma) \).

Finally, when \( S = S_B \) and the auditor unqualifies (scenario 4), investors’ expected payoff from undertaking the project net of the initial investment is

\[
\Pr(G \mid \hat{R} = \hat{R}_G, AO = U, S = S_b, \gamma) \cdot RK + \Pr(B, R_B \mid \hat{R} = \hat{R}_G, AO = U, S = S_b, \gamma) \cdot \alpha K - K = \frac{\mu (1 - p) [(1 - q)(1 - \gamma) + q\gamma]}{\mu (1 - p)[(1 - q)(1 - \gamma) + q\gamma] + (1 - \mu)p[q(1 - \gamma) + (1 - q)\gamma]} \cdot \frac{K}{\mu} + \frac{\alpha K}{(1 - \mu) pq (1 - \gamma)} \cdot \frac{(2q - 1)\gamma + 1 - q}{1 - \alpha q (1 - \gamma)} \geq 0,
\]

if and only if, \( p \leq \bar{p}(\gamma) \equiv \frac{(2q - 1)\gamma + 1 - q}{1 - \alpha q (1 - \gamma)} \).

Q.E.D.

Proof of Proposition 2

(a) Given (7), the auditor’s expected loss when choosing an effort level \( e \) is

\[
e \Pr(\text{audit failure} \mid \gamma_h) \cdot \alpha K + (1 - e) \Pr(\text{audit failure} \mid \gamma_i) \cdot \alpha K + C(e).
\]
Taking a first-order derivative on (15) with respect to $e$ and sets it to zero, we obtain

$$\alpha K[l(q, \gamma_l, \mu) - l(q, \gamma_h, \mu)]p(\gamma)^2 = C'(e),$$

(16)

where $l(q, \gamma, \mu) \equiv (1 - \mu)(1 - \gamma)q$.

Since (15) is strictly convex in $e$, the solution to (16) is indeed a global minimizer for the auditor. Finally, imposing the requirement that investors’ conjecture is confirmed in equilibrium and replacing $\hat{e}$ in (16) with the actual effort choice $e$, the equilibrium auditor’s effort is determined by (12):

$$\alpha K[l(q, \gamma_l, \mu) - l(q, \gamma_h, \mu)]p(e\gamma + (1 - e)\gamma)^2 = C'(e).$$

Clearly, both sides of above expression are continuous in $e$. Also, for the RHS, $C''(0) = 0$ and $C'(1) = +\infty$, while the LHS is bounded and strictly positive for all $e$. Thus, there must exist at least one solution to (12) and all solutions to (12) must lie strictly between 0 and 1.

(b) Note that at $e = 0$, $LHS > RHS$ of (12). Also, at each $e^*$, $LHS = RHS$. Suppose there does not exist any stable equilibrium, i.e., whenever $\frac{\partial[RHS of (12)]}{\partial e}|_{e=e^*} \geq C''(e^*)$. Then, it must be that $LHS \geq RHS, \forall e$. But this contradicts the fact that $C''(1) = +\infty$ and LHS is bounded.

(c) Taking a total derivative on (12) with respect to $q$, we obtain

$$\frac{\partial (LHS of (12))}{\partial q} + \frac{\partial (LHS of (12))}{\partial e} \frac{de}{dq} = C''(e) \frac{de}{dq} \implies \frac{de}{dq} = \frac{\frac{\partial (LHS of (12))}{\partial q}}{C''(e) - \frac{\partial (LHS of (12))}{\partial e}}.$$

Note

$$\frac{\partial (LHS of (12))}{\partial q} = \alpha K(1-\mu)(\gamma_h-\gamma_l)p(\gamma_h+(1-e)\gamma_l)^2 + \alpha K(1-\mu)q(\gamma_h-\gamma_l)2p(e\gamma_h + (1 - e)\gamma_l) \frac{\partial p(e\gamma_h + (1 - e)\gamma_l)}{\partial q}.$$

As the first term is clearly positive and

$$\frac{\partial p(e\gamma_h + (1 - e)\gamma_l)}{\partial q} = 2[e\gamma_h + (1 - e)\gamma_l] - 1 + \alpha \beta \{1 - [e\gamma_h + (1 - e)\gamma_l]\}^2 > 0,$$
we have $\frac{\partial (LHS \text{ of } (12))}{\partial q} > 0$. Finally, recall that, by definition, in a stable equilibrium $\frac{\partial \left( LHS \text{ of } (12) \right)}{\partial e} |_{e=e^*} < C''(e^*)$. Thus $\frac{\partial e}{\partial q} > 0$.

(d) Note that

$$IE \equiv -\mu \Pr(\text{Project Rejected} | G)(RI - I) - (1 - \mu) \Pr(\text{Project Undertaken} | B)I$$

$$= -\mu[1 - \Pr(\text{Project Undertaken} | G)] \left(\frac{1}{\mu} - I\right) - (1 - \mu) \Pr(\text{Project Undertaken} | B)I$$

$$= (1 - \mu) I [\Pr(\text{Project Undertaken} | G) - \Pr(\text{Project Undertaken} | B) - 1],$$

where the second equality obtains because $R = \frac{1}{\mu}$ and $\Pr(\text{Project Rejected} | G) = 1 - \Pr(\text{Project Undertaken} | G)$. Define

$$\Pi \equiv \Pr(\text{Project Undertaken} | G) - \Pr(\text{Project Undertaken} | B).$$

Clearly, our comparative static analysis on $IE$ with respect to $q$ can be equivalently performed on $\Pi$. With a slight abuse of notation, in what follows let’s use $p$ as a shorthand for $\bar{p}(e\gamma_h + (1 - e)\gamma_l)$ to save space and use subscript $ND$ to denote the No Disclosure regime.

$$\Pi_{ND} = e_{ND}^* [q\gamma_h + (1 - q)(1 - \gamma_h)] \left[\int_{1/2}^{1} 2pdp + \int_{1/2}^{P} 2(1 - p)dp\right]$$

$$+ (1 - e_{ND}^*) [q\gamma_l + (1 - q)(1 - \gamma_l)] \left[\int_{1/2}^{1} 2pdp + \int_{1/2}^{P} 2(1 - p)dp\right]$$

$$+ e_{ND}^* [(1 - q)\gamma_h + q(1 - \gamma_h)] \int_{t_h}^{1} 2pdp + (1 - e_{ND}^*) [(1 - q)\gamma_l + q(1 - \gamma_l)] \int_{t_l}^{1} 2pdp$$

$$- e_{ND}^* [q\gamma_h + (1 - q)(1 - \gamma_h)] \int_{t_h}^{1} 2(1 - p)dp - (1 - e_{ND}^*) [q\gamma_l + (1 - q)(1 - \gamma_l)] \int_{t_l}^{1} 2(1 - p)dp$$

$$- e_{ND}^* [(1 - q)\gamma_h + q(1 - \gamma_h)] \left[\int_{1/2}^{1} 2(1 - p)dp + \int_{1/2}^{P} 2pdp\right]$$

$$- (1 - e_{ND}^*) [(1 - q)\gamma_l + q(1 - \gamma_l)] \left[\int_{1/2}^{1} 2(1 - p)dp + \int_{1/2}^{P} 2pdp\right],$$

where $t_h \equiv (2q - 1)\gamma_h + 1 - q$ and $t_l \equiv (2q - 1)\gamma_l + 1 - q$.

Note that

$$\frac{d\Pi_{ND}}{dq} = \frac{\partial \Pi_{ND}}{\partial q} + \frac{\partial \Pi_{ND}}{\partial e_{ND}^*} \frac{de_{ND}^*}{dq}. $$

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The first term $\frac{\partial \Pi_{ND}}{\partial q}$ is

$$\frac{\partial \Pi_{ND}}{\partial q} = e^*_{ND} \{(4\gamma_h - 2)p + [(4q - 2)\gamma_h + 2 - 2q - 2p] \frac{\partial p}{\partial q} + (4\gamma_h - 2) [q \gamma_h + (1 - q)(1 - \gamma_h) - 1]\}
\quad + (1 - e^*_{ND}) \{(4\gamma_l - 2)p + [(4q - 2)\gamma_l + 2 - 2q - 2p] \frac{\partial p}{\partial q} + (4\gamma_l - 2) [q \gamma_l + (1 - q)(1 - \gamma_l) - 1]\}.$$ 

At $\gamma_h = \gamma_l = 1/2$, 

$$\frac{\partial \Pi_{ND}}{\partial q} = (1 - 2p) \frac{\partial p}{\partial q} = -\frac{1}{2} \frac{\alpha q}{1 - \frac{1}{2} \alpha q} \frac{\alpha}{2 - \alpha q^2} < 0.$$ 

From the proof to part (c), we have

$$\frac{de_{ND}}{dq} = \frac{\partial \text{LHS of (12)}}{\partial q} \frac{\partial \text{LHS of (12)}}{\partial e} = \alpha I(\gamma_h - \gamma_l)^2 p^2 + 2(1 - \mu)q\alpha I(\gamma_h - \gamma_l)^2 \frac{\partial p}{\partial q}.$$ 

Note that when $\gamma_h = \gamma_l = 1/2$, the numerator equals zero while the denominator is strictly positive due to $C''(0) > 0$ by assumption. Hence, $\frac{\partial \Pi_{ND}}{\partial q} < 0$ when $\gamma_h = 1/2$. 

By continuity, there must exist a $\gamma_0 > 0$ such that $\forall \gamma_h < \gamma_0$, $\frac{\partial \Pi_{ND}}{\partial q} < 0$. Q.E.D.

**Proof of Proposition 3** The auditor’s objective function when choosing an effort level $e$ is

$$e \left[ l(q, \gamma_h, \mu)p^2(\gamma_h) - l(q, \gamma_l, \mu)p^2(\gamma_l) \right] \alpha K + l(q, \gamma_l, \mu)p^2(\gamma_l) + C(e). \quad (17)$$

Taking a first-order derivative on (17) with respect to $e$ and sets it to zero, we obtain

$$\max \left\{ \alpha K[l(q, \gamma_l, \mu)p(\gamma_l)^2 - l(q, \gamma_h, \mu)p(\gamma_h)^2], 0 \right\} = C'(e), \quad (18)$$

Since (17) is strictly convex in $e$, the solution to (18) is indeed a global minimizer for the auditor. Note that (18) is free of investors’ conjecture $\hat{e}$ and $\hat{\gamma}$. This is because observing the realization of $\gamma$ is a sufficient statistic for investors’ investment decision. 

As such, (18) is also the equilibrium condition for the auditor’s effort under the Disclosure regime. Finally, since only the RHS of (18) is a function of $e$ with $C'(0) = 0$ and $C'(1) = +\infty$ and the LFS is a non-negative constant independent of $e$, there is only one solution to (18). Q.E.D.
Proof of Proposition 4 Our strategy of proving the proposition is to compare the two equilibrium conditions under the two regimes: (12) versus (13). Note that
\[
\bar{p}(\gamma_h)^2 > \bar{p}(\gamma_l)^2 \iff \bar{p}(\gamma_h) > \bar{p}(\gamma_l) \iff \bar{p}'(\gamma) > 0.
\]

As
\[
\bar{p}'(\gamma) = \frac{2q - 1 - \alpha q^2}{[1 - \alpha(1 - \gamma)q]^2},
\]
\[
\bar{p}'(\gamma) > 0 \text{ if and only if } 2q - 1 - \alpha q^2 > 0.
\]

Note that \(2q - 1 - \alpha q^2 = 0\) only admits one solution between \(1/2\) and 1. Thus, there exists a \(q^*\) such that
\[
\bar{p}'(\gamma) > 0 \text{ if and only if } q > q^*,
\]
where \(q^*\) is the unique solution to
\[
2q - 1 - \alpha q^2 = 0 \text{ s.t. } q \in [1/2, 1].
\]

Consider the case \(q \in [\frac{1}{2}, q^*)\) which implies \(\bar{p}'(\gamma) < 0\). As Proposition 2 has established \(e^*_{ND} \in (0, 1)\), we have
\[
\gamma_h > e^*_{ND} \gamma_h + (1 - e^*_{ND}) \gamma_l \implies \bar{p}(\gamma_h) < \bar{p}(e^*_{ND} \gamma_h + (1 - e^*_{ND}) \gamma_l) < \bar{p}(\gamma_l).
\]

Recall \(l(q, \gamma, \mu) \equiv (1 - \mu)(1 - \gamma)q\) is decreasing in \(\gamma\), we have
\[
l(q, \gamma_l, \mu)\bar{p}(e^*_{ND} \gamma_h + (1 - e^*_{ND}) \gamma_l)^2 - l(q, \gamma_h, \mu)\bar{p}(e^*_{ND} \gamma_h + (1 - e^*_{ND}) \gamma_l)^2 <
\]
\[
l(q, \gamma_l, \mu)\bar{p}(\gamma_l)^2 - l(q, \gamma_h, \mu)\bar{p}(\gamma_h)^2 \implies \text{LHS of (12) } < l(q, \gamma_l, \mu)\bar{p}(\gamma_h)^2 - l(q, \gamma_h, \mu)\bar{p}(\gamma_l)^2
\]

Note \(\text{LHS of (12)} > 0\). Thus,
\[
\text{LHS of (12)} < \max \{\alpha K[l(q, \gamma_l, \mu)\bar{p}(\gamma_h)^2 - l(q, \gamma_h, \mu)\bar{p}(\gamma_l)^2], 0\} = \text{LHS of (13)} \implies
\]
\[
e^*_{ND} < e^*_D.
\]
Next consider the case \( q \in (q^*, 1] \) which implies \( p'(\gamma) > 0 \). As Proposition 2 has established \( e_{ND}^* \in (0, 1) \), we have
\[
\gamma_h > e_{ND}^* \gamma_h + (1 - e_{ND}^*) \gamma_l \implies \gamma_l < e_{ND}^* \gamma_l + (1 - e_{ND}^*) \gamma_h \implies \gamma_l > \gamma_l
\]
Thus,
\[
l(q, \gamma_l, \mu) p(e_{ND}^* \gamma_h + (1 - e_{ND}^*) \gamma_l)^2 - l(q, \gamma_h, \mu) p(e_{ND}^* \gamma_h + (1 - e_{ND}^*) \gamma_l)^2 >
l(q, \gamma_l, \mu) p(\gamma_l)^2 - l(q, \gamma_h, \mu) p(\gamma_h)^2 \implies
LHS \text{ of } (12) > l(q, \gamma_l, \mu) p(\gamma_h)^2 - l(q, \gamma_h, \mu) p(\gamma_l)^2
\]
Note LHS of (12) > 0. Thus,
\[
LHS \text{ of } (12) > \max \{ \alpha K [l(q, \gamma_l, \mu) p(\gamma_h)^2 - l(q, \gamma_h, \mu) p(\gamma_l)^2], 0 \} = LHS \text{ of } (13) \implies

e_{ND}^* > e_D^*.
\]
Finally, when \( q = q^* \), \( p'(\gamma) > 0 \). Hence,
\[
l(q, \gamma_l, \mu) p(e_{ND}^* \gamma_h + (1 - e_{ND}^*) \gamma_l)^2 - l(q, \gamma_h, \mu) p(e_{ND}^* \gamma_h + (1 - e_{ND}^*) \gamma_l)^2 =
l(q, \gamma_l, \mu) p(\gamma_l)^2 - l(q, \gamma_h, \mu) p(\gamma_h)^2 \implies

e_{ND}^* = e_D^*.
Q.E.D.

Proof of Proposition 5

(a) We first show that when \( q \) is sufficiently small \( e_D^* > 0 \). Clearly, the equilibrium effort level is strictly positive, iff,
\[
(1 - \gamma_l) p(\gamma_l)^2 - (1 - \gamma_h) p(\gamma_h)^2 > 0.
\]
Thus, a sufficient condition for \( e_D^* > 0 \) is for \( (1 - \gamma) p(\gamma)^2 \) to be decreasing in \( \gamma \).
\[
\frac{\partial [(1 - \gamma) p(\gamma)^2]}{\partial \gamma} = \frac{[(2q - 1) \gamma + 1 - q]}{[1 - \alpha q(1 - \gamma)]^3} v(\gamma, q),
\]
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where \( v(\gamma, q, \alpha) = -3(1-\gamma) + [5 + \alpha(1-\gamma)^2 - 6\gamma]q - \alpha(1-\gamma)(3-2\gamma)q^2 \). Since \( \frac{(2q-1)\gamma+1-q}{1-\alpha q(1-\gamma)^2} > 0 \), the sign of \( \frac{\partial v(\gamma, q, \alpha)}{\partial \gamma} \) is determined by \( v(\gamma, q, \alpha) \). Note that

\[
\frac{\partial v(\gamma, q, \alpha)}{\partial \alpha} = q(1-\gamma)^2 - q^2(1-\gamma)(3-2\gamma) \leq 0.
\]

To see the last inequality, note

\[
\frac{\partial}{\partial q} \left[ q(1-\gamma)^2 - q^2(1-\gamma)(3-2\gamma) \right] = (1-\gamma)^2 - 2q(1-\gamma)(3-2\gamma)
\]

\[
= (1-\gamma)[1-\gamma - 2q(3-2\gamma)] \leq (1-\gamma)[1-\gamma - (3-2\gamma)]
\]

\[
= -(1-\gamma)[2-\gamma] < 0.
\]

Thus,

\[
q(1-\gamma)^2 - q^2(1-\gamma)(3-2\gamma)
\]

\[
\leq \frac{1}{2}(1-\gamma)^2 - \frac{1}{4}(1-\gamma)(3-2\gamma)
\]

\[
= -\frac{1}{4}(1-\gamma) \leq 0.
\]

Since \( \frac{\partial v(\gamma, q, \alpha)}{\partial \alpha} \leq 0 \), we have \( v(\gamma, q, \alpha) \leq -3(1-\gamma) + (5 - 6\gamma)q \). Note that \(-3(1-\gamma) + (5 - 6\gamma)q < 0 \) if and only if \( q < \frac{3(1-\gamma)}{5-6\gamma} \). Since \( \frac{3(1-\gamma)}{5-6\gamma} \) is increasing in \( \gamma \), a sufficient condition for \( e_D^* > 0 \) is \( q < \frac{3(1-\gamma)}{5-6\gamma} \).

Next, we show that when \( \alpha \) and \( \gamma_h \) sufficiently small, \( q \left[ (1 - \gamma_i)\bar{p}(\gamma_i)^2 - (1 - \gamma_h)\bar{p}(\gamma_h)^2 \right] \) decreases with respect to \( q \). This, together with the already established result that effort is strictly positive when \( q \) sufficiently small, implies that effort strictly decreases with respect to \( q \). Define

\[
g(\gamma, q, \alpha) = \frac{\partial}{\partial q} \left[ q(1-\gamma)\bar{p}(\gamma)^2 \right] = (1-\gamma)\bar{p}(\gamma)^2 + 2q(1-\gamma)\bar{p}(\gamma)\frac{\partial \bar{p}(\gamma)}{\partial q}.
\]

A sufficient condition for \( q \left[ (1 - \gamma_i)\bar{p}(\gamma_i)^2 - (1 - \gamma_h)\bar{p}(\gamma_h)^2 \right] \) to decrease with respect to \( q \) is that \( \frac{\partial g(\gamma, q, \alpha)}{\partial \gamma} > 0 \). After some tedious algebra, we obtain

\[
\frac{\partial g(\gamma, q, \alpha)}{\partial \gamma} = \frac{h(\gamma, q, \alpha)}{[1 - \alpha (1-\gamma)q]^2}.
\]

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Recall that in the proof to Proposition 2(d) we have defined

\[ h(\gamma, q, \alpha) \equiv -3(1 - \gamma)^2 + 4(1 - \gamma)[4 - \alpha(1 - \gamma)^2 - 6\gamma]q \\
-15 + 4\alpha(1 - \gamma)^2 - \alpha^2\beta^2(1 - \gamma)^4 - 48\gamma + 36\gamma^2]q^2 \\
-4\alpha(1 - \gamma)(3 - 2\gamma)(2\gamma - 1)q^3 - \alpha^2(1 - \gamma)^2[3 - 4(2 - \gamma)\gamma]q^4. \]

When \( \alpha = 0 \), we have

\[ h(\gamma, q, 0) = -3(1 - \gamma)^2 + 8(1 - \gamma)(2 - 3\gamma)q + 3(2\gamma - 1)(5 - 6\gamma)q^2. \]

\( h(\gamma, q, 0) \) is clearly increasing in \( q \) when \( \gamma \leq \frac{2}{3} \); and \( h(\gamma, \frac{1}{2}, 0) = \frac{5}{4} - 2\gamma > 0 \) when \( \gamma < \frac{5}{8} \).

Thus, by continuity, when \( \alpha \) and \( \gamma \) are sufficiently small, \( q [(1 - \gamma_l)p(\gamma_l)^2 - (1 - \gamma_h)p(\gamma_h)^2] \)

decreases with respect to \( q \). Lastly, in order for \( \frac{de^*_D}{dq} < 0 \), we not only need \( \alpha \) and \( \gamma \) sufficiently small but also \( q \) sufficiently small to make sure \( e^*_D > 0 \) (i.e., \( q [(1 - \gamma_l)p(\gamma_l)^2 - (1 - \gamma_h)p(\gamma_h)^2] > 0 \)) as shown at the beginning of the proof.

(b) Recall that in the proof to Proposition 2(d) we have defined

\[ \Pi \equiv \Pr(\text{Project Undertaken} \mid G) - \Pr(\text{Project Undertaken} \mid B), \]

\[ t_h \equiv (2q - 1)\gamma_h + 1 - q \text{ and } t_l \equiv (2q - 1)\gamma_l + 1 - q, \]

and shown the comparative static analysis on \( IE \) with respect to \( q \) can be equivalently performed on \( \Pi \). Particularly, under the Disclosure regime (denoted by a subscript \( D \)),

\[ \Pi_D = e^*_D[q\gamma_h + (1 - q)(1 - \gamma_h)] \left[ \int_{1/2}^{1} 2pdp + \int_{1/2}^{p(\gamma_h)} 2(1 - p) dp \right] \\
+ (1 - e^*_D) [q\gamma_l + (1 - q)(1 - \gamma_l)] \left[ \int_{1/2}^{1} 2pdp + \int_{1/2}^{p(\gamma_l)} 2(1 - p) dp \right] \\
+ e^*_D [(1 - q)\gamma_h + q(1 - \gamma_h)] \int_{t_h}^{1} 2pdp + (1 - e^*_D) [(1 - q)\gamma_l + q(1 - \gamma_l)] \int_{t_l}^{1} 2pdp \\
- e^*_D [(1 - q)\gamma_h + (1 - q)(1 - \gamma_h)] \int_{t_h}^{1} 2(1 - p) dp - (1 - e^*_D) [(1 - q)\gamma_l + (1 - q)(1 - \gamma_l)] \int_{t_l}^{1} 2(1 - p) dp \\
- e^*_D [(1 - q)\gamma_h + q(1 - \gamma_h)] \left[ \int_{1/2}^{1} 2(1 - p) dp + \int_{1/2}^{p(\gamma_h)} 2pdp \right] \\
- (1 - e^*_D) [(1 - q)\gamma_l + q(1 - \gamma_l)] \left[ \int_{1/2}^{1} 2(1 - p) dp + \int_{1/2}^{p(\gamma_l)} 2pdp \right]. \]
Obviously,
\[
\frac{d\Pi_D}{dq} = \frac{\partial \Pi_D}{\partial q} + \frac{\partial \Pi_D}{\partial e^*_D} \frac{de^*_D}{dq},
\]
let’s go through the three expressions in \( \frac{d\Pi_D}{dq} \) one by one. Part (a) of the proposition has already established that \( \frac{de^*_D}{dq} < 0 \) when \( \alpha, q \) and \( \gamma_h \) are sufficiently small. Next, note that when \( \alpha = 0 \),
\[
\frac{\partial \Pi_D}{\partial e^*_D} = 2 (t_h - t_l) (t_h + t_l - 1) > 0,
\]
which implies \( \frac{\partial \Pi_D}{\partial e^*_D} > 0 \) when \( \alpha \) is sufficiently small. Finally, using the same proof technique in Proposition 2(d), it is easy to show that at \( \gamma_h = \gamma_l = 1/2 \),
\[
\frac{\partial \Pi_D}{\partial q} = -\frac{1}{2} \alpha q \frac{\alpha}{1 - \frac{1}{2} \alpha q [2 - \alpha q]^2} < 0,
\]
which implies that \( \frac{\partial \Pi_D}{\partial q} < 0 \), when \( \gamma_h \) is sufficiently small. Thus, \( \frac{d\Pi_D}{dq} < 0 \) under the condition specified in the proposition. Q.E.D.

**Proof of Claim 1** When the auditor’s effort is the same under the two regimes, intense algebra can show that taking the second order derivative on \( f \), we have
\[
f''(t) = \frac{32(1 - 2q + \alpha q^2)[(1 - 2q) + \alpha q(4t - 2) + \alpha q^2(3 - 8t) + \alpha^2 q^3(2t - 1)]}{[2 - \alpha q (1 - 2t)]^3}
\]
We now claim that \( (1 - 2q) + \alpha q(4t - 2) + \alpha q^2(3 - 8t) + \alpha^2 q^3(2t - 1) < 0 \). To see this, note that it is obvious when \( t \geq \frac{3}{8} \) as very term is negative. Next consider the case \( 0 \leq t < \frac{3}{8} \). Since the expression is linear with respect to \( t \), if we can show that the expression is negative at both \( 0 \) and \( \frac{3}{8} \), then we are done. When \( t = 0 \), the expression is \( 1 - 2q - 2\alpha q + 3\alpha q^2 - \alpha^2 q^3 \). Standard maximization techniques can show that this function achieves its maximum of \( \frac{1}{8}(-2\alpha - \alpha^2) < 0 \) when \( \alpha > 0 \). When \( t = \frac{3}{8} \), the expression is obviously negative. Therefore, when \( q < q^* \), \( 1 - 2q + \alpha q^2 > 0 \) and \( f''(t) < 0 \), which implies \( f(t) \) is concave and the insurance effect dominates Blackwell effect. Similarly, when \( q > q^* \), \( 1 - 2q + \alpha q^2 < 0 \) and \( f''(t) > 0 \). Thus, \( f(t) \) is convex, implying that Blackwell effect dominates insurance effect. When \( q = q^* \), \( f''(t) = 0 \), implying \( IE_D = IE_{ND} \). Q.E.D.
Sketch Proof of Proposition 6 To ease exposition, here we only provide a sketch proof for the proposition. A complete proof is available from the authors upon request.

(a) Suppose \( e_h \) needs to be motivated. Setting \( \alpha = \frac{C}{(1-\mu)(e_h-e_l)(\gamma_h-\frac{1}{2})(1-\frac{1}{2}p_h)qI} \),

\[
\bar{p} \left( e_h\gamma_h + \frac{1}{2} (1 - e_h) \right) = \frac{(2q - 1) \left( e_h\gamma_h + \frac{1}{2} (1 - e_h) \right) + 1 - q}{1 - \alpha \left[ 1 - e_h\gamma_h - \frac{1}{2} (1 - e_h) \right]} \\
> (2q - 1) \left( e_h\gamma_h + \frac{1}{2} (1 - e_h) \right) + 1 - q \\
> p_l \text{ (as } q > \hat{q} \text{ and } e_h \text{ sufficiently big)}. 
\]

Also, when \( G \) is sufficiently small, \( \alpha \) is sufficiently small and thus \( \bar{p} (e_h\gamma_h + (1 - e_h) \gamma_l) < p_h \). Since \( \bar{p} (e_h\gamma_h + (1 - e_h) \gamma_l) \in (p_l, p_h) \), the auditor’s expected loss from choosing \( e_h \) and \( e_l \) is \( \alpha I (1-\mu) q \left[ e_h (1 - \gamma_h) + (1 - e_h) \frac{1}{2} \right] (1 - \frac{1}{2} p_h) + C \) and \( \alpha I (1-\mu) q \left[ e_l (1 - \gamma_h) + (1 - e_l) \frac{1}{2} \right] \) respectively. Hence, at \( \alpha = \frac{C}{(1-\mu)(e_h-e_l)(\gamma_h-\frac{1}{2})(1-\frac{1}{2}p_h)qI} \), the auditor (weakly) prefers to exert \( e_h \). The only other meaningful alternative to motivate \( e_h \) is to raise \( \alpha \) high enough so that \( \bar{p} (e_h\gamma_h + (1 - e_h) \gamma_l) \geq p_h \). But this is not optimal because setting \( \bar{p} (e_h\gamma_h + (1 - e_h) \gamma_l) > p_h \) involves more inefficient use of information by investors (i.e., ignoring a more informative signal). Finally, straightforward algebra also shows that not motivating the auditor to exert \( e_h \) is not optimal. Intuitively, this is because when \( e_h (e_l) \) is sufficiently big (small) there is not too much efficiency loss from not being able to identify the realized \( \gamma \)'s. Finally, investment efficiency can be shown to strictly increases with \( q \) because a more informative \( q \) doesn’t cause much distortion in investors’ investment decision.

(b) We first establish that setting \( \alpha \) such that \( \bar{p} (\gamma_l) < p_l \) cannot motivate \( e_h \). Suppose otherwise. A necessary condition to induce \( e_h \) is\(^\text{13}\)

\[
\alpha K (1-\mu) q \frac{1}{2} \left[ \frac{1}{2} (1 - p_l) + \frac{1}{2} (1 - p_h) \right] \geq \alpha K (1-\mu) q (1 - \gamma_h) \left[ \frac{1}{2} + \frac{1}{2} (1 - p_h) \right] + C, 
\]

which is not possible given \( p_h > \frac{4q_h-p_h}{2\gamma_h-1} \). Next, set \( \alpha = 2 - \frac{1}{p_l} \) such that \( \bar{p} (\gamma_l) = p_l \).

When \( p_h > \frac{(2q-1)\gamma_h+1-q}{1-(2-\frac{1}{p_l})}\gamma_h \) (which is equivalent to \( q < q^{**} \), where \( q^{**} < 1 \) is such that

\(^{13}\)It is a necessary condition because \( \bar{p} (\gamma_h) \) could be larger than \( p_h \), which would make it more difficult to motivate \( e_h \).
\[ p_h = \frac{(2q^{**} - 1)\gamma_h + 1 - q^{**}}{1 - (2 - \frac{1}{p_l})(1 - \gamma_h)}, \quad \bar{p}(\gamma_h) < p_h. \] Clearly, \( e_h \) can be motivated because with \( C \) being sufficiently small

\[ \alpha K (1 - \mu) q \left[ \frac{1}{2} + \frac{1}{2} (1 - p_h) \right] > \alpha K (1 - \mu) q (1 - \gamma_h) \left[ \frac{1}{2} + \frac{1}{2} (1 - p_h) \right] + C. \]

Straightforward but tedious algebra shows that setting \( \alpha = 2 - \frac{1}{p_l} \) weakly dominates other arrangements that may have \( \bar{p}(\gamma_h) > p_h \) and lead to more inefficient use of information by investors or that only motivates \( e_l \) when \( e_h \) (\( e_l \)) is sufficiently big (small).

Next, in the case where \( p_h \leq \frac{(2q - 1)\gamma_h + 1 - q}{1 - (2 - \frac{1}{p_l})(1 - \gamma_h)} \) (which is equivalent to \( q \geq q^{**} \)) such that \( \bar{p}(\gamma_h) \geq p_h \) at \( \alpha = 2 - \frac{1}{p_l} \), the highest possible investment efficiency from motivating \( e_h \) is to have \( \bar{p}(\gamma_i) = p_l \) and \( \bar{p}(\gamma_h) > p_h \), which is dominated by setting \( \alpha = 0 \) and not motivating \( e_h \) when \( p_h > 3\gamma_h - 2p_l \). Because auditor’s effort drops discretely at \( q^{**} \), investment efficiency also decreases with \( q \) at \( q^{**} \).

(c) When \( q > q^{**} \), the Disclosure regime generates strictly lower effort than the No Disclosure regime. When \( e_h \) is sufficiently big, the efficiency loss stemming from uncertainty regarding the realized \( \gamma_i \)'s is more than compensated by the increased effort under the No Disclosure regime. Hence, the investment efficiency is higher under the No Disclosure regime. When \( q \leq q^{**} \), both regimes motivate \( e_h \). But with \( e_h < 1 \), the Disclosure regime has strictly higher efficiency than the No Disclosure regime because the former discloses the realized \( \gamma_i \)'s to investors. Q.E.D.