A Theory of Social Coupons

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Abstract

“Daily deals” or online discount vouchers have emerged as an increasingly popular means of conducting promotions for many small and local businesses. Social coupons are daily deals that display voucher sales information in real-time and may require that a minimum number be sold for the deal to be valid. While conventional wisdom suggests that firms offer social coupons to motivate referrals through the use of minimum limits, many social coupons are also offered without minimum limits. We examine when and why it may be profitable for a firm to do so. We analyze a setting where voucher sales information may influence consumer beliefs about the promoted product’s appeal and consumers can strategically wait to check this information before they buy. We allow for the fact that consumers may not always return when they choose to wait. Interestingly, while a social coupon’s key feature is that consumers can check its progress, its profitability may be decreasing in the probability that consumers indeed return to do so. Thus, for instance, providing consumers reminders to check the deal’s progress may prove counterproductive. We further examine the role of minimum limits in this setting. Our results might explain why their use may have declined.

(Keywords: Daily Deals, Social Coupons, Pricing, Promotions, Observational Learning)

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1 Introduction

"Daily deals" are an increasingly popular means of conducting online promotions. Daily deals are online discount vouchers that target consumers in a given city and are typically offered by small and local businesses in that city, such as restaurants, spas, salons and gyms. Businesses make these deals available through a daily deal website. Daily deals are so called because new deals from different businesses are announced on the website on a daily basis. Each deal is available for a specified time, usually between two to four days, and may be availed by buying a voucher that is redeemable at the business offering the deal. The usage of daily deals has been growing at a considerable pace in many countries across the world: in the U.S. consumer spending on these deals was expected to grow from $873 million in 2010 to nearly $2 billion in 2011 and exceed $4 billion by 2015 (BIA/Kelsey 2011b).

Daily deals have thus emerged as an important promotional tool, especially for many small and local businesses (BIA/Kelsey 2011a, Dholakia 2011); they enable these businesses to reach out beyond their existing clientele and introduce their offerings to a much larger audience of potential consumers (Bloomberg Businessweek 2009, Reuters 2009). In this regard, daily deals may perform a function similar to that of promotional advertisements in local media such as newspapers, yellow pages, circulars or mailers; such promotional advertisements not only create awareness about the business, but also frequently include coupons, discounts or other offers to entice new consumers. Indeed, daily deals are expected to increasingly displace such traditional promotions (BIA/Kelsey 2011a, Dholakia 2011).

Interestingly, however, unlike traditional coupons or vouchers, daily deals can track the number of vouchers sold and display this information prominently on the deal’s web page in real-time. Such deals are often referred to as “social coupons”; they may also require that a minimum number of vouchers be sold for the deal to be valid, in which case, consumers who buy the vouchers are charged for it if and when the minimum limit is reached. Daily deals can therefore be broadly classified into two types: social coupons, which display real-time sales information, and “regular” (daily) deals, which do not reveal sales information. Table 1 provides a non-exhaustive list of both types of U.S. daily deal websites through which businesses can offer daily deals. Hence, when promoting its product through a daily deal, a firm must decide which type of daily deal to offer. Our objective in this paper is to study a firm’s incentives to promote its product using a social coupon instead of a regular (daily) deal.

The conventional rationale for offering social coupons is that they may motivate referrals through the use of minimum limits: the minimum limit may induce consumers interested in the deal to recommend it to others, in order to ensure that the minimum limit is reached and the deal is valid (Financial Times 1

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1While it used to be that only one new deal was announced daily on a daily deal website, this is no longer the case.
A running tally of vouchers sold may then keep consumers updated about the deal’s progress. Furthermore, by setting a minimum limit, the firm can provide a steeper discount than otherwise, since the deal will be valid only if it generates sufficient sales to make it worthwhile for the firm (Reuters 2009, New Media Age 2010). Thus, a social coupon with a minimum limit may also function as a quantity discount.

### Table 1: Examples of Social Coupon and Regular Daily Deal Websites in the U.S. (May 2012)

<table>
<thead>
<tr>
<th>Social Coupon Websites</th>
<th>Regular Daily Deal Websites</th>
</tr>
</thead>
<tbody>
<tr>
<td>AmazonLocal</td>
<td>AdPages ABDailyDeals</td>
</tr>
<tr>
<td>Dealsaver**</td>
<td>Angie’s List Big Deals</td>
</tr>
<tr>
<td>Eversave**</td>
<td>AT&amp;T Yellowpages</td>
</tr>
<tr>
<td>Groupon†</td>
<td>Gilt City</td>
</tr>
<tr>
<td>KGB Deals</td>
<td>Google Offers</td>
</tr>
<tr>
<td>LivingSocial</td>
<td>Restaurant.com</td>
</tr>
<tr>
<td>StarTribune Steals</td>
<td>TimesLimited</td>
</tr>
<tr>
<td>Tippr*</td>
<td>Valpak</td>
</tr>
<tr>
<td>Travelzoo Daily Deals †**</td>
<td>Woot</td>
</tr>
<tr>
<td>Village Voice Daily Deals</td>
<td>Yelp</td>
</tr>
<tr>
<td>Washington Post Capitol Deal</td>
<td>YourBestDeals</td>
</tr>
</tbody>
</table>

* - Deals offered may have minimum limits
† - Deals offered used to have minimum limits
** - Regular deals can also be offered

However, social coupons are also offered without minimum limits. In fact, as can be seen from Table 1, at present, we found that only deals offered on one of the social coupon websites had minimum limits on a consistent basis. Arguably, without a minimum limit, there is limited incentive for referral behavior. Moreover, without a minimum limit, a social coupon also does not function as a quantity discount. Thus, the rationale for offering social coupons may require further investigation. To this end, in this paper we conduct a model-based examination of an alternative mechanism for social coupons: we analyze a setting where consumers may be uncertain about the promoted product’s appeal and knowing how many others bought the deal may influence their beliefs about the same, i.e., it may facilitate observational learning. Prior empirical research has shown that revealing popularity information can facilitate observational learning and influence consumer choices in both online as well as offline settings (e.g., Cai, Chen & Fang 2009, Chen, Wang & Xie 2011, Tucker & Zhang 2011, Zhang & Liu 2012). We examine the implications of such observational learning in the context of a firm’s promotion strategy.

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2The FAQ sections on Groupon and Travelzoo still mention that deals may have minimum limits. But as of May 2012, none of the deals offered through these websites had minimum limits. In particular, in the case of Groupon, a "deal is on" message is displayed as soon as the first purchase is made. The minimum limit for Groupon deals was also verified by using the website’s application programming interface to retrieve the deal parameters.

3We review this literature in §1.1.
Similar mechanisms may be employed in other settings such as in group funding of projects and in micro-lending. The use of popularity information to promote products is also seen in other contexts. For instance, online retailers such as Amazon may reveal best sellers, sales ranks, popular choices of people who viewed a particular product, etc. While similar forces may be at work in these other settings, to make our analysis more concrete, in this paper we focus on social coupons.

We consider that consumers can strategically delay their buying decision to observe the progress of a social coupon before they buy, but allow for the fact that they may not always return when they wait (i.e., the probability that they return may be less than 1). Interestingly, while a key feature of a social coupon is that it allows consumers to check the deal’s progress, we find that the profitability of a social coupon may follow an inverted-U relationship with the probability that consumers indeed return to do so. Thus, for instance, sending reminders to consumers to check the deal’s progress, in order to increase their return probability, may prove counterproductive. This is because observational learning can be a double-edged sword: while it enables the firm to sell to consumers even when the price exceeds their private valuation of the deal, it also limits the ability of the firm to extract surplus from consumers whose private valuation is higher than the price. We find that a social coupon can be more profitable than a regular deal only if the return probability of consumers is not too high and, in some instances, not too low.

We further investigate the role played by a minimum limit in this context. We find that the minimum limit may act as a form of insurance that protects consumers against some of the downside they face when they buy based only on their private information. As a result, the minimum limit can accelerate purchases and enable the firm to extract more surplus from consumers. We find that setting a minimum limit for a social coupon may be profitable only if the return probability of consumers is sufficiently high. Thus, social coupons with and without minimum limits may be profitable in different situations. Our findings may be of interest to managers formulating daily deal promotions. Our results might explain why firms offer social coupons without minimum limits, and why few social coupon websites allow consumers to set reminders to check the deal’s progress. Our results further provide an alternative rationale for the use of a minimum limit, and, at the same time, might explain why some firms may no longer set minimum limits.

1.1 Related Literature

Song 2004, Lu & Moorthy 2007), by examining a new format, namely social coupons, that has been enabled by advances in information technology. Our work also adds to the understanding of marketing strategies firms adopt to leverage social interactions (e.g., Biyalogorsky, Gerstner & Libai 2001, Amaldoss & Jain 2005, Godes et. al. 2005, Chen & Xie 2005, 2008, Chevalier & Mazylin 2006, Mazylin 2006, Joshi, Reibstein & Zhang 2009, Jing 2011, Godes 2012). More specifically, our work is related to research in three areas: daily deals, group buying, and observational learning.

Daily Deals
There is growing literature on daily deals. In ongoing work, Edelman, Jaffer & Kominers (2011) examine whether or not a firm will offer a "daily deal" discount voucher: they consider that such a voucher may serve as a price-discrimination device and may also make consumers aware of the existence of the business conducting the promotion. But they do not examine the case of social coupons. In a survey of businesses that offered daily deals (including social coupons), Dholakia (2011) finds that nearly half the businesses would offer a daily deal again. He also finds that nearly eighty percent of deal users are first-time customers to the business, and eighty percent of deal users do not return to buy at the firm's regular price. He further finds that daily deals led to a considerable decrease in spending on traditional promotional programs such as direct mail, Yellow pages, print and local media advertising. Byers, Mitzenmacher & Zervas (2012) collect deal data from Groupon and LivingSocial and relate deal characteristics with total sales. They find that early morning sales are significantly correlated with final sales. In a separate regression, they also find that final sales are correlated with the final number of Facebook likes. As such, their results are consistent with observational learning as well as other mechanisms such as referrals or word-of-mouth. But they do not examine causal relationships.

Group Buying
Researchers have examined group-buying schemes where the product promoted is always sold, but the price charged to all consumers may decrease when more of them buy. Anand & Aaron (2003) show that such a scheme may function as a quantity-discount schedule in markets where there is uncertainty about the level of demand. Chen, Chen & Song (2007) show that a seller may offer such a scheme when there are economies of scale or the seller is risk-seeking. Researchers have also examined group-buying deals similar to social coupons with minimum limits, where the product promoted is sold to consumers only if a sufficient number of them buy. Jing & Xie (2011) show that such a scheme can be more profitable than traditional individual-selling or a referral-rewards program since it may motivate informed consumers to act as “sales agents” and acquire less-informed consumers in order to ensure that the minimum limit is met. However, revealing the deal’s progress by itself does not play any role in their setting. Thus, in effect, they do not distinguish between a regular daily deal and a social
coupon without a minimum limit. In ongoing work, Hu, Shi & Wu (2011) consider that consumers are uncertain about whether a group-buying deal will reach its minimum limit and that consumers face an opportunity cost when they subscribe to a group-buying deal due to the delay before they know whether the deal will be valid. They show that demand is always higher if later buyers can observe how many early buyers bought the deal (than if they cannot) because consumers face lesser uncertainty about whether the deal will reach the minimum limit. However in their setting, in the absence of a minimum limit, consumers do not benefit from knowing the deal’s progress. In particular, as the authors acknowledge, there is no scope for social (observational) learning in their setting, since consumers are certain about the quality of the product being promoted. Our work complements and extends prior work by examining when and why a firm might offer a social coupon without a minimum limit in a setting where a social coupon facilitates observational learning. We also offer an alternative rationale for why a firm might set a minimum limit.

**Observational Learning**

Starting with the pioneering works of Banerjee (1992), Bikchandani, Hirshleifer & Welch (1992) and Welch (1992), the literature on observational learning has by and large examined whether such learning can lead to efficient revelation of information or to rational herding behavior. Chamley and Gale (1994) and Zhang (1997) examine the efficiency implications of herding behavior when investors can strategically delay their investment decisions to observe that of others. They find that such strategic delay may contribute to inefficient investments. We too consider that consumers may strategically delay their buying decisions, but allow the firm to influence the extent to which they do so through its price. We show that such strategic delay may either be a boon or a bane for the firm. We further show that by setting a minimum limit a firm may be able to mitigate the downside of strategic delay and may also induce more efficient information revelation.

Researchers have also examined firm strategies to influence what a future generation of consumers may learn by observing a previous generation, but when consumers cannot strategically delay their buying decisions. Caminal & Vives (1996) show that in a duopoly setting, where a future generation of consumers can infer a firm’s quality from its past market share, firms may compete more aggressively for market share in order to signal-jam consumer inferences. Taylor (1999) shows that in housing markets, an individual house seller may either price inordinately high in order to minimize the negative inferences that potential buyers draw when her house remains unsold, or may price inordinately low in order to make an early sale and avoid the possibility of negative inferences. Bose, Orosel, Ottaviani & Vesterlund (2006) study dynamic pricing and herding when a monopolist sells to a sequence of buyers, and show that the firm may distort its price to current buyers to facilitate information revelation to
future buyers in order to extract more rent from the latter. In ongoing work, Miklos-Thal & Zhang (2011) show that a monopolist may visibly de-market their product to early adopters to improve the product’s quality image amongst late adopters. Our work adds to this literature by studying whether and when a firm may offer a deal that reveals sales information in a setting where there is a single generation of consumers who may strategically delay their buying decisions. We find that a firm may induce some consumers to strategically wait in order to extract more surplus from them. But it may also be restricted in extracting surplus from other consumers since it must induce them not to wait.

Empirical research has shown that observational learning may significantly influence consumer behavior in online as well as offline settings. Cai, Chen & Fang (2009) conduct a randomized natural field experiment at a restaurant. They find that when consumers are given a ranking of the five most popular dishes, demand for those dishes increased. Chen, Wang & Xie (2011) use a natural experiment on online retailer Amazon to show that, in addition to being influenced by word-of-mouth through consumer reviews, product sales are also influenced by observational learning when Amazon reveals the percentage of consumers who bought the product after considering it. Tucker & Zhang (2011) provide evidence of observational learning from a field experiment on a website listing wedding service vendors. They find that displaying the number of past webpage visits for a vendor’s listing influenced future visits in a manner consistent with observational learning. They also find that narrow-appeal vendors receive more visits than equally popular broad-appeal vendors. In the context of online microloan markets, where individual borrowers raise funds from multiple lenders and prospective lenders can observe a borrower’s funding level, Zhang & Liu (2012) find evidence, consistent with observational learning and rational herding, that well-funded loans tend to attract more funding.

2 Model

We consider a firm that is interested in offering a daily deal to introduce its product (or service) to a group of \( N \geq 3 \) potential new consumers. We are interested in the firm’s trade-off between offering a regular (daily) deal and a social coupon to promote to these consumers. To begin with, to develop our intuition, we analyze the case when there are \( N = 3 \) consumers. Later, in §5.2, we extend our analysis for any \( N \). We assume that each consumer may buy at most one unit of the product. We assume that a consumer’s utility from buying the product at a price \( p \) is given by \( q - p \), where \( q \) denotes the product’s appeal or quality. Since these consumers are not the firm’s regular buyers, we assume that they are uncertain about \( q \). This may especially be the case for a small or local business whose product may not be widely known. This may also be the case for products that consumers tend to buy infrequently (e.g., cosmetic surgeries, home renovations). To capture this uncertainty, we assume that \( q \) can either
be 1 (high quality) or 0 (low quality). As in Bose, Orosel, Ottaviani & Vesterlund (2006), we assume that the firm shares this uncertainty about how potential new consumers will evaluate its product. We note that quality could also be probabilistic in some settings (e.g., in the case of a service), with the product delivering higher utility on an average when its quality is high than when it is low; what will be relevant for our analysis in this case are the expected utilities of these two quality levels, which we have normalized to 0 and 1. Let \( \theta \in (0, 1) \) denote the probability that the product quality is high.

The firm may promote its product by offering a regular deal. In this case, the number of consumers who have bought the voucher is not reported. Let \( p^{rd} \) be the price at which the firm offers its product with a regular deal. Alternatively, the firm may promote its product using a social coupon, in which case the number of consumers who have bought the voucher is publicly reported. Let \( p^{sc} \) be the price that the firm charges consumers for the social coupon. Later, in §5.1, we consider that the firm may also set a minimum limit when offering a social coupon.

![Figure 1: Sequence of the Game](image)

We assume that either type of daily deal is available for two periods, namely periods 1 and 2. Figure 1 lays out the sequence of the game. At the start of period 1, the firm announces the deal through a daily deal website. Consumers visit the website to learn about the deal. Each consumer also obtains a noisy private signal about the product’s quality. For instance, consumers may read reviews about the firm and its products; consumers may differ in their private signals since they may refer to different sources of reviews or may sample only a subset of the reviews as they may be time-constrained (for instance, as in Mazylin 2006). Let consumers be indexed by \( j \in \{1, 2, 3\} \). Let \( s_j \in \{H, L\} \) be the private signal that consumer \( j \) receives. We will refer to \( H \) and \( L \), respectively, as "high" and "low" signals. We will refer to consumers as "high-types" and "low-types" respectively, depending on whether they observed a high signal or a low signal. Let \( t_j \) denote the type of consumer \( j \), such that \( t_j = s_j \).

We assume the following conditional distribution for \( s_j \):

\[
\begin{align*}
\Pr(s_j = H \mid q = 1) &= \alpha; & \Pr(s_j = L \mid q = 1) &= 1 - \alpha; \\
\Pr(s_j = H \mid q = 0) &= 1 - \alpha; & \Pr(s_j = L \mid q = 0) &= \alpha,
\end{align*}
\] (1)
where \( \Pr (X) \) denotes the probability of event \( X \) and \( \alpha \in (\frac{1}{2}, 1) \) represents the accuracy of the signal. Since \( \alpha > \frac{1}{2} \), a high signal is more indicative of a high quality product, and a low signal of a low quality product, with the signals being "correct" \( \alpha \) proportion of the time. As \( \alpha \to \frac{1}{2} \), the signals become non-informative, while as \( \alpha \to 1 \), the signals become perfectly informative. We assume that the signals are independent conditional on product quality, i.e. for \( j, j' \in \{1, 2, 3\}, j \neq j' \):

\[
\Pr (s_j s_{j'} \mid q) = \Pr (s_j \mid q) \Pr (s_{j'} \mid q).
\] (2)

Let \( E[q \mid s] \) be the expected value of \( q \) conditional on having observed a signal \( s \in \{H, L\} \). We note that \( E[q \mid H] > E[q \mid L] \). Let \( E[q \mid S] \) be the expected value of \( q \) conditional on observing a profile of signals \( S \subseteq \bigcup_{k \in \{2, 3\}} \{H, L\}_k \). Since the conditional distribution of the signals is symmetric, we have:\(^5\)

\[
E[q \mid \{H, L\}] = E[q] \quad , \quad E[q \mid \{H, H, L\}] = E[q \mid H] \quad , \quad E[q \mid \{H, L, L\}] = E[q \mid L].
\] (3)

Loosely speaking, a high signal "cancels" a low signal. This feature of the signal distribution is primarily useful to develop our intuition.

After observing their private signals, consumers may choose to either buy the deal in period 1 or to wait until period 2. In the case of a social coupon, the number of vouchers sold in period 1 is displayed on the website at the start of period 2; let \( n^{sc} \) denote the number of vouchers sold in period 1. In the case of a regular deal, this information is not revealed. In either case, we allow for the fact that a consumer who waits in period 1 may not return to the website in period 2 for reasons unrelated to the deal. For instance, the consumer may have become too busy or may have forgotten about the deal. Let \( \delta \in (0, 1) \) denote the probability that the consumer returns in period 2 if she waits, which we will refer to as consumer return probability. We assume that consumers are aware that they may not always return if they wait in period 1. A consumer derives zero utility when she does not return in period 2. A consumer cannot wait beyond period 2, and derives zero utility if she does not buy in period 2.

We note that either type of daily deal is not likely to be profitable if a sufficient number of the firm’s regular buyers avail the deal and pay a lower price than they normally do. Since our main interest in this paper is to examine the firm’s trade-off between offering a social coupon and a regular deal, as opposed to whether or not a firm should offer a daily deal, we assume that the firm’s regular buyers are not able to avail the daily deal and do not model them explicitly. Our assumption holds exactly in some cases: businesses such as dentists, beauty treatment clinics, gyms, home maintenance services and limousine services, can and often do restrict their daily deals to new clients since they can identify their existing clients. Our assumption may also be a reasonable approximation in the case of a small

\(^5\)The conditional distribution is symmetric in that the probability that it is correct is the same whether the quality is high or low: \( \Pr (s_j = h \mid q = 1) = \Pr (s_j = l \mid q = 0) = \alpha \).
business whose existing clientele is relatively small compared to the market reach of a daily deal (i.e., \(N\) is relatively large compared to the size of firm’s existing clientele). For instance, anecdotal evidence suggests that a daily deal may attract more new consumers than a business may usually serve in the course of several months or even a year (e.g., Bloomberg Businessweek 2009). Further, firms typically do not announce their daily deals to regular customers. Hence, a consumer would have to constantly monitor deal websites for a deal from a business where she is a regular customer.\(^6\) Therefore, a daily deal may not attract many of the firm’s regular buyers as they may not be willing to incur the hassle of finding and using daily deals. Later, in §6, we consider the implications if some of the firm’s regular buyers also use the daily deal. The intention behind our parsimonious approach is to focus on the main differences between the two types of daily deals as opposed to factors that may be common to both.

### Table 2: Model Notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(N)</td>
<td>Number of consumers</td>
</tr>
<tr>
<td>(q)</td>
<td>Promoted product’s appeal or quality, (q = 0) (low) or (q = 1) (high)</td>
</tr>
<tr>
<td>(\theta)</td>
<td>Probability that the product quality is high</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>Accuracy of consumers’ private signals</td>
</tr>
<tr>
<td>(\delta)</td>
<td>Consumer return probability</td>
</tr>
<tr>
<td>(k)</td>
<td>Minimum limit for social coupon</td>
</tr>
</tbody>
</table>

subscript \(t\): Private signal observed by consumer / Consumer’s type, \(t \in \{H, L\}\)

subscript \(T\): Period \(T \in \{1, 2\}\)

\(\hat{p}_t\): Private valuation of a type \(t\) consumer

\(n^{sc}\): Number of social coupons sold in period 1

\(p^{rd}, p^{sc}\): Price of regular deal \((p^{rd})\), social coupon \((p^{sc})\)

\(b_{1t}\): Buying probability of type \(t\) consumer in period 1, \(b_{1t} \in [0, 1]\)

\(b_{2t}\): Buying decision of type \(t\) consumer in period 2, \(b_{2t} \in \{0, 1\}\)

\(n_t^*\): Threshold \(n^{sc}\) for type \(t\) consumer to buy in period 2

\(E[q \mid t]\): Expected product valuation of a type \(t\) consumer

\(E[q \mid t, n^{sc} = n]\): Expected product valuation of type \(t\) consumer who observes \(n^{sc} = n\)

\(U_{sc}^{sc}\): Expected utility of buying a social coupon in period \(T\) for type \(t\) consumer.

\(Q^{rd}, Q^{sc}\): Demand for regular deal \((Q^{rd})\), social coupon \((Q^{sc})\)

We assume that firms maximize their profits from voucher sales (as is, for instance, common in the literature on coupons and discount promotions) and do not consider repeat sales. This may be the case,\(^6\) We also note that firms also typically do not offer daily deals on a regular basis. Firms that do offer deals repeatedly space them apart by several months, making it difficult to predict when they will offer the next deal.
for instance, for products or services that consumers buy infrequently (e.g., cosmetic surgeries, home renovations). This may also be the case when firms offer deals primarily to manage lean periods in their business, or if those who buy the deal are mostly "bargain hunters" who may not buy the product at the regular price. For instance, Dholakia (2011) finds that, on an average, eighty percent of deal users do not return to buy at the firm’s regular price. Later, in §6, we consider the implications when deal users may become regular buyers after using a deal and make a repeat purchase at the regular price.

Since daily deals are typically available only for a short duration (between two to four days), we do not assume any discounting for firms or for consumers over this time-frame. We assume that consumers redeem vouchers at the end of period 2 and do not save them for a future occasion. We further assume the following: consumers are rational, forward-looking, risk-neutral and maximize their expected utility; the firm is forward-looking, risk-neutral and maximizes its expected profits; \( N, \theta, \alpha \) and \( \delta \) are common knowledge to the firm and to consumers. Table 2 summarizes our model notation.

### 3 Consumer Buying Behavior and Market Demand

**3.1 Firm Offers a Regular Deal**

As such, a consumer has no incentive to wait till period 2 to make her buying decision; if she does not buy in period 1, then she will not buy in period 2 either. Let \( \tilde{p}_t = E[q | t] \) indicate the willingness to pay of a type \( t \) consumer based on her private information. We will refer to \( \tilde{p}_t \) as the *private valuation* of the product by a consumer of type \( t \). Then all consumers will buy when \( p_{rd} \leq \tilde{p}_L \), and only high-type consumers will buy when \( p_{rd} \in (\tilde{p}_L, \tilde{p}_H) \). Let \( Q_{rd} \) denote the total market demand when the firm uses a regular deal. The expected market demand is given by:

\[
E[Q_{rd}] = \begin{cases} 
3 & \text{if } p_{rd} \leq \tilde{p}_L; \\
3[\theta \alpha + (1 - \theta)(1 - \alpha)] & \text{if } \tilde{p}_L < p_{rd} \leq \tilde{p}_H; \\
0 & \text{otherwise.}
\end{cases}
\]  (4)

**3.2 Firm Offers a Social Coupon**

To develop our intuition, consider the price range \( p_{sc} \in (\tilde{p}_L, \tilde{p}_H) \). Suppose that, similar to the scenario with a regular deal, only high-type consumers buy in period 1. As before, a low-type consumer will not buy in period 1 since the price exceeds her private valuation. Now, in period 2, she can observe the deal’s progress, \( n_{sc} \), which will perfectly reveal the private information of other consumers since only high-type consumers buy in period 1. If \( n_{sc} < 2 \), then her valuation would still be less than
the price and it would not be worthwhile for her to buy the deal.\footnote{We have $E[q \mid L, n^{sc} = 1] = E[q \mid \{H,L,L\}] = \bar{p}_L$ and $E[q \mid L, n^{sc} = 0] = E[q \mid \{L,L,L\}] < \bar{p}_L$.} However, if $n^{sc} = 2$, then both the other consumers observed a high signal, and the low-type consumer’s valuation would be given by $E[q \mid L, n^{sc} = 2] = \bar{p}_H > p^{sc}$.\footnote{We have $E[q \mid L, n^{sc} = 2] = E[q \mid \{H,H,L\}] = \bar{p}_H$.} Hence, it would be worthwhile for her to buy the deal. Thus, the low-type consumer benefits from observing the deal’s progress and may buy the deal even though the price exceeds her private valuation.

Interestingly, even a high-type consumer may potentially benefit from observing the deal’s progress even though the price is below her private valuation. For, suppose that a given consumer, say consumer 1, observes a high signal, but decides to wait till period 2 instead of buying in period 1 (i.e., she alone deviates from buying in period 1 when she observes a high signal). In period 2, if she observes that $n^{sc} > 0$, then it would still be worthwhile for her to buy the deal.\footnote{We have $E[q \mid \{H,H,L\}] = \bar{p}_H$ and $E[q \mid \{H,H,H\}] > \bar{p}_H$.} However, if she observes that $n^{sc} = 0$, then this would indicate that both the other consumers observed a low signal, and her valuation would be given by $E[q \mid H, n^{sc} = 0] = \bar{p}_L < p^{sc}$. Hence, she will not buy the deal. Thus, by waiting till period 2, the high-type consumer can make a more informed decision and avoid the downside of buying the deal when both the other consumers observed a low signal. In fact, conditional on returning in period 2, the expected utility of making a more informed decision in period 2 would be necessarily higher than the expected utility of buying the deal in period 1 based only on her private information. Now, when making a decision in period 1, the consumer must also take into account that she will not always return if she waits. But if the consumer return probability ($\delta$) is sufficiently high, then it would be attractive even for a high-type consumer to wait, even though the price is below her private valuation.

Thus, unlike in the case of a regular deal, it cannot always be that high-type consumers buy in period 1, for this would not be consistent with consumers having rational beliefs about how other consumers buy. We now formally describe our solution approach. We analyze the following extensive-form game of incomplete information for consumers. In period 1, a consumer’s information set consists of the type of deal offered by the firm, the deal price and her private signal. In period 2, her information set additionally includes the number of other consumers who bought the deal in period 1. Let $A = \{0,1,2\}$ be the set of possible values of $n^{sc}$ at the consumer’s period 2 information set. Let $b^j_t = \left(b^j_{1t}, b^j_{2t}(n^{sc})\right)$ denote the strategy of consumer $j$ when she is of type $t \in \{H,L\}$, where:

- $b^j_{1t}$ denotes her period 1 action: $b^j_{1t} = 1$ if she buys, and $b^j_{1t} = 0$ if she waits till period 2.
- $b^j_{2t}(n^{sc})$ denotes her period 2 action at information set $n^{sc} \in A$: $b^j_{2t}(n^{sc}) = 1$ if she buys and $b^j_{2t}(n^{sc}) = 0$ if she does not buy.
We note that while a consumer’s period 2 information sets are reached only if she chooses to wait, the beliefs at these information sets can be uniquely determined (per Bayes rule) even when she chooses not to wait, by considering a small tremble in her period 1 action. We use sequential equilibrium as our solution concept. We restrict attention to symmetric equilibria where all consumers of a particular type follow identical strategies, i.e., \( b_t^j = b_t \). Let \( b = (b_H, b_L) \) denote such a symmetric strategy profile. Mixed strategies consist of consumers mixing between their actions at each information set. It will be sufficient to consider mixed strategies where either type may mix only between their period 1 actions.

To conserve on notation, we redefine \( b_t = (b_1, b_2 (n^{sc})) \) to denote the possibly mixed strategy of a type \( t \) consumer, where \( b_1 \in [0, 1] \) is the probability that she buys in period 1.

Let \( U_{1t}^{sc} \) denote the utility of a type \( t \) consumer if she buys the deal in period 1, given by,

\[
U_{1t}^{sc} = E [q - p^{sc} | t].
\]

Given the strategy profile \( b \), let \( \Pr (n^{sc} = n | t) \) denote the probability that a consumer of type \( t \) reaches information set \( n^{sc} = n \) in period 2 if she decides to wait in period 1 (i.e., she finds that \( n \) consumers bought the deal in period 1), and let \( E [q - p^{sc} | t, n^{sc} = n] \) denote her expected utility if she buys the deal at this information set. Given the strategy profile \( b \), let \( U_{2t}^{sc} \) denote the expected utility of a type \( t \) consumer making her buying decision in period 2, conditional on returning to make the decision. \( U_{2t}^{sc} \) is given by,

\[
U_{2t}^{sc} = \sum_{n \in A} E [q - p^{sc} | t, n^{sc} = n] \Pr (n^{sc} = n | t) b_2 (n),
\]

**Equilibrium Buying Behavior:** In lemma 1 in Appendix A, we show that in any equilibrium, (i) if high-type consumers weakly prefer to wait till period 2, then low-type consumers must strictly prefer to wait till period 2, and (ii) if low-type consumers weakly prefer to buy in period 1, then high-type consumers must strictly prefer to buy in period 1. Intuitively, low-type consumers have more to lose by not waiting to observe the deal’s progress. Then, in lemma 2 in Appendix A, we show that in any equilibrium consumers will follow a threshold strategy in period 2: a type \( t \) consumer will buy in period 2 if \( n^{sc} \geq n_t^* \), where \( n_t^* \in \{0, 1, 2, 3\} \) denotes the threshold for a type \( t \) consumer. These conditions limit the possible equilibrium scenarios. In Theorem 1 in Appendix A, we solve for the equilibrium by construction. Here we only discuss the outcomes. For comparison with a regular deal, it is useful to partition the price range into three regions: (i) \( p^{sc} \geq \bar{p}_H \), (ii) \( \bar{p}_L < p^{sc} < \bar{p}_H \), and (iii) \( p^{sc} \leq \bar{p}_L \).

(i) \( p^{sc} \geq \bar{p}_H \): All consumers wait till period 2 in equilibrium. In period 2, \( n^{sc} \) is not informative and consumers rely only on their private information. High-type consumers buy only when \( p^{sc} = \bar{p}_H \), whereas low-type consumers never buy.\(^{10}\)

\(^{10}\)It is not an equilibrium for high-type consumers to buy in period 1 when \( p^{sc} = \bar{p}_H \) since \( n^{sc} \) would then be perfectly
(ii) \( p^{sc} \in (\bar{p}_L, \bar{p}_H) \): A low-type consumer always waits till period 2 since \( U^{sc}_{1L} < 0 \). We find that there are two equilibrium scenarios depending on the behavior of high-type consumers: (a) they always buy in period 1, (b) they mix between buying in period 1 and waiting till period 2. Consider the first scenario. Suppose a given high-type consumer deviates and waits till period 1.

...and that this equilibrium occurs only when the price is sufficiently below the private valuation of a high-type consumer. Specifically, we have,

\[
U^{sc}_{1H} \geq \delta U^{sc}_{2H} \implies p^{sc} \leq \hat{p}_{11} = \frac{[1 - \delta] \bar{p}_H + \delta \Pr(n^{sc} = 0 \mid H) \bar{p}_L}{1 - \delta + \delta \Pr(n^{sc} = 0 \mid H)},
\]

(7)

where \( \hat{p}_{11} < \bar{p}_H \). \( \hat{p}_{11} \) has an intuitive form: it is the weighted average of two factors, \( \bar{p}_H \) and \( \bar{p}_L \). \( \bar{p}_H \) is the private valuation of a high-type consumer and reflects the value of acting solely based on her private information, whereas \( \bar{p}_L \) is her valuation when \( n^{sc} = 0 \) and reflects the downside that she can avoid if she waits. The balance between these two factors depends on the consumer return probability \( (\delta) \) as well as the scope for avoiding the downside by waiting, \( \Pr(n^{sc} = 0 \mid H) \). In (7), these aspects are captured by the weights of \( \bar{p}_H \) and \( \bar{p}_L \). \( \hat{p}_{11} \) is decreasing in \( \delta \) and \( \Pr(n^{sc} = 0 \mid H) \) since waiting becomes more attractive. In the case of low-type consumers, it follows from our earlier discussion that they buy in period 2 if \( n^{sc} = 2 \). Hence, their threshold \( n^*_L = 2 \).

Consider the second scenario, where a high-type consumer mixes between buying the deal in period 1 and waiting till period 2. We find that this equilibrium occurs when \( p^{sc} \in (\hat{p}_{11}, \bar{p}_H) \). As such, we note from (7) that, when \( n^{sc} \) is perfectly informative, \( U^{sc}_{1H} < \delta U^{sc}_{2H} \) if \( p^{sc} > \hat{p}_{11} \). But when high-type consumers wait with positive probability, the equilibrium is only partially separating between the two consumer types and \( n^{sc} \) is not perfectly informative. Hence, the value of waiting to make a more informed decision is lower, and a high-type consumer is indifferent between buying in period 1 and waiting till period 2. In equilibrium, we have,

\[
U^{sc}_{1H} = \delta U^{sc}_{2H} \implies p^{sc} = \frac{[1 - \delta] \bar{p}_H + \delta \Pr(n^{sc} = 0 \mid H) E[q \mid H, n^{sc} = 0]}{1 - \delta + \delta \Pr(n^{sc} = 0 \mid H)}.
\]

(8)

Again, the RHS of (8) can be thought of as a weighted average of \( \bar{p}_H \) and \( E[q \mid H, n^{sc} = 0] \), where the latter reflects the downside that can be averted by waiting. We find the probability that high-type consumers buy in period 1 is decreasing in price, since it is more imperative to avoid the downside of buying early when the price is higher.

informative and high-type consumers can derive strictly positive utility by waiting to observe \( n^{sc} \), which is a contradiction.
(ii) \( p^{sc} \leq \bar{p}_L \): A high-type consumer always buys the deal in period 1, since it would be worthwhile to do so even if she knew that both the other consumers observed a low signal. We find that there are three possible equilibrium scenarios depending on the behavior of low-type consumers: (a) they always buy in period 1, (b) they mix between buying in period 1 and waiting till period 2, and (c) they always wait till period 2. Depending on the beliefs of low-type consumers at their period 2 information sets, there can be multiple equilibria in some instances. To select an equilibrium, we assume that consumers are able to coordinate their beliefs such that the Pareto-dominant equilibrium prevails.

We note that it is always an equilibrium for all low-type consumers to buy in period 1 when \( p^{sc} \leq \bar{p}_L \); in this case, \( n^{sc} \) is not informative and, thus, there is no benefit of waiting. It can also be an equilibrium for low-type consumers to wait till period 2; low-type customers will buy in period 2 only if \( n^{sc} > 0 \), which allows them to avoid the downside that both the other consumers observed a low signal. For this to be an equilibrium strategy, we require that:

\[
U^{sc}_{L1} \leq \delta U^{sc}_{2L} \implies p^{sc} \geq \hat{p}_{10} = \frac{[1 - \delta] \bar{p}_L + [\delta \Pr(n^{sc} = 0 | L)] E[q | \{L, L, L\}]}{1 - \delta + \delta \Pr(n^{sc} = 0 | L)}.
\] (9)

There also exists mixed strategy equilibria where low-type consumers mix between buying in period 1 and waiting till period 2, when \( p^{sc} \in (\hat{p}_{10}, \bar{p}_L) \). Hence, multiple equilibria occur over the price range \([\hat{p}_{10}, \bar{p}_L]\). High-type consumers are equally better off in all such equilibria, since they always buy in period 1. Low-type consumers are better off in an equilibrium where they strictly prefer to wait till period 2 (and hence derive higher utility than buying in period 1), than in equilibria where they buy in period 1 with positive probability. Essentially, the equilibrium where \( n^{sc} \) is perfectly informative Pareto-dominates the others. Therefore, for \( p^{sc} \in (\hat{p}_{10}, \bar{p}_L) \), we select the equilibrium where low-type consumers always wait. Thus, we find that low-type consumers too may delay their buying decision even when the price is below their private valuation.

**Market Demand:** The observational learning facilitated by a social coupon can be a double-edged sword. On the one hand, a social coupon may cause some consumers to buy the deal even though the price is higher than their private valuation; these consumers would not have bought the deal if it were a regular deal (at the same price). On the other hand, it may also cause some consumers to wait even when the price is below their private valuation; these consumers would have always bought the deal in period 1 if it were a regular deal. Such a delay causes a loss in demand for three reasons: (i) a consumer who delays her decision may not return in period 2, (ii) a consumer who returns in period 2 may decide not to buy after observing others, and (iii) in instances where a high-type consumer delays her buying decision, it reduces the chance that \( n^{sc} \) will exceed the threshold for low-type consumers in period 2, thereby lowering the probability that they buy. Theorem 1 in Appendix A also describes
the expected market demand with a social coupon. For a given price, we say that a social coupon "stimulates" demand if it leads to higher expected demand than a regular deal, and it "suppresses" demand if it leads to lower expected demand than a regular deal. We find that, depending on the price, a social coupon may either stimulate or suppress demand. Figure 2 illustrates the outcomes for a specific instance. The following proposition describes our findings.

**Proposition 1.** Compared to a regular deal, a social coupon stimulates demand in the price range \((\tilde{p}_L, \tilde{p}_{12})\) and suppresses demand in the price ranges \((\tilde{p}_{10}, \tilde{p}_L)\) and \((\tilde{p}_{12}, \tilde{p}_H)\), where \(\tilde{p}_{12} \in (\tilde{p}_{11}, \tilde{p}_H)\).

Therefore, a social coupon can be effective in stimulating demand from low-type consumers only if the price is sufficiently below the private valuation of high-type consumers. In particular, \(\tilde{p}_{11}\) is the highest price up to which the firm can leverage the full demand from high-type consumers to stimulate demand from low-type consumers. At this price, the social coupon stimulates demand by \(3\delta \Pr(\{H, H, L\})\), which is the probability that there is a low-type consumer who returns in period 2 to observe that \(n^{sc} = 2\). As we will see in §4, \(\tilde{p}_{11}\) plays an important role in determining the firm’s optimal promotional strategy. It is thus useful to define \(\eta = \frac{\tilde{p}_H - \tilde{p}_{11}}{\tilde{p}_H}\) as the *pricing loss ratio* for a social coupon, which measures how much lower the firm should price the social coupon relative to a regular coupon to ensure that high-type consumers always buy in period 1. We also define \(\gamma = \frac{3\delta \Pr(\{H, H, L\})}{3 \Pr(H)} = \alpha (1 - \alpha) \delta\) as the *volume gain ratio* for a social coupon, which measures how much the firm can stimulate demand by leveraging the full demand from high-type consumers as compared to a regular coupon when it generates demand only from high-type consumers. \(\eta\) and \(\gamma\) capture competing effects of a social coupon. The following corollaries describe how they are influenced by market factors.

![Figure 2: Market Demand with Regular Deal and Social Coupon](image)

\(\theta = 0.5, \alpha = 0.75, \delta = 0.3\)
**Corollary 1.** The volume gain ratio ($\gamma$) for a social coupon is independent of the probability that the product quality is high ($\theta$), is decreasing in the accuracy of consumer signals ($\alpha$), and is increasing in the consumer return probability ($\delta$).

When $\theta$ is higher, both the expected demand from high-type consumers and the demand stimulated from low-type consumers increase proportionately; the former because there is higher probability that a consumer is of high-type, and the latter because there is higher probability that a low-type consumer observes that the coupon is sufficiently popular. Thus, $\gamma$ is unaffected by $\theta$. When $\alpha$ is higher, the signals observed by consumers are more correlated. Consequently, it is less likely that one consumer observed a high signal while another observed a low signal, and the possibility for a social coupon to stimulate demand is lower. Lastly, $\gamma$ is higher when $\delta$ is higher, since low-type consumers are more likely to return in period 2.

**Corollary 2.** The pricing loss ratio ($\eta$) for a social coupon is decreasing in the probability that product quality is high ($\theta$), follows an inverted-U relationship with the accuracy of consumer signals ($\alpha$), and is increasing in the consumer return probability ($\delta$).

When $\theta$ is higher, from the perspective of a high-type consumer, there is lower probability that other consumers observed a low signal and hence a lesser incentive to observe their decisions. Thus, $\eta$ is lower. When $\alpha$ is higher, on the one hand, a high-type consumer’s own signal is more accurate and is also more correlated with that of others, which makes it less imperative to wait. On the other hand, the signals of other consumers are also more accurate, which makes it more attractive to wait. The latter aspect dominates when $\alpha$ is low. Consequently, we find that $\eta$ is initially increasing and then decreasing with $\alpha$. Lastly, $\eta$ is higher when $\delta$ is higher since the benefit of waiting for high-type consumers is higher.

4 **When Should a Firm Offer a Social Coupon**

When the firm promotes the product using a regular deal, it may be optimal to either charge a price of $\tilde{p}_H$, such that only high-type consumers buy, or a price of $\tilde{p}_L$, such that all consumers buy. We will refer to these respective strategies (of charging $\tilde{p}_H$ and $\tilde{p}_L$) as "low-discount strategy" and "deep-discount strategy". We find that a low-discount strategy is optimal iff the accuracy of the consumers’ private signals is sufficiently high (see lemma 3 in Appendix A). In such instances, the difference between the prices $\tilde{p}_H$ and $\tilde{p}_L$ is sufficiently high, which makes deep-discounting unattractive. This is even more so when $\theta$ is lower, since the the private valuation of a low-type consumer is then lower, making the low-discount strategy more attractive. Figure 3 depicts the instances where a low-discount strategy (dark-shaded region) and a deep-discount strategy (unshaded region) are optimal for a regular deal.
In either case, we find that offering a social coupon can be more profitable under certain conditions. Figure 3 also depicts instances (hatched region) where a social coupon is more profitable than a regular deal for some value of $\delta$. Given that a social coupon stimulates demand for $p^{sc} \in (\hat{p}_L, \hat{p}_{12})$ and demand is constant for $p^{sc} \in (\hat{p}_L, \hat{p}_{11}]$, a social coupon can be more profitable only when $p^{sc} \in [\hat{p}_{11}, \hat{p}_{12}]$. Since demand in this price range is highly non-linear in price (see Theorem 1 in Appendix A), we proceed to analyze the firm’s optimal promotional strategy numerically.\textsuperscript{11} We find that in most instances where a social coupon is more profitable, the optimal price to charge is, in fact, $p^{sc} = \hat{p}_{11}$, such that all high-type consumers buy in period 1. This is depicted in the left panel of Figure B.1 in Appendix B, which shows instances where the optimal price is higher than $\hat{p}_{11}$ for some $\delta$ when the social coupon is more profitable. Even in this region, we find that a price of $\hat{p}_{11}$ is optimal if $\delta$ is not too high; this is depicted in the right panel of the same figure for a specific instance of $\theta$ ($= 0.95$). Thus, for the purpose of our discussion, to understand how a social coupon can be more profitable, we focus on instances where it is optimal to charge $\hat{p}_{11}$, noting that the intuition in the other instances is similar.

Figure 3: Firm’s Promotion Strategy

Whenever the low-discount strategy is optimal for a regular deal, such that a firm charges $\bar{p}_H$ to sell only to high-type consumers, a social coupon can be potentially more profitable since it leverages demand from high-type consumers to stimulate demand from low-type consumers. We note that when low-type consumers buy the social coupon in period 2, their willingness to pay is, in fact, $E[q \mid L, n^{sc} = 2] = \bar{p}_H > \hat{p}_{11}$. However, the firm cannot charge $\bar{p}_H$ since it must induce high-type consumers to buy in period 1 by setting a price sufficiently below their private valuation. Thus, even as observational learning enables the firm to stimulate demand, it limits the firm’s ability to extract\textsuperscript{11} We were in fact able to verify the results analytically using Mathematica. Mathematica resolves expressions involving higher order polynomials using advanced computational algebra techniques such as Groebner basis and cylindrical algebraic decomposition (Mathematica 2011). Mathematica also does not provide a step-by-step breakdown of its analysis.
surplus from consumers. Consequently, we find that a social coupon can be more profitable only if $\delta$ is not too high; otherwise, the pricing loss ratio is too high. Further, we also require that $\alpha$ is not too high when $\theta$ is sufficiently low ($\theta < \frac{1}{2}$). This is because the pricing loss ratio is higher when $\theta$ is low and volume gain ratio is decreasing in $\alpha$; hence the pricing loss offsets the volume gain when $\alpha$ is high.

Whenever the deep-discount strategy is optimal for a regular deal, such that a firm charges $p_L$ to ensure that low-type consumers also buy, a social coupon can be potentially more profitable since it leverages demand from high-type consumers to increase the valuation of low-type consumers. In other words, a firm may potentially benefit from inducing low-type consumers to wait in order to charge them a higher price. In this case, we find that a social coupon can be more profitable only if $\delta$ is neither too low nor too high. $\delta$ must be sufficiently high to ensure that sufficient number of low-type consumers return when the firm induces them to wait, i.e., volume gain ratio must be sufficiently high. In particular, as $\delta \to 0$, we note that the demand from a social coupon converges to that of a regular coupon, and its profits in the price range $[\hat{p}_{11}, \hat{p}_{12}]$ converge to that of the low-discount strategy, which in this case, is strictly lower than that of a deep-discount strategy. However, if $\delta$ is too high, then the pricing loss ratio is too high for the social coupon to be profitable. We also require that $\alpha$ is sufficiently high. Otherwise, the private valuation of a low-type consumer would be sufficiently high and close to that of a high-type consumer so as to make it more profitable to sell to all consumers by deep-discounting a regular deal.\footnote{In the case of a social coupon, selling to all consumers would require that the firm charge a price sufficiently below the private valuation of low-type consumers (see Figure 2), which is strictly dominated by deep-discounting a regular deal.}

Proposition 2. (i) When the low-discount strategy is optimal for a regular deal, offering a social coupon is more profitable iff the consumer return probability ($\delta$) is not too high, and the accuracy of the consumers’ signal ($\alpha$) is not too high when the probability that the product is of high quality ($\theta$) is low. (ii) When the deep-discount strategy is optimal for a regular deal, offering a social coupon is more profitable iff the consumer return probability ($\delta$) is neither too high nor too low, and the accuracy of the consumers’ signal ($\alpha$) is sufficiently high.

Conventional wisdom suggests that firms offer social coupons and set minimum limits to motivate consumers interested in the deal to recommend it to others. However, without a minimum limit, arguably, there is limited incentive for referral behavior. Interestingly, we find that social coupons can be profitable even without a minimum limit and in the absence of referral behavior; social coupons may enable a firm to leverage demand from consumers with favorable beliefs about their product to stimulate demand from those with unfavorable beliefs. However, social coupons may also cause consumers with
favorable beliefs to delay their buying decision. We find that a necessary condition for a social coupon to be more profitable is that the consumer return probability is not too high and, in some instances, not too low.

Our results further shed light on the interplay between the probability that a firm’s product appeal or quality is high and the accuracy of consumers’ signals. Interestingly, a firm can benefit from offering a social coupon even when $\theta$ is relatively low. This, however, requires that the accuracy of consumers’ signals is neither too high nor too low. For instance, limited information (e.g., limited online reviews) may be available about the products of firms that are relatively new to the market. Such firms may benefit from a social coupon even if there is relatively low probability that their product appeal is low. However, such firms are better off offering a regular deal (using a deep-discount strategy), if there is relatively high probability that their product appeal is high. Similarly, more established firms with more widely available product information can benefit from a social coupon only if there is relatively high probability that their product appeal is high. Otherwise, such firms are better off offering a regular deal (using a low-discount strategy).

**Influencing Consumer Return Probability**

While a number of factors may affect consumer return probability, a firm may also be able to influence this to some degree. For instance, a firm can send reminders to consumers to prompt them to reconsider a coupon. Or, it can shorten the time period for which the deal is available, thereby effectively reducing the time window or opportunity available for consumers to return to reconsider the coupon. This raises the question of whether a firm should increase or decrease the consumer return probability. Consumers benefit when $\delta$ is higher, since this provides them more opportunity to make a better informed decision. Does this mean that a firm should strive to increase $\delta$? Interestingly, we find that this may not always be the case. While, on the one hand, a higher $\delta$ leads to more demand from low-type consumers (higher volume gain ratio), on the other hand, it also makes waiting more attractive for high-type consumers (higher pricing loss ratio). We find that a firm has to strike a balance between these opposing factors, and increasing $\delta$ beyond a point may prove counter-productive.

**Proposition 3.** In instances where it is more profitable to offer a social coupon, the profitability of a social coupon follows an inverted-U relationship with consumer return probability.

A distinguishing characteristic of a social coupon is that it enables consumers to check the progress of the deal. However, from the firm’s perspective, making this aspect more salient and providing consumers more opportunity to check the deal’s progress may not always be sensible. Indeed, of the social coupon websites listed in Table 1, we found that presently only one (KGB Deals) allowed...
consumers to set reminders to check the deal’s progress. To the extent that such reminders significantly
increase the consumer return probability, our results suggest that they may reduce the value firms gain
from offering social coupons.

5 Extensions

To gain further insight regarding the use of social coupons, we analyze two separate extensions to our
main model. We first investigate what role the use of a minimum limit may play in this context, and
whether and when it may be profitable to set one. We then examine consumer buying behavior when
there are more consumers \((N \geq 3)\) in the market, and analyze the firm’s optimal strategy when there
are a large number of consumers \((N \to \infty)\).

5.1 On the Use of a Minimum Limit

Let \(k\) denote the minimum limit when the firm sets one. The deal is valid only if the minimum limit is
reached, and consumers committing to buy the deal are charged for it if and when the minimum limit
is reached. We note that \(k = 1\) is equivalent to not having a minimum limit. Thus, \(k = 2\) or \(k = 3\).
Theorems 2 and 3 in Appendix A describe the equilibrium buying behavior in these respective cases.

When Minimum Limit is \(k = 2\): Consider the price range \(p^{sc} \in (\bar{p}_L, \bar{p}_H)\). We find that high-type
consumers always (commit to) buy in period 1, whereas, depending on the price, low-type consumers
may always wait till period 2, or mix between buying in period 1 and waiting till period 2. In the first
equilibrium scenario, \(n^{sc}\) is perfectly informative. A low-type consumer will buy in period 2 only if
she observes that \(n^{sc} = 2\). Now a high-type consumer who buys in period 1 will be charged for the
deal only if at least one other consumer also buys in period 1 or 2. But in period 2, only a low-type
consumer may buy and will do so only if she observes that both the other consumers bought in period
1. Hence, effectively, a high-type consumer buying in period 1 will be charged for the deal only if at
least one other consumer also buys in period 1, i.e., if there is at least one other high-type consumer.
But this implies that there is no incentive for a high-type consumer to wait; for even if she does so,
she will again buy only if there was at least one other high-type consumer (i.e., \(n^{sc} \geq 1\) since \(n^*_H = 1\)).
In fact, we have \(U_{1H}^{sc} = U_{2H}^{sc}\), and the consumer strictly prefers to buy in period 1. Thus, while in the
absence of a minimum limit, a high-type consumer would have had to wait till period 2 to avoid the
downside when both the other consumers observed a low signal, this is no longer necessary when there
is a minimum limit. The minimum limit makes waiting redundant by essentially enabling her to make
her actions costlessly contingent on that of other consumers. Stated differently, the minimum limit
acts as a form of insurance that protects a high-type consumer against the risk she faces when acting
solely on the basis of her private information, thus eliminating her incentive to wait.
In fact, even a low-type consumer can benefit to some extent from the minimum limit. If a low-type consumer were to buy in period 1, then the minimum limit would also protect her against the possibility that both the other consumers were of low-type (in which case n_{sc} = 1, and neither low-type consumers would buy even if they returned in period 2). However, it does not protect her against the possibility that only one of the other consumers is of low-type. Thus, the minimum limit does not make waiting redundant for low-type consumers, but it offers some protection against the downside of buying early. Consequently, low-type consumers’ willingness to pay for the deal in period 1 is higher than their private valuation of \( \bar{p}_L \). We find that when the price is sufficiently high, it is still unattractive for a low-type consumer to buy in period 1. Whereas, when the price is not as high, low-type consumers mix between buying in period 1 and waiting till period 2. In the latter scenario, while \( n_{sc} \) is no longer perfectly informative, the minimum limit still affords high-type consumers protection against the downside that they may avoid by waiting.\(^{14}\) Hence, waiting is again redundant for high-type consumers. Thus, in this price range, the minimum limit eliminates waiting by high-type consumers, and may even cause low-type consumers to buy in period 1 with positive probability.

Interestingly, we find that high-type consumers may buy even when \( p_{sc} > \bar{p}_H \). Since the minimum limit continues to protect a high-type consumer against the possibility that both the other consumers observed a low-signal, her effective valuation is higher than her private valuation of \( \bar{p}_H \). We find that high-type consumers always buy in period 1 when the price is not much higher than \( \bar{p}_H \). At higher prices, high-type consumers mix between buying in period 1 and waiting till period 2. If \( p_{sc} > E[q | \{H, H\}] \), then none of the consumers buy. Thus, the use of a minimum limit leads to positive demand even beyond the private valuation of high-type consumers.

Finally, in the price range \( p_{sc} \leq \bar{p}_L \), we find that high-type consumers always buy in period 1, but there may be multiple equilibria in some instances that differ in the behavior of low-type consumers. We again select the equilibrium that is Pareto-dominant for consumers.\(^{15}\) We find that when the price is sufficiently close to \( \bar{p}_L \), low-type consumers may wait till period 2 with positive probability and buy if \( n_{sc} \geq 1 \). Now, it might appear that waiting should be redundant; for if low-type consumers will buy in period 2 only if \( n_{sc} \geq 1 \), then they should instead buy in period 1 since the minimum limit will be reached only if at least one other consumer buys in period 1. However, if a low-type consumer buys in period 1, then it may cause another low-type consumer who waits till period 2 to buy the deal, thereby making the deal reach its minimum limit. This can occur when there are three low-type consumers and

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\(^{14}\)In equilibrium, \( n_{H} = 1 \). Moreover, as in lemma 1, it can be shown that if low-type consumers are indifferent between buying in period 1 and waiting till period 2, high-type consumers strictly prefer to buy in period 1.

\(^{15}\)In this case, even a partially separating equilibrium Pareto-dominates one where low-type consumers always buy in period 1. When \( n_{sc} \) is non-informative, the minimum limit affords no protection and \( U_{1H}^{sc} = \hat{p}_H - p_{sc} \). However, when \( n_{sc} \) is partially informative, the minimum limit offers some protection and \( U_{1H}^{sc} > \hat{p}_H - p_{sc} \).
consumers, one of whom buys in period 1.\textsuperscript{16} If it occurs, then the low-type consumer’s valuation of the deal is $E[q | \{L, L, L\}] < \bar{p}_L$. Hence, when the price is sufficiently close to $\bar{p}_L$, the prospect of influencing a low-type consumer in period 2 counteracts the protection offered by the minimum limit, which motivates low-type consumers to wait with positive probability. In turn, this ensures that $n^{sc}$ is informative. Essentially, low-type consumers internalize the informational externality that their actions may impose on other low-type consumers, and this induces them to make decisions that reveal more information. Nevertheless, $U^{sc}_{1L}$ is higher when there is a minimum limit since it still offers some protection. Therefore, we find that low-type consumers wait to a lesser extent than they do without a minimum limit. When the price is sufficiently below $\bar{p}_L$ all consumers buy in period 1.

\textbf{When Minimum Limit is } $k = 3$: We find that setting a higher minimum limit offers even more protection against an unpopular deal for consumers who buy in period 1; when $k = 3$, two other consumers must also buy for the deal to be valid, which is even less likely when one or both the other consumers observed a low signal. When $p^{sc} \in (\bar{p}_L, \bar{p}_H)$, we again find that high-type consumers always buy in period 1, while low-type consumers may always wait till period 2 or mix between buying in period 1 and waiting till period 2; the latter occurs over a larger range of prices than when $k = 2$. Again, low-type consumers wait with positive probability even though $n^{sc}_L = 2$, since the prospect of influencing a low-type consumer in period 2 counteracts the protection offered by the minimum limit. When $p^{sc} \geq \bar{p}_H$, high-type consumers buy over a larger range of prices than when $k = 2$, and always do so in period 1. Lastly, when $p^{sc} \leq \bar{p}_L$, low-type consumers may wait over a smaller range of prices than when $k = 2$ and, when they do so, the probability that they wait is lower than before.

\textbf{Implications of Setting a Minimum Limit:} The minimum limit essentially raises the valuation of consumers buying in period 1, since they will be charged for the deal only if it is sufficiently popular. This impacts consumer buying behavior in two ways. Firstly, it extends the maximum price up to which consumers may buy. Secondly, it "accelerates" purchases by either eliminating or reducing the extent of waiting compared to when there is no minimum limit. Consequently, all else being equal, the expected number of consumers committing to buy the social coupon is higher. Further, these effects are more pronounced when the minimum limit is higher. The following proposition describes our findings.

\textbf{Proposition 4.} All else being equal, setting a minimum limit for a social coupon: (i) increases the maximum price up to which there is positive demand, (ii) weakly reduces the probability that consumers delay their buying decisions, and (iii) weakly increases the expected number of consumers committing

\textsuperscript{16}If there are two low-type consumers and one high-type consumer, then the high-type consumer will buy in period 1. Hence, if a low-type consumer buys in period 1, this per-se will not influence the buying decision of the other low-type consumer.
to buy the deal. These effects are even more pronounced when the minimum limit is higher.

Conventional wisdom suggests that firms may set a minimum limit in order to motivate referrals from consumers interested in the deal. Such referrals are likely to accelerate buying decisions and lead to higher demand. Interestingly, we find that even in the absence of referral behavior, setting a minimum limit can have a similar effect; in a context where consumers can postpone their buying decision to learn by observing others, the minimum limit acts as a form of insurance that reduces the risk that consumers face when acting solely on the basis of their private information. This suggests that if one were to empirically observe that a minimum limit accelerates purchases, this would be consistent with either mechanism. Additional information would then be needed to determine the extent to which either mechanism is at work.

Figure 4: Market Demand under Different Deal Types

We had previously noted that a downside of offering a social coupon is that it may motivate some consumers to postpone their buying decisions. We now find that setting a minimum limit can mitigate this to some extent and accelerate purchases. However, using a minimum limit also has its downside: the minimum limit may not be reached, resulting in no sales. Further, the probability that the minimum limit is reached, and that demand is realized, is lower when the minimum limit is higher. This can offset the effect of purchase acceleration. Figure 4 provides a comparison of expected market demand across different types of daily deals for two different values of \( \delta \) for a given \( \theta \) and \( \alpha \). As can be seen, setting a minimum limit extends the price range over which there is positive demand. Moreover, in the price range \([\bar{p}_L, \bar{p}_H]\), setting a minimum limit can also stimulate demand depending on the price. When \( \delta \) is low, setting a minimum limit can stimulate demand at the lower end of the price range by accelerating purchases from low-type consumers; it induces them to buy in period 1 even though the price exceeds their private valuation. Whereas, when \( \delta \) is high, setting a minimum limit can stimulate demand at the higher end of the price range since low-type consumers are more likely to return to buy in period 2 (and high-type consumers always buy in period 1).
Firm’s Optimal Promotion Strategy: We analyze the firm’s optimal promotion strategy when it can choose between offering a regular deal, a social coupon without a minimum limit and a social coupon with a minimum limit. We find that it can indeed be profitable to offer a social coupon with a minimum limit; a necessary condition for this is that the consumer return probability should be sufficiently high. Figure 5 depicts the instances (hatched region) when a social coupon with a minimum limit is most profitable for some $\delta$. In all such instances, we find that it is always optimal to offer a social coupon with a minimum limit of $k = 2$ at a price $p^{sc} = \bar{p}H$, such that high-type consumers always buy in period 1 while low-type consumers wait till period 2 and buy if $n^{sc} = 2$. For clarity, in Figure 5, we do not depict instances where a social coupon without a minimum limit is still the most profitable; this region is the same as in Figure 3.\footnote{For a given $\theta$, $\alpha$, offering a social coupon with a minimum limit may be more profitable than offering one without a minimum limit for some $\delta$. However, the former never fully displaces the latter as the optimal strategy for all $\delta$.} The following proposition describes when a social coupon with a minimum limit is more profitable.

**Proposition 5.** A social coupon with a minimum sign-up limit is more profitable than a regular deal and a social coupon without a minimum limit iff the probability that the product is of high quality ($\theta$) is sufficiently high, the accuracy of the consumers’ signal ($\alpha$) is sufficiently high, and the consumer return probability ($\delta$) is sufficiently high.

Thus, the use of a minimum limit can broaden the scope for offering a social coupon. Whereas without a minimum limit, a social coupon can be more profitable only if the consumer return probability is not too high, with a minimum limit, a necessary condition for a social coupon to be more profitable is that the consumer return probability is sufficiently high. In this sense, the two forms of social coupons may complement each other. In particular, setting a minimum limit essentially enables the
firm to leverage the demand stimulating effect of observational learning without causing consumers with favorable beliefs to delay their purchase. This is especially relevant when $\delta$ is high, since, without a minimum limit, high-type consumers have a higher propensity to wait till period 2. Further, the probability that the minimum limit is reached is also higher when $\delta$ is high, since there is then higher probability that low-type consumers return in period 2. Moreover, when $\theta$ and $\alpha$ are high, there is higher probability that low-type consumers observe that the deal is popular, and hence they are more likely to buy the deal; these conditions again increase the likelihood that the minimum limit is met, thereby increasing expected demand. As before, a social coupon without a minimum limit can be profitable only if $\delta$ is not too high and, in instances where a deep-discount strategy is optimal for a regular deal, only if $\delta$ is not too low.

Our findings might explain why the use of minimum limits may have declined over time. In the initial days of social coupons there were relatively few daily deal websites, and each daily deal website offered only a few or, at times only one, new deal every day. Furthermore, the novelty factor of social coupons was also relatively high. Thus, from the perspective of a given deal, consumers were more likely to remember to check its progress, i.e., $\delta$ was relatively high. It would have then been worthwhile to offer a social coupon with a minimum limit. However, over time the number of daily deal websites and the number of new daily deals announced every day have both increased substantially. Moreover, the novelty factor of social coupons has also declined. With multiple deals competing for a consumer’s attention, consumers these days may be less likely to remember to check the progress of a given deal. Then, to the extent that consumer return probability has declined, the use of minimum limits for social coupons would be relatively less attractive.

5.2 When There are More Consumers in the Market

Observational learning could potentially play a larger role when there are more consumers, since there is more information available across consumers. However, since observational learning can be a double-edged sword, it is not immediately apparent how this might affect the firm. We show in §B of the Supplemental Appendix that when there are $N$ consumers, there exists a unique Pareto-dominant equilibrium and the buying behavior follows a similar pattern as before. Specifically, when prices are sufficiently low ($p^{sc} \leq \bar{p}_1$), all consumers buy in period 1. When prices are moderate ($p^{sc} \in (\bar{p}_1, \bar{p}_2)$), high-type consumers always buy in period 1, whereas low-type consumers wait till period 2 and buy if $n^{sc}$ exceeds a certain threshold that depends on the price. When prices are higher ($p^{sc} \in (\bar{p}_2, \bar{p}_H)$), high-type consumers delay their buying decision with positive probability, while low-type consumers always wait. If $p^{sc} = \bar{p}_H$ all consumers wait till period 2 and high-type consumers always buy in period 2. Lastly, if $p^{sc} > \bar{p}_H$, none of the consumers buy in either period.
We find that a social coupon can stimulate demand from low-type consumers to a greater extent when there are more consumers; in §B.1 of the Supplemental Appendix we show that when \( p^{sc} > \tilde{p}_L \), the buying probability of low-type consumers in period 2 can be higher when \( N \) is higher. This is especially the case when \( \theta \) is high; when there are more consumers, \( n^{sc} \) is more informative, and it is more likely that the low-type consumers are positively influenced. In fact, if the deal is sufficiently popular in period 1, then it can lead to a buying frenzy in period 2 as all consumers who return will buy. However, a social coupon can also suppress demand from high-type consumers to a greater extent when there are more consumers; for if \( n^{sc} \) is more informative, then high-type consumers also have a higher incentive to wait. In fact, \( \tilde{p}_2 \), the maximum price up to which high-type consumers buy in period 1, is decreasing in \( N \) and may even be less than \( \tilde{p}_L \).\(^{18}\) Thus, both the demand stimulating and suppressing tendencies of a social coupon can be more pronounced when there are more consumers. This raises the question whether social coupons continue to be viable when there are a large number of consumers. Therefore, in §B.2 of the Supplemental Appendix, we derive sufficient conditions for a social coupon to be more profitable in the limiting case when \( N \to \infty \); we analyze when a social coupon offered at a price \( \tilde{p}_2 \) is more profitable than a regular deal. We find that a social coupon can indeed be more profitable if \( \theta \) and \( \alpha \) are sufficiently high and \( \delta \) is not too high and, in some instances, not too low. Figure 6 depicts our results.

![Figure 6: Use of Social Coupon When There Are Large Number of Consumers](image)

6 Conclusion

Daily deals have emerged as an increasingly popular and important promotional tool, especially for many small and local businesses. Such deals may perform a function similar to that of promotional

\(^{18}\)We show in §B.2 of the Supplemental Appendix that as \( N \to \infty \), \( \tilde{p}_2 < \tilde{p}_L \) if \( \alpha \) is sufficiently low.
advertisements in that they not only create awareness but also entice new customers through discounts. But unlike traditional promotions, daily deals that are social coupons also display the number of deals sold to consumers in real-time. While conventional wisdom suggests that firms offer social coupons to motivate referrals by setting minimum limits, one observes that many social coupons are also offered without minimum limits. In this paper, we examine when and why a firm may do so. Interestingly, while a key feature of a social coupon is that it allows consumers to check the deal’s progress, we find that the profitability of a social coupon may follow an inverted-U relationship with the probability that consumers indeed return to do so. Thus, for instance, sending consumers reminders to check the deal’s progress, in order to increase their return probability, may prove counterproductive. We also find that a social coupon without a minimum limit can be more profitable than a regular deal only if the consumer return probability is not too high and, in some instances, not too low.

We further examine the role played by a minimum limit in this context. We find that it may accelerate purchases and enable the firm to extract more surplus from consumers. We also find that offering a social coupon with a minimum limit can be optimal only if the consumer return probability is sufficiently high. In this sense, social coupons with and without minimum limits may be profitable in complementary situations. Our findings may be of interest to managers formulating daily deal promotions. Our results might explain why firms offer social coupons without minimum limits, and why few social coupon websites allow consumers to set reminders to check the deal’s progress. Our analysis also provides an alternative rationale for the use of a minimum limit: it may act as a form of insurance that protects consumers against some of the downside they face when they buy based only on their private information. At the same time, our findings might explain why some firms may no longer set minimum limits: as the number of daily deals being offered has increased over time, and to the extent that consumers are therefore less likely to remember to check the progress of a given deal, the need to use a minimum limit may have declined.

We assumed that the firm maximizes its profits from voucher sales and that consumers who buy the deal do not make repeat purchases at the regular price. This may be appropriate in instances where firms offer deals primarily to manage lean periods in their business, or when consumers make infrequent purchases, or most deal users are bargain hunters who may not buy at the regular price (e.g., Dholakia 2011). To understand the ramifications of repeat purchases, in Appendix C we examine a setting where deal buyers realize the true appeal of the product when they consume it and may make a repeat purchase at the regular price. As such, this makes it more attractive for the firm to offer a regular deal at a deep-discount to encourage repeat purchases. Nevertheless, we find that offering a social coupon without a minimum limit may still be profitable and this is so only if the consumer
return probability is not too high, and in some cases, not too low. Figure C.1 in Appendix C depicts the firm’s optimal promotion strategy.

We assumed that the firm’s regular buyers do not avail the daily deal. This assumption holds exactly when the daily deal is restricted to new customers, and may also approximate situations where there are many more potential new customers than regular buyers or when most regular buyers are not willing to incur the hassle of finding and using the deal. To understand the implications when some of the firm’s regular buyers also avail the deal, in Appendix C we conduct some limited analysis by examining a specific situation: we consider that in addition to the three potential new customers, one of the firm’s loyal regular buyer may also buy the deal.\textsuperscript{19} As such, this makes either type of daily deal less profitable since the regular buyer pays less than the regular price. Furthermore, a deal that offers a deeper discount is now relatively less attractive than one that offers a lower discount. Figure C.2 in Appendix C depicts the firm’s optimal promotion strategy. We find that, when $\theta$ is sufficiently low, offering a regular deal is unprofitable since even a low-discount strategy entails too low a price compared to the firm’s regular price. Interestingly, in some of these instances, it may in fact be profitable to offer a social coupon.\textsuperscript{20} Further, a social coupon may also be more profitable than a regular deal in instances where the latter is profitable. In either case, the consumer return probability should not be too high, and, in some instances, not too low.

We now briefly discuss some limitations of our work. We focused on the interactions within a single cohort of consumers all of whom learn about the deal at the same time. One could extend the analysis to examine interactions between cohorts of consumers who learn about the deal at different times. The actions of the earlier cohort is then likely to influence the actions of the later cohort through observational learning, and may lead to herding behavior that can stimulate or suppress demand. We focused on one possible mechanism that may distinguish social coupons from regular deals even in the absence of minimum limits. We acknowledge however that there may be other forces that affect both regular daily deals and social coupons, such as word-of-mouth (e.g., Jing & Xie 2011). It may be interesting to develop an integrated model that incorporates multiple mechanisms. Lastly, while we focused on the context of social coupons to make our analysis more concrete, our findings may be relevant for other settings where revealing popularity information can influence consumer decisions, such as in crowd-funding, micro-lending or in retailing. However, there may also be additional forces at work specific to these institutional settings. We leave it for future research to examine an entrepreneur’s or a firm’s optimal strategy in these other contexts.

\textsuperscript{19}For instance, Dholakia (2011) finds that, on an average, close to eighty percent of deal users are new customers. Our setting is meant to approximate such a situation.

\textsuperscript{20}This occurs in a small set of instances and is not discernable in Figure C.2.
References


Since, by definition, \( \text{LHS of (A.3)} \) is strictly larger than that of \( \text{LHS of (A.4)} \). If where we have used (A.2). For low-type consumers to strictly prefer to buy in period 1, then high-type consumers strictly prefer to wait till period 2.

Proof. We prove the first part of the lemma. The second part is the contrapositive. Given an equilibrium strategy profile \( b \), let \( N_t = \{ n \in A : E[q|n^{sc} = n, t] > p^{sc} \} \) denote the set of values of \( n^{sc} \) for which type \( t \) consumers buy in period 2. Let \( \hat{U}_{2L}^{sc} \) denote the expected utility of low-type consumers if they were to follow the same period 2 strategy as high-type consumers. \( \hat{U}_{2L}^{sc} \) is given by,

\[
\hat{U}_{2L}^{sc} = \sum_{n \in N_H} (E[q|n^{sc} = n, L] - p^{sc}) \Pr(n^{sc} = n|L).
\] (A.1)

Since, by definition, \( \hat{U}_{2L}^{sc} \leq U_{2L}^{sc} \), it will be sufficient to show \( U_{1H}^{sc} \leq \delta \hat{U}_{2L}^{sc} \Rightarrow U_{1L}^{sc} < \delta \hat{U}_{2L}^{sc} \). We have,

\[
E[q|t, n^{sc} \in N_H] = \frac{\Pr(n^{sc} \in N_H|q=1) \Pr(t|q=1) \Pr(q=1)}{\Pr(n^{sc} \in N_H|t) \Pr(t)} = E[q|t] \frac{\Pr(n^{sc} \in N_H|q=1)}{\Pr(n^{sc} \in N_H|t)}.
\] (A.2)

Since high-type consumers weakly prefer buying in period 2, we require,

\[
U_{1H}^{sc} \leq \delta U_{2H}^{sc}, \iff E[q|H][1 - \delta \Pr(n^{sc} \in N_H|q = 1)] \leq p^{sc} [1 - \delta \Pr(n^{sc} \in N_H|H)],
\] (A.3)

where we have used (A.2). For low-type consumers to strictly prefer to buy in period 2, we require,

\[
U_{1L}^{sc} < \delta \hat{U}_{2L}^{sc}, \iff E[q|L][1 - \delta \Pr(n^{sc} \in N_H|q = 1)] < p^{sc} [1 - \delta \Pr(n^{sc} \in N_H|L)],
\] (A.4)

where we have used (A.2). The LHS of (A.3) is strictly larger than that of (A.4). If \( N_H = \{0, 1, 2\} \), then \( \Pr(n^{sc} \in N_H|t) = 1 \), and the RHS of (A.3) equals that of (A.4). Therefore, (A.3) is sufficient for (A.4) to hold. Instead, if \( N_H \subset \{0, 1, 2\} \), then high-type consumer avoid some downside of buying the coupon in period 2. We then have,

\[
E[q - p^{sc}|H, n^{sc} \in N_H] > E[q - p^{sc}|H] \implies E[q|H] \frac{\Pr(n^{sc} \in N_H|q=1)}{\Pr(n^{sc} \in N_H|H)} > E[q|H],
\]

\[
\implies \Pr(n^{sc} \in N_H|q = 1) > \Pr(n^{sc} \in N_H|q = 1) \Pr(q = 1|H) + \Pr(n^{sc} \in N_H|q = 0) \Pr(q = 0|H),
\]

\[
\implies \Pr(n^{sc} \in N_H|q = 1) > \Pr(n^{sc} \in N_H|q = 0),
\] (A.5)
where we have used (A.2) in the first step. We also have,

$$\Pr (n^{sc} \in N_H|L) = \Pr (n^{sc} \in N_H|q = 1) \Pr (q = 1|L) + \Pr (n^{sc} \in N_H|q = 0) \Pr (q = 0|L),$$

$$< \Pr (n^{sc} \in N_H|q = 1) \Pr (q = 1|H) + \Pr (n^{sc} \in N_H|q = 0) \Pr (q = 0|H),$$

$$= \Pr (n^{sc} \in N_H|H), \quad (A.6)$$

where we have used that $\Pr (q = 1|H) > \Pr (q = 1|L)$ and (A.5). Hence, the RHS of (A.4) is strictly larger than the RHS of (A.3), and thus (A.3) is sufficient for (A.4) to hold.

**Lemma 2.** In any equilibrium, in period 2, consumers follow a threshold strategy of the following form:

$$b_{2t} (n^{sc}) = \begin{cases} 
0 & \text{if } n^{sc} < n^*_t; \\
1 & \text{otherwise,}
\end{cases}$$

for some $n^*_t \in \{0, 1, 2, 3\}$. The threshold for high-type consumers is weakly lower than that for low-type consumers ($n^*_H \leq n^*_L$).

**Proof.** From Lemma 1, we note that either $b_{1H} > b_{1L}$, or $b_{1H} = b_{1L} = 0$, or $b_{1H} = b_{1L} = 1$. In the latter two scenarios, $n^{sc}$ is not informative, and the optimal period 2 strategy is independent of $n^{sc}$. Thus, the result holds trivially with either $n^*_t = 0$ (consumers always buy) or $n^*_t = 3$ (consumers never buy). When $b_{1H} > b_{1L}$, if a consumer is observed purchasing in period 1, then this increases the likelihood that this consumer is of high-type, and therefore the likelihood that the product is of higher quality. Hence, the conditional expected utility of buying a coupon in period 2, $E [q - p^{sc}|n^{sc}, t]$, is increasing in $n^{sc}$. It follows that the optimal period 2 strategy must be of the form described in the lemma. Further, $n^*_H \leq n^*_L$ since for any $n \in A$, $E [q - p^{sc}|n^{sc} = n, H] > E [q - p^{sc}|n^{sc} = n, L]$. \qed

**Lemma 3.** For a regular deal, a low-discount strategy is optimal iff $\alpha > \alpha_1$ where $\alpha_1 = \frac{1+\theta-\sqrt{(1-\theta)(5-\theta)}}{2(2\theta-1)}$ and $\alpha_1 \to \frac{2}{3}$ when $\theta \to \frac{1}{2}$.

**Proof.** The firm’s profits from a low-discount strategy is given by $3\tilde{p}_H \Pr (H)$ and from a deep-discount strategy is given by $3\tilde{p}_L$. It is straightforward to verify that $\tilde{p}_H \Pr (H) > \tilde{p}_L$ iff $\alpha > \alpha_1$. \qed

**Theorem 1.** When the firm offers a social coupon, for $t \in \{H, L\}$,

(i) when $p^{sc} \geq \tilde{p}_H$, there is a unique equilibrium where $b_{1t} = 0$, $n^*_t = 3$, $n^*_H = 0$ if $p^{sc} = \tilde{p}_H$ and $n^*_H = 3$ otherwise;

(ii) when $p^{sc} \in (\tilde{p}_{11}, \tilde{p}_H)$, there is a unique equilibrium where $b_{1L} = 0$, $n^*_L = 2$, $n^*_H = 1$, $b_{1H} \in (0, 1)$ is the root of the following quadratic equation in (A.17) and $\tilde{p}_{11}$ is given by (A.10);

(iii) when $p^{sc} \in (\tilde{p}_L, p_H)$, there is a unique equilibrium where $b_{1H} = 1$, $n^*_H = 1$, $b_{1L} = 0$, $n^*_L = 2$;

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(iv) when \( p^s_{ct} \in [\hat{p}_{10}, \bar{p}_L] \), there is a unique Pareto dominant equilibrium where \( b_{1H} = 1 \), \( n^*_H = 0 \), \( b_{1L} = 0 \), \( n^*_L = 1 \), and \( \hat{p}_{10} \) is given by (A.13);

(v) when \( p^s_{ct} < \hat{p}_{10} \), there is a unique equilibrium where \( b_{1H} = 1 \), \( n^*_L = 0 \).

The expected market demand is given by,

\[
E [Q^{sc}] = \begin{cases} 
0 & \text{if } p^s_{ct} > \bar{p}_H; \\
3\delta & \text{if } p^s_{ct} = \bar{p}_H; \\
3S_3b_{1H} [1 + \delta (1 - b_{1H}) (2 - b_{1H})] + S_2b_{1H} [2 + \delta (2 - b_{1H})] + S_1b_{1H} & \text{if } p^s_{ct} \in (\hat{p}_{11}, \bar{p}_H); \\
3 \Pr(H) + S_2\delta & \text{if } p^s_{ct} \in (\bar{p}_L, \hat{p}_{11}); \\
3 \Pr(H) + S_2\delta + 2S_1\delta & \text{if } p^s_{ct} \in [\hat{p}_{10}, \bar{p}_L]; \\
3 & \text{otherwise;}
\end{cases}
\]

where \( S_m = \binom{3}{m} \left[ \theta \alpha^m (1 - \alpha)^{3-m} + (1 - \theta) (1 - \alpha)^m \alpha^{3-m} \right] \) denotes the probability that there are \( m \) high-type consumers in the market.

**Proof.** Let \( R^t_m \) denote the probability that when a given consumer is of type \( t \), \( m \) of the other consumers are of high-type, given by

\[
P^t_m = \binom{2}{m} \frac{\left[ \theta \alpha^m (1 - \alpha)^{2-m} \Pr(t|q=1) + (1 - \theta) (1 - \alpha)^m \alpha^{2-m} \Pr(t|q=0) \right]}{\Pr(t)}.
\]

There are ten possible equilibrium scenarios consistent with lemmas 1 and 2. We enumerate these and solve for the price range over which each may exist.

(i) \( b_{1H} = 0 \), \( b_{1L} = 0 \): Buying in period 2 is strictly dominated by buying in period 1 since \( n^s_{ct} \) is non-informative. Therefore, consumers do not buy in either period. Hence we require \( p^s_{ct} > \bar{p}_H \).

(ii) \( b_{1H} = 1 \), \( b_{1L} = 0 \), \( n^*_L = 3 \): For \( n^*_L = 3 \) to be optimal, we require \( p^s_{ct} > \bar{p}_H \) so that low-type consumers never buy in period 2. But then high-type consumers will not buy in period 1, which is a contradiction.

(iii) \( b_{1H} = 1 \), \( b_{1L} = 0 \), \( n^*_L = 2 \): We require \( p^s_{ct} \in [\bar{p}_L, \bar{p}_H] \) for \( n^*_L = 2 \) to be optimal. Then, in this price range, \( n^*_H = 1 \). We have,

\[
U^{sc}_{1H} = E [q|H] - p^s_{ct},
\]

\[
U^{sc}_{2H} = [E [q|\{H, H, H\}] - p^s_{ct}] R^H_2 + [E [q|\{H, H, L\}] - p^s_{ct}] R^H_1.
\]

We require,

\[
U^{sc}_{1H} \geq \delta U^{sc}_{2H} \implies p^s_{ct} \leq \hat{p}_{11} = \frac{\theta \alpha(1 - \alpha(2 - \alpha))}{\theta \alpha(1 - \alpha(2 - \alpha)) + (1 - \theta)(1 - \alpha)(1 - (1 - \alpha)(2 + \alpha))}.
\]

Low-type consumers will not buy in period 1 since \( U^{sc}_{1L} \leq 0 \) and \( U^{sc}_{2L} > 0 \).
(iv) $b_{1L} = 1, b_{1H} = 0, n_{L}^* = 1$: We require $p^{sc} \in [E[q[L,L,L]], \bar{p}_L]$ for $n_{L}^* = 1$ to be optimal. Then, in this price range, $n_{H}^* = 0$ and high-type consumers never buy in period 1. But $U_{1H}^{sc} < 0$ when $p^{sc} > \bar{p}_H$, which is a contradiction. 

(vi) $b_{1H} \in (0, 1), b_{1L} = 0$ and $n_{L}^* = 3$: For $n_{L}^* = 3$ to be optimal, we require $p^{sc} > \bar{p}_H$ so that low-type consumers never buy in period 2. Since high-type consumers follow a mixed strategy, they must buy in both periods. But $U_{1H}^{sc} < 0$ when $p^{sc} > \bar{p}_H$, which is a contradiction. 

Then, $n_{H}^* \leq 1$. If $n_{H}^* = 0$, then waiting does not affect the consumer’s buying decision and hence is strictly dominated by buying in period 1, which is a contradiction. Therefore, $n_{H}^* = 1$, which implies that $p^{sc} > \bar{p}_L$. We have,

$$U_{1H}^{sc} = E[q[H]] - p^{sc},$$

$$U_{2H}^{sc} = [E[q[H,H,H]] - p^{sc}] R_2^H \left[1 - (1 - b_{1H})^2\right] + \left[E[q[H,H,L]] - p^{sc}\right] R_1^H b_{1H}. \quad (A.15)$$

We require,

$$U_{1H}^{sc} - \delta U_{2H}^{sc} = 0, \quad (A.16)$$

We note that the LHS above is decreasing in $b_{1H}$, since $U_{2H}^{sc}$ is increasing in $b_{1H}$. The LHS is also decreasing in $p^{sc}$ since $R_2^H + R_1^H < 1$. Further, (A.16) is satisfied when $p^{sc} = \bar{p}_H$ and $b_{1H} = 0$, and when $p^{sc} = \bar{p}_{11}$ and $b_{1H} = 1$ (see (A.10)). Thus, for $p^{sc} \in (\bar{p}_{11}, \bar{p}_H)$ there is a unique $b_{1H} \in (0, 1)$ that satisfies (A.16) and vice-versa. (A.16) can be expressed as

$$p^{sc} = \frac{\theta_0 (1 - \alpha b_{1H} (2 - \alpha b_{1H}))}{\theta_0 (1 - \alpha b_{1H} (2 - \alpha b_{1H})) + (1 - \theta)(1 - \alpha b_{1H} (2 - \alpha b_{1H}))}. \quad (A.17)$$

We note that low-type consumers will not buy in period 1 since $U_{1L}^{sc} < 0$ when $p^{sc} > \bar{p}_L$. To verify that $n_{L}^* = 1$ is not optimal, we require that,

$$2 \left[E[q[H,H,L]] - p^{sc}\right] R_2^L b_{1H} (1 - b_{1H}) + \left[E[q[H,L,L]] - p^{sc}\right] R_1^L b_{1H} < 0. \quad (A.18)$$

Substituting for $p^{sc}$ from (A.17) and simplifying, we require,

$$-1 + \delta b_{1H} \left(2 - b_{1H} + 2 \alpha (1 - \alpha) (1 - b_{1H})^2\right) < 0. \quad (A.19)$$
We require that the LHS is concave in \( b_{1H} \), attains its maximum at \( b_{1H} = 1 \), and is negative for \( b_{1H} = 1 \). Therefore, \( n_L^* = 1 \) is not optimal.

(vii) \( b_{1H} \in (0, 1), b_{1L} = 0 \) and \( n_L^* = 1 \): We require \( p^{sc} \leq \bar{p}_H \) so that \( E[q | n^{sc} = n_L^*, L] - p^{sc} \geq 0 \). From lemma 2, we require that \( n_H^* \leq 1 \). As in scenario (iii) above, \( n_H^* = 0 \) leads to a contradiction. Further, the indifference condition for high-type consumers is the same as in (A.17) in scenario (vi). However, \( n_L^* = 1 \) is not optimal for low-type consumers in such an equilibrium, which is a contradiction.

(ix) \( b_{1H} = 1, b_{1L} \in (0, 1), n_L^* = 1 \): We require \( p^{sc} < \bar{p}_L \) so that low-type consumers buy in period 1. We proceed as in (vi) to show that such an equilibrium can exist when \( p^{sc} \in (\hat{p}_{10}, \bar{p}_L) \). But this equilibrium is strictly Pareto-dominated by that in (iv), since in the latter low-type consumers strictly prefer to wait when \( p^{sc} \in (\hat{p}_{10}, \bar{p}_L) \) and hence must derive a higher utility.

Theorem 2. When a firm offers a social coupon with a minimum limit \( k = 2 \), for \( t \in \{H, L\} \),

(i) when \( p^{sc} \geq E[q | \{H, H\}] \), there is a unique equilibrium where \( b_{11} = 0, n_t^* = 3 \);

(ii) when \( p^{sc} \in (\hat{p}_{23}, E[q | \{H, H\}]) \), there is a unique Pareto-dominant equilibrium, where \( b_{1L} = 0, n_L^* = 3, n_H^* = 2, b_{1H} \in (0, 1) \) is given by (SA.3) and \( \hat{p}_{23} \) is given by (SA.4);

(iii) when \( p^{sc} \in (\bar{p}_H, \hat{p}_{23}] \), there is a unique Pareto-dominant equilibrium, where \( b_{1H} = 1, n_H^* = 2, b_{1L} = 0, n_L^* = 3 \);

(iv) when \( p^{sc} \in [\hat{p}_{22}, \bar{p}_H] \), there is a unique equilibrium, where \( b_{1H} = 1, n_H^* = 1, b_{1L} = 0, n_L^* = 2 \) and \( \hat{p}_{22} \) is given by (SA.10);

(v) when \( p^{sc} \in (p_L, \hat{p}_{22}) \), there is a unique equilibrium, where \( b_{1H} = 1, n_H^* = 1, n_L^* = 2 \) and \( b_{1L} \in (0, 1) \) is the root of the quadratic equation in (SA.15);

(vi) when \( \delta \leq \frac{3 - \sqrt{5}}{2} \) and \( p^{sc} \in [\hat{p}_{20}, p_L] \) or \( \delta > \frac{3 - \sqrt{5}}{2} \) and \( p^{sc} \in [\hat{p}_{20}, \hat{p}_{21}] \), there is a unique Pareto-dominant equilibrium where \( b_{1H} = 1, n_H^* = 0, n_L^* = 2 \) and \( b_{1L} \in (0, 1) \) is the smaller root of the quadratic equation in (SA.15), \( \hat{p}_{20} \) is given by (SA.15) when \( b_{1L} \) is the root of the equation in (SA.16), and \( \hat{p}_{21} \) is given by (SA.13);
Theorem 3. When a firm offers a social coupon with a minimum limit $k = 3$, for $t \in \{H, L\}$,

(i) when $p^{sc} > E[q|\{H,H,H\}]$, there is a unique equilibrium where $b_{1L} = 0$, $n^*_H = 3$;

(ii) when $p^{sc} \in (\hat{p}_H, E[q|\{H,H\}])$, there is a unique Pareto-dominant equilibrium, where $b_{1H} = 1$, $n^*_H = 2$, $b_{1L} = 0$, $n^*_L = 3$;

(iii) when $p^{sc} \in (\hat{p}_L, \hat{p}_H]$, there is a unique Pareto-dominant equilibrium, where $b_{1H} = 1$, $n^*_H = 1$, $b_{1L} = 0$, $n^*_L = 2$, and $\hat{p}_32$ is given by (SA.21);

(iv) when $p^{sc} \in (p_L, \hat{p}_32)$, there is a unique equilibrium, where $b_{1H} = 1$, $n^*_H = 1$, $n^*_L = 2$ and $b_{1L} \in (0,1)$ is the root of the quadratic equation in (SA.27);

(v) when $\delta \in \left[\frac{1}{2}, \frac{\sqrt{5} - 1}{2}\right]$ and $p^{sc} \in [\hat{p}_{30}, p_L]$ or $\delta > \frac{\sqrt{5} - 1}{2}$, $\hat{p}_{30} < \hat{p}_{31}$ and $p^{sc} \in [\hat{p}_{30}, \hat{p}_{31}]$, there is a Pareto-dominant equilibrium, where $b_{1H} = 1$, $n^*_H = 0$, $n^*_L = 2$ and $b_{1L} \in (0,1)$ is the larger root of the quadratic equation in (SA.27), $\hat{p}_{30}$ is given by (SA.27) when $b_{1L} \in (0,1)$ is the root of the equation in (SA.28), and $\hat{p}_{31}$ is given by (SA.24);

(vi) when $\delta > \frac{\sqrt{5} - 1}{2}$ and $p^{sc} \in [\hat{p}_{31}, p_L]$, there is a unique Pareto-dominant equilibrium where $b_{1H} = 1$, $n^*_H = 0$, $b_{1L} = 0$, $n^*_L = 1$;

(vii) when $\delta < \frac{1}{2}$ and $p^{sc} < \hat{p}_L$, or $\delta \in \left[\frac{1}{2}, \frac{\sqrt{5} - 1}{2}\right]$ and $p < \hat{p}_{30}$, or $\delta > \frac{\sqrt{5} - 1}{2}$ and $p < \min(\hat{p}_{30}, \hat{p}_{31})$, there is a unique equilibrium where $b_{1t} = 1$, $n^*_t = 0$. 

Proof. Refer to Supplemental Appendix. □

The expected market demand is given by,

$E[Q^{sc}] = \begin{cases} 
0 & \text{if } p^{sc} \geq E[q|\{H,H\}]; \\
3S_3b_1^2[1 + (1 - b_{1H})(1 + \delta)] + 2S_2b_1^2 & \text{if } p^{sc} \in (\hat{p}_{23}, E[q|\{H,H\}]); \\
3S_3 + 2S_2 & \text{if } p^{sc} \in (\hat{p}_H, \hat{p}_{23}); \\
3S_3 + (2 + \delta)S_2 & \text{if } p^{sc} \in [\hat{p}_{22}, \hat{p}_H]; \\
3S_3 + S_2[3 - (1 - b_{1L})(1 - \delta)] + & S_1[3b_1^2 + 2(2 + \delta)b_{1L}(1 - b_{1L})] + S_0[3b_0^2 + 2(2 + \delta)b_{1L}^2(1 - b_{1L})] & \text{if } \delta < \frac{3 - \sqrt{5}}{2} \text{ and } p^{sc} \in [\hat{p}_{20}, \hat{p}_{22}); or \\
3S_3 + (2 + \delta)S_2 + S_1(3\delta^2 + 4\delta(1 - \delta)) & \text{if } \delta > \frac{3 - \sqrt{5}}{2} \text{ and } p^{sc} \in [\hat{p}_{21}, \hat{p}_L]; \\
3 & \text{otherwise.}
\end{cases}$

where $S_m$ denotes the probability that there are $m$ high-type consumers in the market.
The expected market demand is given by,

\[
E[Q^{sc}] = \begin{cases} 
0 & \text{if } p^H > E[q|\{H,H,H\}] \text{;}
3S_3 & \text{if } p^{sc} \in (\bar{p}_{H}, E[q|\{H,H,H\}) \text{;}
3[S_3 + S_2\delta] & \text{if } p^{sc} \in [\bar{p}_{32}, \bar{p}_H] \text{;}
3S_3 + 3S_2(1 - (1 - b_{1L})(1 - \delta)) + 3S_1(b_{1L}^2 + 2b_{1L}(1 - b_{1L})\delta) + 3S_0((b_{1L}^3 + 3b_{1L}^2(1 - b_{1L})\delta) & \text{if } p^{sc} \in (\bar{p}_L, \bar{p}_{32}) \text{;}
3[S_3 + S_2\delta + S_1\delta^2] & \text{if } \frac{1}{2} < \delta \leq \frac{\sqrt{\gamma} - 1}{2} \text{ and } p^{sc} \in [\bar{p}_{30}, \bar{p}_L] \text{;}
3 & \text{if } \delta > \frac{\sqrt{\gamma} - 1}{2}, \bar{p}_{30} < \bar{p}_{31} \text{ and } p^{sc} \in [\bar{p}_{30}, \bar{p}_{31}] \text{;}
\end{cases}
\]

where \(S_m\) denotes the probability that there are \(m\) high-type consumers in the market.

**Proof.** Refer to Supplemental Appendix.

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**Appendix B: Propositions and Corollaries**

**Proof for Proposition 1:** From Theorem 1, we have that a social coupon stimulates demand for \(p^{sc} \in (\bar{p}_L, \bar{p}_{11})\) and suppresses demand for \(p^{sc} \in [\bar{p}_{10}, \bar{p}_L]\). We also have that demand is continuously decreasing for \(p^{sc} \in (\bar{p}_{11}, \bar{p}_H)\) and goes to zero when \(p^{sc} \to \bar{p}_H\). It follows that there exists \(\bar{p}_{12} \in (\bar{p}_{11}, \bar{p}_H)\) as described in the proposition.

**Proof for Corollary 1:** We have \(\gamma = \alpha (1 - \alpha) \delta\), and the results immediately follow.

**Proof for Corollary 2:** We have \(\eta = \frac{(1 - \alpha)(1 - \theta)(2\alpha - 1)\delta}{(2\alpha - 1)(1 - (1 + \alpha)(1 - \alpha))\delta + (1 - \alpha)(1 - (1 - \alpha^2))\delta}\). Therefore,

\[
\frac{d\eta}{d\theta} = \frac{-\alpha(1 - \alpha)(2\alpha - 1)\delta(1 - (2\alpha)\alpha\delta)}{(2\alpha - 1)(1 - (1 + \alpha)(1 - \alpha))\delta + (1 - \alpha)(1 - (1 - \alpha^2))\delta^2}, \tag{B.1}
\]

which is strictly negative.

\[
\frac{d\eta}{d\delta} = \frac{(1 - \alpha)(2\alpha - 1)(1 - \delta)(1 - \theta)(2\alpha - 1)}{(2\alpha - 1)(1 - (1 + \alpha)(1 - \alpha))\delta + (1 - \alpha)(1 - (1 - \alpha^2))\delta^2}, \tag{B.2}
\]

which is strictly positive.

\[
\frac{d\eta}{d\alpha} = \frac{\delta(1 - \theta)[2(1 - \alpha)^2(1 - (1 - \alpha^2))\delta - \theta(2\alpha - 1)^2(1 - (2 - \alpha(2 - \alpha))\delta)]}{(2\alpha - 1)(1 - (1 + \alpha)(1 - \alpha))\delta + (1 - \alpha)(1 - (1 - \alpha^2))\delta^2}, \tag{B.3}
\]

where the numerator is strictly decreasing in \(\alpha\), is positive for \(\alpha = \frac{1}{2}\) and negative for \(\alpha = 1\).

**Verifying Proposition 2:** The optimal price for a social coupon when it is more profitable than a regular deal must be in the price range \([\bar{p}_{11}, \bar{p}_{12}]\). Since the demand for a social coupon in this price range is highly non-linear, we resort to numerical optimization. We note that the parameter space and this price range are bounded. Thus, it is straightforward to numerically analyze the outcomes. Figure
3 in the main text depicts our results. As explained in the main text, Figure B.1 depicts instances where the social coupon is more profitable and the optimal price may be higher than $\hat{p}_{11}$.

![Figure B.1: Optimal Price When Social Coupon is More Profitable](image)

**Verifying Proposition 3:** We verify this proposition numerically.

**Proof for Proposition 4:** From Theorems 2 and 3, a social coupon with a minimum limit of $k = 2$ has positive demand up to a price of $E[q|\{H, H\}] > \bar{p}_H$ and a social coupon with a minimum limit of $k = 3$ has positive demand up to $E[q|\{H, H, H\}] > \bar{p}_H$. We note that $E[q|\{H, H\}] < E[q|\{H, H, H\}]$, and thus a higher minimum limit extends demand to a higher range of prices.

A social coupon with a minimum limit of $k = 2$ eliminates waiting by high-type consumers for $p^{sc} \leq \bar{p}_H$, compared to a social coupon without a minimum limit. In the case of low-type consumers, the social coupon with the minimum limit induces them to buy in period 1 with positive probability instead of strictly waiting when $p^{sc} \in (\bar{p}_L, \bar{p}_{22})$. Similarly, when $p^{sc} \leq \bar{p}_L$, it induces them to buy in period 1 instead of strictly waiting, except when $\delta > \frac{3-\sqrt{5}}{2}$ and $p^{sc} \in [\hat{p}_{21}, \bar{p}_L]$, in which case low-type consumers wait as in the case of a social coupon without a minimum limit. Thus, a social coupon with a minimum limit of $k = 2$ weakly reduces the delay in buying decisions. It follows that the expected number of consumers buying the deal is higher, since a delay causes a loss of demand either because consumers do not return or, if they return, they may choose not to buy depending on $n^{sc}$.

Similarly, a social coupon of minimum limit $k = 3$ also eliminates waiting by high-type consumers, and weakly reduces the delay in buying decisions of low-type consumers compared to a social coupon without a minimum limit when $p^{sc} \in (\hat{p}_{30}, \hat{p}_{32})$ or $p^{sc} \in (\hat{p}_{31}, \hat{p}_{32})$ (depending on $\delta$). Further, compared to a social coupon with minimum limit $k = 2$, it eliminates waiting by high-type consumers when $p^{sc} \in (\hat{p}_{23}, E[q|\{H, H\}])$, and weakly reduces the delay in buying decisions of low-type consumers when $p^{sc} \in (\hat{p}_{20}, \hat{p}_{32})$. It follows therefore that the expected number of consumers buying the deal is higher when the minimum limit is higher.
Proof for Proposition 5: Theorems 2 and 3 describe the demand for a social coupon with a minimum limit. We numerically solve for the optimal promotional strategy, noting that if a social coupon with a minimum limit of \( k = 2 \) is optimal, its price should be in \([\bar{p}_L, E[q|\{H, H\}]]\) and if a social coupon with a minimum limit of \( k = 3 \) is optimal, its price should be in \([\bar{p}_L, E[q|\{H, H, H\}]]\). Figure 5 in the main text depicts our results. We find that a social coupon with a minimum limit of \( k = 3 \) is never optimal and whenever a social coupon with a minimum limit of \( k = 2 \) is optimal, its optimal price is \( p^{sc} = \bar{p}_H \).

Appendix C: Additional Extensions

C.1 When Consumers Who Buy the Deal Make Repeat Purchases

We assume that the firm has a group of loyal regular buyers whose reservation price for the firm’s product is 1, such that the firm’s regular price is 1. Consumers who buy the deal and find that the product appeal is low \( (q = 0) \) will have a reservation price of 0 and do not become repeat buyers. Consumers who buy the deal and find that the product appeal is high \( (q = 1) \) will have a reservation price of 1 and will be willing to buy at the firm’s regular price; we assume that all such consumers return to make one repeat purchase. We further assume that the firm does not discount future profits. If not all consumers make repeat purchases or if the firm discounts future profits, then the effect of repeat purchases would be weaker than in our setting. If consumers make multiple repeat purchases, then the effect of repeat purchases will be more pronounced.

![Figure C.1: Use of Social Coupon When There Are Repeat Purchases](image)

We note that the consumer buying behavior for either type of deal is the same as before. In particular, since a consumer’s expected utility from any future purchases is zero, the prospect of repeat purchase does not influence her current buying behavior. The expected number of repeat buyers is given by the probability that a consumer buys the deal and the product appeal is high. This can be
obtained directly from our results in Theorem 1. For instance, when \( p^{sc} = \hat{p}_{11} \), the expected number of
consumers buying the deal is given by \( 3 \Pr(H) + \delta \Pr(\{H, H, L\}) \), and the expected number of repeat
buyers is given by \( 3 \Pr(H, q = 1) + \delta \Pr(\{H, H, L\}, q = 1) \). The firm’s profits from a repeat buyer is
1. We then proceed as before to analyze when a social coupon is more profitable than a regular deal. Figure C.1 depicts our results.

C.2 When Regular Buyers Can Buy the Deal
We assume that the firm has a group of loyal regular buyers whose reservation price for the firm’s
product is 1, such that the firm’s regular price is 1. We consider that one of these loyal regular buyers
is able to (or is willing to incur the hassle to) avail the deal. If the firm does not offer a daily deal,
then this regular buyer would have bought the product at the firm’s regular price. Whenever a deal
is offered, the regular buyer will always buy in period 1. Other consumers will, however, discount
her purchases when they consider the number of deals sold in period 1. Consequently, their buying
behavior is essentially the same as before. We then proceed as before to analyze the firm’s optimal
promotion strategy. Figure C.2 depicts our results.

![Figure C.2: Use of Social Coupon When Regular Buyers Can Avail The Deal](image-url)
Supplemental Appendix

A Proofs for Theorems 2 & 3

Proof for Theorem 2: $S_m$ and $R_m^t$ are as defined in Theorem 1. It can be shown that the necessary conditions from lemmas 1 and 2 continue to apply. There are ten possible equilibrium scenarios. The approach is essentially the same as in Theorem 1. We therefore focus only on the equilibrium scenarios that exist and are Pareto-dominant.

(i) $b_{1H} = 0$, $b_{1L} = 0$: Buying in period 2 is strictly dominated by buying in period 1 since $n^{sc}$ is non-informative. Therefore, consumers do not buy in either period. Now, as such, in period 2, any given consumer is indifferent between buying and not buying the deal since the minimum limit will never be reached. However, for a small tremble in the period 2 decisions (such that the minimum limit can be reached with a small positive probability) we require that $p^{sc} > \hat{p}_H$ in order for neither high-type nor low-type consumers to have an incentive to buy in period 2. Further, this equilibrium will be Pareto-dominated for $p^{sc} \in (\hat{p}_H, E[q|\{H,H\}])$ by those of scenarios (ii) and (v).

(ii) $b_{1H} \in (0,1)$, $b_{1L} = 0$ and $n^{*}_L = 3$: We require $p^{sc} > \hat{p}_H$ for $n^{*}_L = 3$ to be optimal. Since high-type consumers follow a mixed strategy, they must buy in both periods. Then $n^{*}_H = 1$ or 2. In either case, we require that $p^{sc} < E[q|\{H,H\}]$. Suppose $n^{*}_H = 2$. We have,

\[
U^{sc}_{1H} = [E[q|\{H,H\}]] - p^{sc} + R^H \left[1 - (1 - b_{1H})^2\right] + [E[q|\{H,H,L\}]] - p^{sc} + R^H b_{1H}, \quad (SA.1)
\]

\[
U^{sc}_{2H} = [E[q|\{H,H\}]] - p^{sc} + R^H b_{1H}^2. \quad (SA.2)
\]

We require that $U^{sc}_{1H} = \delta U^{sc}_{2H}$, which yields: \(^{21}\)

\[
b_{1H} = \frac{(\alpha^2(2-\alpha(1+\delta))\theta + (1-\alpha)(2-(1-\alpha)(1+\delta))(1-\theta))p^{sc} - \alpha^2(2-\alpha(1+\delta))\theta}{\alpha^2(1+\delta)\theta - (1-\alpha)(1+\delta)(1-\theta)p^{sc}}. \quad (SA.3)
\]

It can be verified that $b_{1H} \in (0,1)$ when $p^{sc} \in (\hat{p}_{23}, E[q|\{H,H\}])$, where $\hat{p}_{23} > \hat{p}_H$ is given by

\[
\hat{p}_{23} = \frac{(2-\alpha(1+\delta))\theta}{(2-\alpha(1+\delta))\theta + (2-\alpha(1+\delta))(1-\theta)(1-\theta)}.
\]

Further, this equilibrium Pareto-dominates that of scenario (i) in this price range.

Suppose $n^{*}_H = 1$. Proceeding as before, such an equilibrium can exist when $p^{sc} \in (\hat{p}_{23}, E[q|\{H,H\}])$, But when it exists, it entails lesser separation between customer types (i.e., $b_{1H}$ is smaller) than in the equilibrium when $n^{*}_H = 2$. It is hence Pareto-dominated\(^{22}\).

(iii) $b_{1H} \in (0,1)$, $b_{1L} = 0$ and $n^{*}_L = 2$: It can be shown that this equilibrium cannot exist.

(iv) $b_{1H} \in (0,1)$, $b_{1L} = 0$ and $n^{*}_L = 1$: It can be shown that this equilibrium cannot exist.

\(^{21}\) We note that $U^{sc}_{1H} = \delta U^{sc}_{2H} \iff U^{sc}_{1H} < \delta U^{sc}_{2H}$.

\(^{22}\) Indeed, it is because $n^{sc}$ is less informative in this equilibrium that $n^{*}_H = 1$ as opposed to $n^{*}_H = 2$ in the previous equilibrium.
(v) \( b_{1H} = 1, b_{1L} = 0, n^*_L = 3 \): \( n^{sc} \) is perfectly informative. It must be that \( p^{sc} > \bar{p}_H \) such that low-type consumers do not buy in period 2. We also require \( p^{sc} \leq E[\{H, H, H\}] \) such that high-type consumers may buy. Also \( n^*_H = 2 \), since \( E[\{n^{sc} = 1, H\}] = \bar{p}_H \). We have,

\[
U^{sc}_{1H} = [E[\{H, H, H\}] - p^{sc}] R^H_2 + [E[\{H, H, L\}] - p^{sc}] R^H_1,  \tag{SA.5}
\]

\[
U^{sc}_{2H} = [E[\{H, H, H\}] - p^{sc}] R^H_2.  \tag{SA.6}
\]

We require that \( U^{sc}_{1H} \geq \delta U^{sc}_{2H} \), which yields \( p^{sc} \leq \bar{p}_{23} \). Thus, this equilibrium exists when \( p^{sc} \in (\bar{p}_H, \bar{p}_{23}] \) and Pareto dominates the equilibrium in (i) in this price range.

(vi) \( b_{1H} = 1, b_{1L} = 0, n^*_L = 2 \): We require \( p^{sc} \in [\bar{p}_L, \bar{p}_H] \) so that \( n^*_L = 2 \) can be optimal. Then, in this price range \( n^*_H = 1 \). We have,

\[
U^{sc}_{1H} = U^{sc}_{2H} = E[\{H\}] - p^{sc} - (E[\{H, L, L\}] - p^{sc}) R^H_0,  \tag{SA.7}
\]

Thus, \( U^{sc}_{1H} > \delta U^{sc}_{2H} \), and high-type consumers do not have an incentive to wait. We also have,

\[
U^{sc}_{1L} = (E[\{H, L, L\}] - p^{sc}) R^L_1 + (E[\{H, H, L\}] - p^{sc}) R^L_2,  \tag{SA.8}
\]

\[
U^{sc}_{2L} = (E[\{H, H, L\}] - p^{sc}) R^L_2.  \tag{SA.9}
\]

For low-type consumers to find it attractive to wait, we require \( U^{sc}_{1L} \leq \delta U^{sc}_{2L} \), which yields,

\[
p^{sc} \geq \frac{(\bar{p}_2 + \delta)}{(2 - \alpha \delta)^\theta}.  \tag{SA.10}
\]

Thus, this equilibrium exists when \( p^{sc} \in [\bar{p}_2, \bar{p}_H] \).

(vii) \( b_{1H} = 1, b_{1L} = 0, n^*_L = 1 \): We require \( p^{sc} \in [E[\{L, L, L\}], \bar{p}_L] \) so that \( n^*_L = 1 \) can be optimal. Then, in this price range, \( n^*_H = 0 \) and high-type consumers have no incentive to wait. We have,

\[
U^{sc}_{1L} = E[\{L\}] - p^{sc} - (E[\{L, L, L\}] - p^{sc}) R^L_0 (1 - \delta)^2,  \tag{SA.11}
\]

\[
U^{sc}_{2L} = E[\{L\}] - p^{sc} - (E[\{L, L, L\}] - p^{sc}) R^L_0.  \tag{SA.12}
\]

For low-type consumers to find it attractive to wait, we require \( U^{sc}_{1L} \leq \delta U^{sc}_{2L} \), which yields,

\[
p^{sc} \geq \frac{\theta (1 - \delta)}{(1 - \alpha \delta)(1 - \delta^2)} R^L_0 (1 - \delta - (1 - \alpha)^2 (1 - \delta + \delta^2)).  \tag{SA.13}
\]

We note that \( \bar{p}_{21} < \bar{p}_L \) provided \( \delta > \frac{3 - \sqrt{5}}{2} \). Thus, this equilibrium exists for \( p^{sc} \in [\bar{p}_{21}, \bar{p}_L] \) iff \( \delta > \frac{3 - \sqrt{5}}{2} \).

(viii) \( b_{1H} = 1, b_{1L} \in (0, 1), n^*_L = 2 \): We require \( p^{sc} < \bar{p}_H \) so that low-type consumers buy. We have,

\[
U^{sc}_{1L} = E[\{L\}] - p^{sc} - (E[\{L, L, L\}] - p^{sc}) R^L_0 (1 - b_{1L})^2,  \tag{SA.14}
\]

\[
U^{sc}_{2L} = E[\{L\}] - p^{sc} - (E[\{H, L, L\}] - p^{sc}) R^L_1 (1 - b_{1L}) - (E[\{L, L, L\}] - p^{sc}) R^L_0 (1 - b_{1L}^2).  \tag{SA.14}
\]
We require that $U_{1L}^{sc} = \delta U_{2L}^{sc}$, which yields,\footnote{We note that $U_{1L}^{sc} = \delta U_{2L}^{sc} \implies U_{1H}^{sc} > \delta U_{2H}^{sc}$.}

$$p^{sc} = \frac{\theta(1-\alpha)(\alpha+b_{1L}(1-\alpha))^{2-(1+\delta)(\alpha+b_{1L}(1-\alpha))}}{\theta(1-\alpha)(\alpha+b_{1L}(1-\alpha))^{2-(1+\delta)(\alpha+b_{1L}(1-\alpha))} + (1-\theta)\alpha(1-\alpha+b_{1L}(1-\alpha))^{2-(1+\delta)(1-\alpha+b_{1L}(1-\alpha))}}, \quad \text{(SA.15)}$$

which is a quadratic equation in $b_{1L}$. It is straightforward to show that when $p^{sc} \in [\hat{p}_L, \hat{p}_{22}]$ there is a unique solution, whereas when $p^{sc} \in (\hat{p}_{20}, \hat{p}_L)$ there are two solutions, the smaller of which is Pareto-dominant since it entails a higher degree of separation, making $n^{sc}$ more informative. Here $\hat{p}_{20}$ is the price satisfying (SA.15) when $b_{1L}$ equals the larger root of the following quadratic equation in $b_{1L}$:

$$\alpha (1 - \alpha) \delta (1 + \delta) b_{1L}^2 - 2(1 - (1 - 2\alpha (1 - \alpha)) \delta) (1 + \delta) b_{1L} + 1 - \delta (1 - \alpha (1 - \alpha) (1 + \delta)) = 0. \quad \text{(SA.16)}$$

Lastly, we note that $\hat{p}_{20} < \hat{p}_{21}$. Therefore, when $\delta > \frac{3-\sqrt{5}}{2}$, the equilibrium in (vii) Pareto-dominates this equilibrium when $p^{sc} \in (\hat{p}_{21}, \hat{p}_L)$.

(ix) $b_{1H} = 1, b_{1L} \in (0,1), n_L^{*} = 1$: It can be shown that this equilibrium exists when $\delta > \frac{3-\sqrt{5}}{2}$ and $p^{sc} \in (\hat{p}_{21}, \hat{p}_L)$, but is Pareto-dominated by the equilibrium in (vii) since the latter entails perfect separation between consumer types making $n^{sc}$ more informative.

(x) $b_{1H} = 1, b_{1L} = 1$: As such, this equilibrium exists when $p^{sc} \leq \hat{p}_L$. However, it is Pareto-dominated by scenario (vii) and (viii) when $p^{sc} \in [\hat{p}_{20}, \hat{p}_L]$. Thus, this equilibrium holds when $p^{sc} \leq \hat{p}_{20}$.

**Proof for Theorem 3:** $S_m$ and $R_{m}^{t}$ are as defined in Theorem 1. It can be shown that the necessary conditions from lemmas 1 and 2 continue to apply. There are ten possible equilibrium scenarios. The approach is essentially the same as in Theorem 1. We therefore focus only on the equilibrium scenarios that exist and are Pareto-dominant.

(i) $b_{1H} = 0, b_{1L} = 0$: As in the case of scenario (i) in Theorem 2, this equilibrium exists when $p^{sc} > \hat{p}_H$. Further, this equilibrium will be Pareto-dominated for $p^{sc} \in (\hat{p}_H, E [q | \{H, H, H\}])$ by that of scenario (v).

(ii) $b_{1H} \in (0,1), b_{1L} = 0$ and $n_L^{*} = 3$: It can be shown that this equilibrium cannot exist.

(iii) $b_{1H} \in (0,1), b_{1L} = 0$ and $n_L^{*} = 2$: It can be shown that this equilibrium cannot exist.

(iv) $b_{1H} \in (0,1), b_{1L} = 0$ and $n_L^{*} = 1$: This equilibrium may exist when $p^{sc} \in [\hat{p}_L, \hat{p}_H]$, But when it does, is Pareto-dominated by that of scenario (vi).

(v) $b_{1H} = 1, b_{1L} = 0, n_L^{*} = 3$: $n^{sc}$ Since $E [q | n^{sc} = 2, L] = \hat{p}_H$, it must be that $p^{sc} > \hat{p}_H$ such that low-type consumers do not buy in period 2. We require that $U_{1H}^{sc} = E [q | \{H, H, H\}] - p^{sc} \geq 0$. Therefore $p^{sc} \leq E [q | \{H, H, H\}]$. It follows that $U_{1H}^{sc} = U_{2H}^{sc}$, and hence $U_{1H}^{sc} > \delta U_{2H}^{sc}$. Therefore, this equilibrium exists when $p^{sc} \in (\hat{p}_H, E [q | \{H, H, H\}])$. Further, it Pareto-dominates that of scenario (i) in this price range.
(vi) $b_{1H} = 1, b_{1L} = 0, n_{L}^* = 2$: We require that $p^{sc} \in [\bar{p}_L, \bar{p}_H]$ for $n_{L}^* = 2$ to be optimal. We have,

$$U_{1H}^{sc} = E[q|H] - p^{sc} - (E[q|\{H, L, L\}] - p^{sc}) R_1^H (1 - \delta) - (E[q|\{H, L, L\}] - p^{sc}) R_0^H,$$

$$U_{2H}^{sc} = E[q|H] - p^{sc} - (E[q|\{H, L, H\}] - p^{sc}) R_1^H - (E[q|\{H, L, L\}] - p^{sc}) R_0^H.$$ (SA.17)

We note that $U_{1H}^{sc} > U_{2H}^{sc}$ and therefore $U_{1H}^{sc} > \delta U_{2H}^{sc}$. Hence high-type consumers do not have an incentive to wait. We also have,

$$U_{1L}^{sc} = (E[q|\{H, L, L\}] - p^{sc}) R_1^L \delta + (E[q|\{H, H, L\}] - p^{sc}) R_2^L,$$ (SA.19)

$$U_{2L}^{sc} = (E[q|\{H, H, L\}] - p^{sc}) R_2^L.$$ (SA.20)

For low-type consumers to find it attractive to wait, we require,

$$U_{1L}^{sc} \leq \delta U_{2L}^{sc} \implies p^{sc} \geq \hat{p}_{32} = \frac{2(\delta - \alpha (1 - \delta))}{3(2\alpha (1 - \delta) + \alpha^2 (1 - \delta + \delta^2))}.$$ (SA.21)

We note that $\hat{p}_{32} \in (\bar{p}_L, \bar{p}_H)$. Thus, this equilibrium exists when $p^{sc} \in [\hat{p}_{32}, \bar{p}_H]$. It is straightforward to verify that the equilibrium in scenario (iv) may exist only when $p^{sc} > \hat{p}_{32}$, but is Pareto-dominated by the current equilibrium since $n^{sc}$ is less informative in the former.

(vi) $b_{1H} = 1, b_{1L} = 0, n_{L}^* = 1$: Since $E[q|n^{sc} = 1, L] = \bar{p}_L$ and $E[q|n^{sc} = 0, L] = E[q|\{L, L, L\}]$, it must be that $p^{sc} \in [E[q|\{L, L, L\}], \bar{p}_L]$. Also, $n_{H}^* = 0$ and hence high-type consumers do not have an incentive to wait. We have,

$$U_{1L}^{sc} = E[q|L] - p^{sc} - (E[q|\{H, L, L\}] - p^{sc}) R_1^L (1 - \delta) - (E[q|\{L, L, L\}] - p^{sc}) R_0^L,$$

$$U_{2L}^{sc} = E[q|L] - p^{sc} - (E[q|\{H, L, L\}] - p^{sc}) R_1^L (1 - \delta) - (E[q|\{L, L, L\}] - p^{sc}) R_0^L.$$ (SA.22)

For low-type consumers to find it attractive to wait, we require,

$$U_{1L}^{sc} \leq \delta U_{2L}^{sc} \implies p^{sc} \geq \hat{p}_{31} = \frac{\theta (1 - \alpha) (2 \alpha (1 - \delta) + \delta^2 + \alpha^2 (1 - \delta + \delta^2))}{\theta (1 - \alpha) (2 \alpha (1 - \delta) + \delta^2 + \alpha^2 (1 - \delta + \delta^2)) + (1 - \theta) \alpha (2 \alpha (1 - \delta) + \delta^2 + (1 - \alpha)^2 (1 - \delta + \delta^2))}.$$ (SA.23)

It is straightforward to verify that $\hat{p}_{31} \leq \bar{p}_L$ provided $\delta \geq \frac{\sqrt{\gamma} - 1}{2}$. Thus, this equilibrium exists for $p^{sc} \in [\bar{p}_{31}, \bar{p}_L]$ iff $\delta \geq \frac{\sqrt{\gamma} - 1}{2}$.

(vii) $b_{1H} = 1, b_{1L} \in (0, 1), n_{L}^* = 2$: We note that $p^{sc} < \bar{p}_H$, since otherwise $n_{L}^* = 2$ cannot be
optimal. We have,

\[
U_{1L}^{sc} = E[ q | L ] - p^{sc} - ( E[ q | \{ H, L, L \} ] - p^{sc} ) R_1^L ( 1 - b_{1L} ) ( 1 - \delta ) -
\]

\[
( E[ q | \{ L, L, L \} ] - p^{sc} ) R_0^L \left( ( 1 - b_{1L} )^2 + 2 b_{1L} ( 1 - b_{1L} ) ( 1 - \delta ) \right),
\]

(SA.25)

\[
U_{2L}^{sc} = E[ q | L ] - p^{sc} - ( E[ q | \{ H, L, L \} ] - p^{sc} ) R_1^L ( 1 - b_{1L} ) -
\]

\[
( E[ q | \{ L, L, L \} ] - p^{sc} ) R_0^L ( 1 - b_{1L}^2 ).
\]

(SA.26)

We require that \( U_{1L}^{sc} = \delta U_{2L}^{sc} \), which yields\(^{24}\)

\[
p^{sc} = \frac{\theta (1-\alpha)(1-(1-b_{1L})(1-\alpha))(1-\delta-(1-b_{1L})(1-\alpha)(1-3\delta))}{(1-\gamma)\alpha[1-(1-b_{1L})\alpha[1-\delta-(1-b_{1L})\alpha(1-3\delta)] + \theta (1-\alpha)(1-(1-b_{1L})(1-\alpha))(1-\delta-(1-b_{1L})(1-\alpha)(1-3\delta)]},
\]

(SA.27)

which is a quadratic equation in \( b_{1L} \). It is straightforward to show that when \( \delta \leq \frac{1}{2} \), a solution to (SA.27) exists only when \( p^{sc} \in (\bar{p}_L, \hat{p}_{32}) \) and is unique. When \( \delta > \frac{1}{2} \), a solution exists for \( p^{sc} \in (\hat{p}_{30}, \hat{p}_{32}) \), where \( \hat{p}_{30} \) is the price in (SA.27) when \( b_{1L} \) is the root of the quadratic equation\(^{25}\),

\[
\alpha (1-\alpha)(3\delta - 1)(2\delta - 1)(1-b_{1L})^2 - (1-\delta)(3\delta - 1)(1-b_{1L}) - (1-\delta)(2\delta - 1) = 0.
\]

(SA.28)

Further, the solution is unique when \( p^{sc} \in [\bar{p}_L, \hat{p}_{32}] \). For \( p^{sc} \in (\hat{p}_{30}, \hat{p}_{32}) \), there are two equilibria corresponding to the two roots of (SA.27). However, the equilibrium corresponding to the smaller root Pareto-dominates since it involves greater separation between the consumer types.

(ix) \( b_{1H} = 1, b_{1L} \in (0,1), n^*_L = 1 \): It can be shown that this equilibrium exists if \( p^{sc} \in (\hat{p}_{31}, \bar{p}_L] \) and \( \delta \geq \frac{\sqrt{5}-1}{2} \), but is Pareto-dominated by that of scenario (vii), since the latter entails better separation between customer types.

(x) \( b_{1H} = 1, b_{1L} = 1 \): As such, this is an equilibrium when \( p^{sc} \leq \bar{p}_L \). However, it is Pareto-dominated by any equilibrium that entails separation between consumer types. Thus, this equilibrium holds under the conditions described in the theorem.

### B When There are \( N \) Consumers

We show that there is a unique equilibrium for \( p^{sc} > \bar{p}_L \) and \( p^{sc} < \hat{p}_1 \), where \( \hat{p}_1 \) is defined below, and a unique Pareto-dominant equilibrium otherwise. Let \( A = \{0, 1, 2 \ldots N-1\} \) denote the set of values of \( n^{sc} \) that a consumer who waits till period 2 may observe. As before, it can be shown that in any equilibrium, (i) if high-type consumers weakly prefer to wait till period 2, then low-type consumers strictly prefer to wait till period 2, (ii) if low-type consumers weakly prefer to buy the coupon in period 1, then high-type consumers strictly prefer to buy the coupon in period 1, and (iii) the optimal buying strategy in period 2 is a threshold-type strategy. Let \( n^*_t \in A \cup \{N\} \) denote the threshold for a type \( t \)

\(^{24}\)We note that \( U_{1L}^{sc} = \delta U_{2L}^{sc} \iff U_{1L}^{sc} > \delta U_{2L}^{sc} \).

\(^{25}\)When \( \delta > \frac{1}{2} \), (SA.15) has a unique root in \( \bar{w}_{1L} \) in the interval \((0,1)\).
We require that for such that the consumer buys if $n^{sc} \geq n^*_t$. By definition, $n^*_t$ is weakly increasing in $p^{sc}$. Further, $n^*_H \leq n^*_L$ since $E[q|H, n^{sc}] > E[q|L, n^{sc}]$. There are five possible equilibrium scenarios.

(i) $b_1 = 0$ and $b_1 = 0$: We require that $U^{sc}_{1H} \leq \delta U^{sc}_{2H}$. Since $n^{sc}$ is not informative, it must be that $p^{sc} \geq \bar{p}_H$ such that $U_{1H} \leq 0$ and $U^{sc}_{2H} = 0$.

(ii) $b_1 = 1$ and $b_1 = 0$: For $b_H$ to be an equilibrium strategy, we require that,$$
^{sc}_{1H} \leq \delta U^{sc}_{2H} \implies p^{sc} \leq \frac{(1-\delta)\bar{p}_H + \delta \Pr(n^{sc} < n^*_H^{sc}|H)E[q|H, n^{sc} < n^*_H^{sc}]}{1-\delta + \delta \Pr(n^{sc} < n^*_H^{sc}|H)}.
$$

For $b_L$ to be an equilibrium strategy, we require that:

$$U^{sc}_{1L} \leq \delta U^{sc}_{2L} \implies p^{sc} \geq \frac{(1-\delta)\bar{p}_L + \delta \Pr(n^{sc} < n^*_L^{sc}|L)E[q|L, n^{sc} < n^*_L^{sc}]}{1-\delta + \delta \Pr(n^{sc} < n^*_L^{sc}|L)}.
$$

Since $n_t^{*}$ varies with $p^{sc}$, (SA.2) and (SA.3) are essentially implicit constraints for $p^{sc}$. We will show that these conditions both hold only for a certain price interval $[\bar{p}_1, \bar{p}_2]$. Since $n^{sc}$ is informative, $E[q|t, n^{sc}]$ is strictly increasing in $n^{sc}$. Let $I_{t,n}$ denote the price interval $(r_{t,n}, r_{t,n+1})$, such that $n_t^{*} = n$ iff $p^{sc} \in I_{t,n}$.

By definition, we require,

$$r_{t,n} = E[q|t, n^{sc} = n - 1] \forall n \in \{1, 2 \ldots N\},
$$

$r_{t,0} = -\infty$ and $r_{t,N+1} = \infty$. Define $G_t(n)$ as follows,

$$G_t(n) = \frac{(1-\delta)\bar{p}_L + \delta \Pr(n^{sc} < n|t)E[q|t, n^{sc} < n]}{1-\delta + \delta \Pr(n^{sc} < n|t)}.
$$

We note that $G_L(n^*_L)$ and $G_H(n^*_H)$ are the RHS of (SA.3) and (SA.2) respectively. Consider the price interval $I_{t,n}$ where $n_t^{*} = n$. Suppose for some $p' \in I_{t,n}$, $p' > G_t(n^*_t) = G_t(n)$. Now, since $n^*_t$ is constant over $I_{t,n}$, $p^{sc} > G_t(n^*_t) \forall p^{sc} \in [p', r_{t,n+1}]$. Further, we have,

$$r_{t,n+1} > G_t(n),$$

$$\implies 1 - \delta + \delta \Pr(n^{sc} < n|t) \cdot r_{t,n+1} > (1 - \delta) \bar{p}_t + \delta \Pr(n^{sc} < n|t) \cdot E[q|t, n^{sc} < n],$$

$$\implies 1 - \delta + \delta \Pr(n^{sc} < n + 1|t) \cdot r_{t,n+1} > (1 - \delta) \bar{p}_t + \delta \Pr(n^{sc} < n + 1|t) \cdot E[q|t, n^{sc} < n + 1],$$

$$\implies r_{t,n+1} > G_t(n + 1)$$

where we have applied (SA.4) in the penultimate step. But this implies that $\forall p^{sc} \in I_{t,n+1}$, $p^{sc} > G_t(n^*_t) = G_t(n + 1)$. Applying this repeatedly, $p^{sc} > G_t(n^*_t) \forall p \geq p'$. Similarly, if for some $p^{sc} = p'$, $p^{sc} < G_t(n^*_t)$, then $p^{sc} < G_t(n^*_t) \forall p^{sc} \leq p'$. Thus, the equation $p^{sc} = G_t(n^*_t)$ has at most one solution.

To show that it always has a solution, we must eliminate two possibilities. The first possibility
is that $p^{sc}$ is always greater than or always less than $G_t(n^*_t)$. We note that $p^{sc} < G_t(0) = \tilde{p}_t$ for some $p^{sc} \in I_{t,0}$ and $p^{sc} > G_t(N) = \tilde{p}_{t+1}$ for some $p^{sc} \in I_{t,N}$, which is the desired contradiction. The second possibility is that, $p^{sc} < G_t(n^*_t)$ when $p^{sc} \leq r_{t,n}$ and $p^{sc} > G_t(n^*_t)$ when $p^{sc} > r_{t,n}$ for some $n \in A \cup \{N\}$, since $G_t(n^*_t)$ may be discontinuous at $p^{sc} = r_{t,n}$. However, if $r_{t,n} < G_t(n - 1)$, then, proceeding as in (SA.4), we have that $r_{t,n} < G_t(n)$. Hence, $p^{sc} < G_t(n^*_t)$ for some $p^{sc} > r_{t,n}$, which is the required contradiction.

Therefore, there exists a unique solution $\tilde{p}_1$ that satisfies $p^{sc} = G_L(n^*_L)$ and a unique solution $\tilde{p}_2$ that satisfies $p^{sc} = G_H(n^*_H)$ such that,

$$\tilde{p}_1 = \frac{(1-\delta)\tilde{p}_L + \delta \Pr(n^{sc} \leq n^*_L[L]) E[q|L, n^{sc} < n^*_L]}{\frac{1}{1-\delta+\delta \Pr(n^{sc} < n^*_L|L)}}, \quad \tilde{p}_2 = \frac{(1-\delta)\tilde{p}_H + \delta \Pr(n^{sc} < n^*_H|H) E[q|H, n^{sc} < n^*_H]}{\frac{1}{1-\delta+\delta \Pr(n^{sc} < n^*_H|H)}}. \quad (SA.7)$$

It follows from our earlier arguments that $p^{sc} > G_L(n^*_L) \forall p > \tilde{p}_1$ and $p^{sc} < G_H(n^*_H) \forall p < \tilde{p}_2$. Further, when $p^{sc} = \tilde{p}_2$, (SA.2) holds as an equality and hence high-type consumers are indifferent between buying in period 1 and waiting till period 2. Then low-type consumers must strictly prefer to wait, and hence (SA.3) holds strictly. Therefore, it must be that $\tilde{p}_1 < \tilde{p}_2$. Thus, this equilibrium scenario occurs iff $p^{sc} \in [\tilde{p}_1, \tilde{p}_2]$. $G_t(n^*_t)$ is a weighted average of $\tilde{p}_t$ and $E[q|t, n^{sc} < n^*_t]$, where the latter is strictly smaller than $\tilde{p}_t$ when $n^*_t < N$. Now, when $p^{sc} = \tilde{p}_t$, $n^*_t < N$ ($n^*_t = \frac{N-1}{2}$ if $N$ is odd and $\frac{N}{2}$ if $N$ is even). Thus, when $p^{sc} = \tilde{p}_t$, $p^{sc} > G_t(n^*_t)$. It follows then that $p^{sc} > G_t(n^*_t) \forall p^{sc} \geq \tilde{p}_t$. Therefore, $\tilde{p}_1 < \tilde{p}_L$ and $\tilde{p}_2 < \tilde{p}_H$. We also note that $G_t(n^*_t)$ is decreasing in $\delta$, since the weight shifts from $\tilde{p}_t$ to $E[q|t, n^{sc} < n^*_t]$. Hence $\tilde{p}_1$ and $\tilde{p}_2$ are decreasing in $\delta$. Lastly, $\tilde{p}_1$ and $\tilde{p}_2$ are decreasing in $N$, since $n^{sc}$ is more informative when $N$ is higher.

(iii) $b_{1H} \in (0, 1)$ and $b_{1L} = 0$: We require that,

$$U_{1H}^{sc} - \delta U_{2H}^{sc} = 0 \implies E[q|H] - p^{sc} - \delta (E[q|H, n^{sc} \geq n^*_H] - p^{sc}) \Pr(n^{sc} \geq n^*_H|H) = 0, \quad (SA.8)$$

While $n^*_H$ is discrete and weakly increasing in $b_{1H}$, $U_{2H}^{sc}$ is continuous in $b_{1H}$ since at the point at which $n^*_H$ changes say from $n$ to $n + 1$, we have that $E[q - p^{sc}|H, n^{sc} = n] = 0$. Further, $U_{2H}^{sc}$ is increasing in $b_{1H}$ since $n^{sc}$ is more informative when $b_{1H}$ is higher and, hence, the value of waiting to use this information is higher. Thus, the LHS of (SA.8) is continuous and strictly decreasing in $b_{1H}$. Now for $p > \tilde{p}_H$, the LHS is negative when $b_{1H} = 0$: $U_{2H}^{sc} = U_{1H}^{sc}$ since $n^{sc}$ is not informative, and $U_{1H}^{sc} < 0$ since $p > \tilde{p}_H$. Since the LHS is decreasing in $b_{1H}$, it is negative for all $b_{1H} \in (0, 1)$. From the proof for scenario (ii), for $p^{sc} \leq \tilde{p}_2$, the LHS is non-negative when $b_{1H} = 1$, and hence strictly positive for all $b_{1H} \in (0, 1)$. Since (SA.8) is satisfied when $p^{sc} = \tilde{p}_2$ and $b_{1H} = 1$, and when $p^{sc} = \tilde{p}_H$ and $b_{1H} = 0$, and the LHS is strictly decreasing in $b_{1H}$ and in $p^{sc}$, it follows that for $p^{sc} \in (\tilde{p}_2, \tilde{p}_H)$, there is a unique $b_{1H} \in (0, 1)$ for which (SA.8) is satisfied, and vice-versa. Further, $b_{1H}$ is decreasing in $p^{sc}$. We also
We require that 

\[ \text{(iv) } b_{1H} = 1 \text{ and } b_{1L} = 1: \text{ We require that } U_{1L}^{sc} \geq \delta U_{2L}^{sc}. \text{ Since } n^{sc} \text{ is not informative, we require } \]

\[ p^{sc} \leq \bar{p}_L. \text{ Since } \hat{p}_1 < \bar{p}_L \text{ and it may be that } \hat{p}_2 < \bar{p}_L, \text{ this scenario overlaps with scenario (ii) and may overlap with scenario (iii). For } p^{sc} \in [\hat{p}_1, \bar{p}_L], \text{ low-type consumers are strictly better off in either scenarios (ii) or (iii), depending on whichever equilibrium exists at the given price, compared to the current scenario; low-type consumers strictly prefer to wait in the former scenarios and hence derive higher expected utilities. High-type consumers are indifferent between these scenarios as they buy in period 1. Thus the current scenario is Pareto dominated for } p^{sc} \in [\hat{p}_1, \bar{p}_L] \text{ and holds for } p^{sc} < \hat{p}_1. \]

\[ \text{(v) } b_{1H} = 1 \text{ and } b_{1L} \in (0,1): \text{ We require that, } \]

\[ U_{1L}^{sc} = \delta U_{2L}^{sc} \implies (E[q|L] - p^{sc}) - \delta (E[q|L, n^{sc} \geq n_L^*] - p^{sc}) \Pr (n^{sc} \geq n_L^*|L) = 0, \text{ (SA.9)} \]

Proceeding as in scenario (iii), it can be shown that this equilibrium occurs iff \( p^{sc} \in (\hat{p}_1, \bar{p}_L) \) and there is a unique equilibrium in this price range. Similar to scenario (iv), this scenario overlaps with scenario (ii) and possibly scenario (iii), but is always Pareto-dominated by them.

### B.1 Buying Probability of Low-type Consumers

We show that the buying probability of low-type consumers in period 2 may be higher when \( N \) is higher. We focus on the equilibrium scenario (i) above, where \( b_{1H} = 1 \text{ and } b_{1L} = 0. \text{ This scenario occurs when } p^{sc} \in [\hat{p}_1, \hat{p}_2]. \text{ We note that } \hat{p}_2 \text{ is decreasing in } N \text{ and can be less than } \bar{p}_L. \text{ We restrict attention to instances when } \hat{p}_2 > \bar{p}_L \text{ and the price range } p^{sc} \in [\bar{p}_L, \hat{p}_2]. \text{ Since a social coupon can potentially outperform a regular coupon only for } p^{sc} > \bar{p}_L, \text{ this price range is relevant for our discussion.}^{26} \text{ Without loss of generality we will focus on } N \text{ even and consider that it increases to } N + 2. \text{ Let } N = 2m. \text{ Let } n_L^*(N) \text{ denote the period 2 threshold strategy of low-type consumers when there are } N \text{ consumers in the market. When } p^{sc} > \bar{p}_L, \text{ } n_L^*(N) = m + 1, \text{ since } E[q|L, n^{sc} = m, N = 2m] = \bar{p}_L \text{ and } E[q|L, n^{sc} = m + 1, N = 2m] = \bar{p}_H. \text{ Similarly, } n_L^*(N + 2) = m + 2. \text{ Let } S_n^N \text{ denote the event that there are } n \text{ out of } N \text{ consumers who are high-type consumers. By considering } N + 2 \text{ consumers as two groups of } N \text{ and } 2 \text{ consumers, we have, } \]

\[ \Pr (n^{sc} \geq m + 2|L, N = 2m + 2) = \Pr (n^{sc} \geq m + 1|L, N = 2m) - \]

\[ \frac{1}{\Pr(L)} \left( \frac{2m - 1}{m + 1} \right) \left[ \theta \alpha^{m+1} (1 - \alpha)^{m-1} (1 - \alpha)^2 + (1 - \theta) (1 - \alpha)^{m+1} \alpha^{m-1} \right] + \]

\[ \frac{1}{\Pr(L)} \left( \frac{2m - 1}{m} \right) \left[ \theta \alpha^m (1 - \alpha)^m \alpha^2 + (1 - \theta) (1 - \alpha)^m \alpha (1 - \alpha)^2 \right]. \]

\[^{26}\text{In §B.2 of the Supplemental Appendix, we derive conditions for } \hat{p}_2 > \bar{p}_L \text{ as } N \to \infty \text{ in (SA.16) and show that a social coupon can indeed outperform a regular coupon when } p^{sc} = \hat{p}_2 \text{ and } \hat{p}_2 > \bar{p}_L. \]
Therefore,
\[
\Pr(n^c \geq m + 2|L, N = 2m + 2) - \Pr(n^c \geq m + 1|L, N = 2m) > 0
\]
\[
\iff \frac{\theta \alpha^2 + (1 - \theta)(1 - \alpha)^2}{\alpha(1 - \alpha)} > \frac{m - 1}{m + 1}.
\]

We note that the RHS is strictly less than 1. The LHS is increasing in \(\theta\) and is strictly larger than 1 when \(\theta \rightarrow 1\). Therefore, if \(\theta\) is sufficiently high, then low-type consumers are more likely to buy when there are more consumers in the market.

**B.2 Large Number of Consumers**

A social coupon can be more profitable than a regular deal only for \(p^c > \bar{p}_L\). Thus, we restrict attention to instances when \(\hat{p}_2 > \bar{p}_L\). From equilibrium scenario (i) above, \(n^c\) follows a binomial distribution conditional on \(q\) when \(p^c = \hat{p}_2\). Without loss of generality, let \(N = 2m + 1\) be odd. We note that \(n^*_H = m\) and \(n^*_L = m + 1\) since \(\bar{p}_H > \hat{p}_2 > \bar{p}_L\). If \(q = 1\), \(n^c\) follows a binomial distribution with mean \(2m\alpha\) and variance \(\sqrt{2m\alpha(1 - \alpha)}\), and if \(q = 0\), \(n^c\) follows a binomial distribution with mean \(2m(1 - \alpha)\) and variance \(\sqrt{2m\alpha(1 - \alpha)}\). From the De Moivre-Laplace theorem, as \(m \rightarrow \infty\), the conditional distribution of \(n^c\) asymptotically approaches a normal distribution. We use this to derive the equilibrium outcomes for the limiting case. We note that,
\[
\lim_{m \rightarrow \infty} \frac{n^*_H - 2m\alpha}{\sqrt{2m\alpha(1 - \alpha)}} = -\infty \quad \text{and} \quad \lim_{m \rightarrow \infty} \frac{n^*_H - 2m(1 - \alpha)}{\sqrt{2m\alpha(1 - \alpha)}} = \infty.
\]

Therefore,
\[
\lim_{m \rightarrow \infty} \Pr(n^c < n^*_H|q = 1) = 0, \quad \lim_{m \rightarrow \infty} \Pr(n^c < n^*_H|q = 0) = 1.
\]

We then obtain,
\[
\lim_{m \rightarrow \infty} \Pr(n^c < n^*_H|H) = \lim_{m \rightarrow \infty} \frac{\theta \alpha \Pr(n^c < n^*_H|q = 1) + (1 - \theta)(1 - \alpha) \Pr(n^c < n^*_H|q = 0)}{\Pr(H)} = \frac{(1 - \theta)(1 - \alpha)}{\theta \alpha(1 - \alpha)} = \frac{(1 - \theta)(1 - \alpha)}{\theta(1 - \alpha) + (1 - \theta)\alpha}.
\]

We also have,
\[
\lim_{m \rightarrow \infty} \Pr(n^c < n^*_L|q = 1) = 0, \quad \lim_{m \rightarrow \infty} \Pr(n^c < n^*_L|q = 0) = 1, \quad \lim_{m \rightarrow \infty} \Pr(n^c < n^*_L|L) = \frac{(1 - \theta)\alpha}{\theta(1 - \alpha) + (1 - \theta)\alpha}.
\]

From (SA.7) and (SA.13), we have,
\[
\hat{p}_2 = \bar{p}_H \frac{1 - \delta + \delta \Pr(n^c < n^*_H|q = 1)}{1 - \delta + \delta \Pr(n^c < n^*_H|H)} = \frac{\theta \alpha(1 - \delta)}{(1 - \theta)(1 - \alpha) + \theta \alpha(1 - \delta)}.
\]

We note that
\[
\hat{p}_2 > \bar{p}_L \implies \frac{\theta \alpha(1 - \delta)}{(1 - \theta)(1 - \alpha) + \theta \alpha(1 - \delta)} > \frac{\theta(1 - \alpha)}{\theta(1 - \alpha) + (1 - \theta)\alpha} \implies \alpha^2 (1 - \delta) > (1 - \alpha)^2.
\]
Therefore, if $\alpha$ is sufficiently high, then $\check{p}_2 > \bar{p}$. The expected demand per consumer when $p^{sc} = \check{p}_2$ is given by,

$$\lim_{m \to \infty} E \left[ \frac{Q^{sc}}{N} \right] = \lim_{m \to \infty} \Pr(H) + \delta \Pr(L) \Pr(n^{sc} \geq n^*_L|L),$$

$$= 1 - \alpha + \theta (2\alpha - 1) + \theta \delta (1 - \alpha). \quad (SA.17)$$

We compare the expected profit per consumer in the case of a regular deal and in the case of a social coupon when $p^{sc} = \check{p}_2$ to obtain sufficient conditions for a social coupon to be more profitable.