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The authors put forth a sales response model to explain the differences in immediate and dynamic effects of promotional prices and regular prices on sales. The model consists of a vector autoregression that is rewritten in error correction format, which allows the authors to disentangle the immediate effects from the dynamic effects. In a second level of the model, the immediate price elasticities, the cumulative promotional price elasticity, and the long-term regular price elasticity are correlated with various brand-specific and category-specific characteristics. The model is applied to seven years of data on weekly sales of 100 different brands in 25 product categories. The authors find many significant moderating effects on the elasticity of price promotions. Brands in categories that are characterized by high price differentiation and that constitute a lower share of budget are less sensitive to price discounts. Deep price discounts increase the immediate price sensitivity of customers. The authors also find significant effects for the cumulative elasticity. The immediate effect of a regular price change is often close to zero. The long-term effect of such a regular price decrease usually amounts to an increase in sales. This is especially true in categories that are characterized by a large price dispersion and frequent price promotions and for hedonic, nonperishable products.

A Hierarchical Bayes Error Correction Model to Explain Dynamic Effects of Price Changes

There is substantial literature on dynamic price effects on sales (see, e.g., Kopalle, Mela, and Marsh 1999; Paap and Franses 2000; Pauwels, Hanssens, and Siddarth 2002; Van Heerde, Leeuwen, and Wittink 2000). The term “dynamic” effect refers to the effect of a current price change on future sales. In the case of dynamic effects, the overall (net) effect of a price change cannot completely be summarized by the immediate price elasticity. In general, ignoring the presence of dynamic effects leads to erroneous conclusions. For example, Kopalle, Mela, and Marsh (1999) find that promotions have positive contemporaneous effects on sales, accompanied by negative future effects, and they rightfully emphasize (p. 317) that “models that do not consider dynamic promotional effects can mislead managers to over-promote.” Note that the net effect of promotions on sales can even be negative. For example, Jedidi, Mela, and Gupta (1999) find that the long-term effects of promotions on sales are negative and, in an absolute sense, that they are about two-fifths of the magnitude of the positive short-term effect. Note that this estimate includes the (negative) effects of competitive reaction and of changes in consumer behavior.

In the literature, there does not seem to be a consensus on the size of the dynamic effect relative to that of the immediate effect. A lack of consensus also appears in the literature on the so-called postpromotional dip. Although several articles provide evidence of postdeal troughs, many others fail to find supporting empirical evidence (e.g., Blattberg, Briensch, and Fox 1995; Grover and Srinivasan 1992). Neslin

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and Stone (1996) first addressed this lack of consensus, and then, Van Heerde, Leeftlang, and Wittink (2000) showed how the pre- and postpromotion dip can be estimated appropriately using time series models. The issue of the existence of postpromotion dips was then resolved by Macé and Neslin (2004), who consider the determinants of such dips.

An explanation for this initial disagreement in the marketing literature is that the effect varies across categories, stores, and brands (Macé and Neslin 2004). For example, people may not be inclined to stockpile brands that are frequently discounted but, instead, may capitalize their storage capacity for other, less frequently promoted brands, because in the former category, consumers may expect new discounts soon. Therefore, researchers may be less likely to find evidence of a postpromotion dip for brands with frequent price promotions. In addition, there may be products that are more difficult to store because they are perishable or because of their large size; again, evidence of a postpromotion dip for such products may be rare.

Furthermore, it is sometimes not clear which marketing action is actually being considered. Most studies consider the effect of a change in actual price. However, this variable captures two different pricing decisions: price discounting and the regular price. Naturally, there is a difference between the effects of a temporary price promotion (price discount) and those of a permanent (regular) price change. All estimated effects are contingent on the chosen definition. Bucklin and Gupta (1999) point out the need for separation of the two pricing decisions from the point of view of practitioners and researchers, and they classify the investigation and estimation of regular price elasticity as unresolved from both perspectives.

Another possible explanation pertains to the definition of the dynamic effect. In the literature, there seems to be some confusion about the definition (and the measurement) of concepts such as “short-term,” “long-term,” “net,” “cumulative,” and “dynamic” effects. For example, some authors analyze the effect of frequent promotions on consumer brand perception and call this the long-term effect of promotion. In other articles, including ours, the focus is on the measurement of postpromotion dynamics in sales. In most mature product categories, sales are stationary. Thus, temporary changes in price cannot have a permanent effect on the future level of sales. Conversely, permanent changes in regular price most likely do have a permanent effect. It is also expected that the immediate and dynamic effects of the two types of price changes differ substantially (Bijmolt, Van Heerde, and Pieters 2005). Following this line of thought, it is important to distinguish between price promotions and changes in the regular price. By definition, price promotions are temporary, whereas changes in the regular price are permanent.

For price promotions and changes in regular price, the immediate effect is of interest. As we discussed, temporary price promotions do not have a permanent effect. Therefore, it is not useful to consider the long-term effect of price promotions. The effect of a price promotion at time t on sales at time t + k for k → ∞ is zero for all (stationary) cases. Thus, for price promotions, it is more relevant to consider the total, or cumulative, effect on current and all future sales, that is, the sum of the effects over all periods (t, t + 1, t + 2, …). The sign of this cumulative effect indicates whether the promotion will be beneficial in the long run. Note that for permanent changes in the regular price, it is likely that a permanent effect indeed exists. In this case, it is interesting to consider the long-term effect of such a change. The cumulative effect of a permanent change is not informative.

In this article, we aim to identify the immediate and dynamic effects of temporary price promotions and permanent changes in the regular price. We use the term “immediate effect” for both price promotions and regular price changes. Throughout this article, we consider the “cumulative effect of a price promotion” and the “long-term effect of regular price change.” Finally, we use the term “dynamic effect” to capture all effects of current changes on future sales.

To investigate these dynamic effects, we consider a new modeling methodology, which we apply to weekly data for a large number of categories. We relate the effects of price changes of multiple brands in various product categories to observable product and category characteristics by using a hierarchical Bayes (HB) error correction model (ECM); hereinafter, we refer to this as the HB-ECM. This model enables us to estimate directly the potential different immediate and dynamic effects of price changes on sales. We relate these effects to characteristics of brands and categories. As mentioned, we explicitly distinguish between promotional price elasticities and regular price elasticities. Although many studies suggest that these elasticities differ, there is a gap in the marketing literature regarding the determinants of regular price elasticities. Our model provides important insights for brand managers and for retailers about these determinants, which can moderate the effect of price changes. Among other things, we analyze whether there are common determinants for the immediate promotional price elasticity and for the cumulative effects of price promotions.

It is important to emphasize that our analysis focuses on the cross-sectional, not the longitudinal, determinants of price elasticities. That is, we analyze the differences in the immediate and cumulative effects of promotional and regular price changes across categories and brands, assuming that the characteristics of the investigated markets do not change. Especially with respect to the determinants of regular price elasticity, this study has an exploratory nature. There are too few studies available to formulate hypotheses.

We organize the remainder of this article as follows: We present a detailed overview of the literature. We then discuss our hypotheses on the relationship between brand and category characteristics and the immediate and dynamic effects of promotional price and regular price. Next, we present our HB-ECM in detail. (We discuss technical derivations of the estimation algorithm in Appendix A.) Finally, we present the empirical results.

**LITERATURE**

In the literature, there are many articles that investigate the relationship between market characteristics and promotional price elasticities. The number of articles on regular price elasticities is much smaller. In Table 1, we present a selection of studies on price elasticities. It is clear that most of these articles focus on relating immediate promotional price elasticities to brand, category, or consumer characteristics. Almost all studies use a two-stage approach for the
explains dynamic effects of price changes. In contrast, our approach uses an HB-ECM to relate the immediate and cumulative effects of promotional price and regular price to brand and category characteristics. Among the articles listed in Table 1, we find only four articles that investigate the determinants of the immediate and the dynamic effects of marketing actions. Foekens, Leeflang, and Wittink (1999) use time-varying parameter models to investigate the effects of the properties of a particular discount (e.g., the size of the discount and the time since the previous discount) on the intercept and price promotion parameters in a sales model. Lim, Currim, and Andrews (2005) explore whether the dynamic effects of price promotions and the duration of the adjustment period differ between light and heavy users and between loyal consumers and switchers. Nijs and colleagues (2001) consider the moderating effect of marketing intensity, competitive reactivity, and competitive structure on the category-demand effect of price promotions. Macé and Neslin (2004) investigate the determinants of pre- and postpromotion dips.

Table 1
OVERVIEW OF THE LITERATURE ON THE DETERMINANTS OF PRICE PROMOTIONS EFFECTIVENESS

<table>
<thead>
<tr>
<th>Study</th>
<th>Explanatory Variables</th>
<th>Immediate and/or Dynamic Effects</th>
<th>Dependent Variable</th>
<th>Approach</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bolton (1989)</td>
<td>Category + brand characteristics</td>
<td>Immediate</td>
<td>Price elasticity</td>
<td>Single equation (two-step)</td>
</tr>
<tr>
<td>Fader and Lodish (1990)</td>
<td>Category characteristics</td>
<td>Immediate</td>
<td>Percentage of volume purchased on price cut/feature activity/display activity/store coupon offer</td>
<td>Factor + cluster analysis</td>
</tr>
<tr>
<td>Raju (1992)</td>
<td>Category characteristics</td>
<td>Immediate</td>
<td>Variability in category sales</td>
<td>Single equation (one-stepa)</td>
</tr>
<tr>
<td>Hoch and colleagues (1995)</td>
<td>Consumer + competitor characteristics</td>
<td>Immediate</td>
<td>Price elasticity</td>
<td>Single equation (two-step)</td>
</tr>
<tr>
<td>Shankar and Krishnamurthi (1996)</td>
<td>Retailer pricing policy + promotional variables</td>
<td>Immediate</td>
<td>Regular price elasticity</td>
<td>Single equation (three-step)</td>
</tr>
<tr>
<td>Narasimhan, Neslin, and Sen (1996)</td>
<td>Category characteristics</td>
<td>Immediate</td>
<td>Nonsupported price elasticities/supported price elasticities with feature and display activity</td>
<td>Single equation (one-stepb)</td>
</tr>
<tr>
<td>Montgomery (1997)</td>
<td>Demographic + competitive characteristics of stores</td>
<td>Immediate</td>
<td>Price sensitivity</td>
<td>System (HB approach)</td>
</tr>
<tr>
<td>Mulhern, Williams, and Leone (1999)</td>
<td>Brand + consumer characteristics</td>
<td>Immediate</td>
<td>Price elasticity</td>
<td>Single equation (two-step)</td>
</tr>
<tr>
<td>Bell, Chiang, and Padmanabhan (1999)</td>
<td>Category, brand, + consumer characteristics</td>
<td>Immediate (primary versus secondary demand effects)</td>
<td>Price elasticity</td>
<td>Single equation (two-stepc)</td>
</tr>
<tr>
<td>Nijs and colleagues (2001)</td>
<td>Category characteristics</td>
<td>Immediate + dynamic</td>
<td>Price elasticity</td>
<td>VAR (two-step)</td>
</tr>
<tr>
<td>Wakefield and Inman (2003)</td>
<td>Product characteristics + social context of purchase</td>
<td>Immediate</td>
<td>Price elasticity</td>
<td>Single equation (one-stepb)</td>
</tr>
<tr>
<td>Macé and Neslin (2004)</td>
<td>Product + category + store trading area characteristics</td>
<td>Immediate + pre- and postpromotion dips</td>
<td>Price elasticity</td>
<td>Single equation (two-stepc)</td>
</tr>
<tr>
<td>Lim, Currim, and Andrews (2005)</td>
<td>Consumer characteristics</td>
<td>Dynamic</td>
<td>Price promotion</td>
<td>VAR (two-step)</td>
</tr>
<tr>
<td>This study</td>
<td>Category + brand characteristics</td>
<td>Immediate + dynamic</td>
<td>Promotional + regular price elasticity</td>
<td>VAR (HB approach)</td>
</tr>
</tbody>
</table>

Notes: VAR = vector autoregression.

aA two-step approach is not needed because the variability in category sales is not captured by a statistical model.
bThis is actually a two-stage approach because we obtained the price elasticities from another study.
cAuthors explicitly account for uncertainty in the first-level estimates in the second stage.
They view brand and category characteristics and characteristics pertaining to the store trading area as moderating factors. Our approach is more in line with the latter two articles because we investigate heterogeneity in the immediate and dynamic effects of price across brands and categories, assuming constant parameters over time.

Table 1 also shows that most studies consider moderators of price elasticity. As we discussed previously, price promotions and changes in regular price are likely to have different effects on sales. In this study, we explicitly distinguish between these two price effects.

Overview of Findings

In this subsection, we give a brief overview of the literature on the effects of price promotions and of regular price. We begin by considering the relative size of the different effects. Next, we consider possible determinants of the price elasticities in more detail.

On the basis of the extensive literature on the postpromotion dip, we expect that the cumulative effect of price promotions is smaller than the immediate effect. The long-term elasticity of regular price can be larger (in absolute value) than the immediate effect. Because of its lasting feature, a regular price cut does not induce consumers to accelerate their purchase and stockpile. Thus, a permanent price reduction has its effect over a longer period. Consumers decide to buy the price-reduced brand during the regular shopping trip, causing no postpromotion dip.

We expect that short-term promotional elasticity is greater than the immediate elasticity of regular price (Bjornm, Van Heerde, and Pieters 2005; Blattberg, Briesch, and Fox 1995). The relationship between the cumulative promotional elasticity and the long-term elasticity of regular price is less clear. Indeed, the cumulative effect of a price promotion measures something completely different from the long-term effect of a decrease in regular price.

Subsequently, we discuss the expected relationship between brand and category characteristics and the price elasticities. We make a selection of variables on the basis of the properties of the available data and the existing literature that relates promotional elasticities to market, category, brand, and/or consumer characteristics (see, e.g., Bell, Chiang, and Padmanabhan 1999; Hoch et al. 1995; Narasimhan, Neslin, and Sen 1996; Raju 1992). We extend the set of already studied explanatory characteristics with additional variables and provide a discussion of the determinants of dynamic effects. Compared with the literature on price promotions, there are significantly fewer articles that examine the determinants of regular price effects (see, e.g., Blattberg and George 1991; Guadagni and Little 1983; Jedidi, Mela, and Gupta 1999; Mitra and Lynch 1995; Shankar and Krishnamurthi 1996). Therefore, we focus mainly on price promotions. To develop our hypotheses, we take as a starting point a utility-maximizing consumer who operates under a budget constraint (Bell, Chiang, and Padmanabhan 1999; Varian 1992). We provide insights into the expected marginal effects of brand and category characteristics.

We distinguish three groups of variables: category-specific variables, brand-specific variables, and variables that can be defined at both levels. For example, both the promotion frequency in a category and the promotion frequency in a brand relative to the category may be important. Note that this distinction contrasts with most of the existing literature. For example, Macé and Neslin (2004) consider only the relative promotion frequency within a category. For each characteristic, we summarize the literature and, if possible, provide hypotheses for the sign of the immediate and cumulative effect of a price promotion. In several cases, however, we cannot formulate hypotheses about how brand- and category-specific characteristics influence cumulative effects. In these cases, our research can be considered an exploratory quest for empirical evidence on whether and how certain category- and brand-specific characteristics are related to the dynamic effects of price.

Category-Specific Characteristics

Average budget share. The budget share of a category captures two distinct dimensions: the general price level in the category and the average quantity purchased in a period. The first dimension leads to the conclusion that expensive product categories are likely to exhibit a smaller immediate effect and a lower and shorter postpromotion dip. Therefore, the difference between the immediate effect and the cumulative effect is relatively small. A consumer with a budget constraint will probably be less inclined to buy extra quantity of an expensive product because the additional purchase will lead to a large grocery bill (Raju 1992). In addition, price promotions may be less effective for expensive products because higher-income (and, thus, less price-sensitive) shoppers may constitute a greater ratio of the consumer population.

The second dimension results in the opposite, that is, a large promotional price effect and a large promotional dip. Macé and Neslin (2004) provide empirical evidence of the latter. The argument is that if the consumer is sure to consume the product in the near future, he or she will be inclined to purchase additional items from the high-budget-share category.

Utilitarian. Bell, Chiang, and Padmanabhan (1999) argue that relatively more necessity (i.e., nonimpulse) products exhibit lower primary demand effects and higher secondary effects. However, when facing a promotion for necessity products, consumers, knowing that they will surely be in the need of such products in the future, may be more inclined to stock up at home, even by postponing purchases of products in other categories. Moreover, shoppers with higher incomes (and, thus, those who are less price sensitive) may constitute a smaller proportion of the consumer population of necessity goods. We conjecture that necessity products have greater immediate effects and also a possibly greater difference between the immediate and the cumulative effect. Wakefield and Inman (2003) make a distinction along a similar dimension and find that consumers are less price sensitive in categories they perceive as primarily hedonic in nature.

A regular price increase in a utilitarian product category is likely to have a lower effect than a similar increase in a more hedonic category. In the case of a price rise of a necessity product, households with a binding budget constraint may be inclined to reduce purchases of other, less essential products.

Perishability. Consumers may favor price promotions in a category with easily storable goods, that is, a category in
which the consumer can allow for purchases at irregular intervals as a response to deals (Narasimhan, Neslin, and Sen 1996; Raju 1992). In the case of easily storable goods, consumers are more likely to use the accumulated stock during a longer period after the promotion. This suggests that promotions in a category with less perishable products lead to higher sales during the offer but also to a larger and longer dip after the promotion than in a category with more perishable products. Therefore, in this context, we expect that the cumulative effect is smaller than the immediate effect.

**Competitive intensity.** Nijs and colleagues (2001) argue that cooperation among brands is easier in markets with a limited number of players. In such markets, the brands can control production and price to end up at a relatively elastic region of the industry demand curve. Conversely, brand proliferation has also been identified as a potential cause of weakened brand loyalty (Narasimhan, Neslin, and Sen 1996), suggesting opposite moderating effects for competitiveness. Narasimhan, Neslin, and Sen (1996) also point out that product differentiation makes brands less exposed to competitors’ actions. Accordingly, in differentiated categories, promotions are expected to induce less brand switching, and the immediate price elasticity is expected to be lower.

The aforementioned theories capture two different aspects of competition. The first focuses on how the division of the market among competing brands influences price elasticities, and the second focuses on how increased competition due to a different degree of differentiation among brands influences promotional effectiveness. We proxy the first aspect by a concentration index and the second by price dispersion.

**Category- and Brand-Specific Characteristics**

We use both the intensity of marketing activities at the category level and the intensity for a brand relative to the category. In some categories, promotions may be more frequent than in others, but within a category, there may also be relevant differences.

**Frequency of price promotional activity.** Theory suggests mixed effects of the frequency of promotions. The theory on price consciousness (Kopalle, Mela, and Marsh 1999; Mela, Gupta, and Jedidi 1998; Mela, Gupta, and Lehmann 1997) leads to a positive effect of the price-promotion frequency on the immediate effect and the size of the postpromotion dip. High price promotional activity leads to price-conscious consumers. These consumers tend to purchase when there are deals, and they may develop a habit of stockpiling.

Conversely, the use of discounts may reduce consumers’ reference prices (Kalaynaram and Winer 1995), resulting in a lower level of effectiveness of discounts. Furthermore, if a category is promoted infrequently, consumers are more likely to use these opportunities to stock up for future consumption (Raju 1992).

Within a category, other processes may also play a role. Brands with relatively frequent price promotions are often considered of lower quality than similar, rarely promoted brands. Intense promotional activity may also influence the mix of consumers for a brand. More specifically, frequently promoted brands may draw a larger proportion of the price-sensitive consumer base (Zenor, Bronnenberg, and McAlister 1998).

Empirical findings about this relationship seem to be ambiguous. Blattberg, Briesch, and Fox (1995) and Raju (1992) find that high levels of promotional activity are associated with a lower height of the deal spike. Bolton (1989) finds no significant relationship between category price activity and the price elasticity. Zenor, Bronnenberg, and McAlister (1998) and Nijs and colleagues (2001) find the opposite effect; they state that the price promotion frequency is positively related to the effect of price promotions. Zenor, Bronnenberg, and McAlister find this effect at the brand level, and Nijs and colleagues find it at the category level. Moreover, Nijs and colleagues find that this correlation disappears for the long-term effect of price. However, Macé and Neslin (2004) report a positive correlation between the relative price-promotion frequency and the size of the postpromotion dip.

These ambiguous findings may partly be due to some researchers trying to determine the consequences for price elasticities of more frequent promotional usage in a category, whereas others focus on the result of frequent promotional usage for brands. In this study, we use the price promotion frequency at both the category and the brand level.

**Average depth of price promotion.** In categories in which price reductions are relatively large, consumers, who expect to obtain a high reward, are probably inclined to accelerate their purchase. As such, they draw sales from the weeks following the promotion, unless consumption increases correspondingly (Foekens, Leeflang, and Wittink 1999). This would result in a high immediate increase in sales and a large difference between the immediate effect and the cumulative effect of the promotion.

In categories in which consumers are used to deep price cuts, a small change in regular price may not trigger a reaction. Thus, we expect a lower effect of regular price changes in categories with relatively large price discounts.

**Frequency of display/feature activity.** Bolton (1989) argues that display activity may be systematically related to own price elasticities. The relative frequency of displays may influence consumers’ beliefs about the popularity and quality of market offerings. This effect is more pronounced within a category than across categories. On the one hand, display activity may encourage customers to apply choice rules that rely less on search for price information, arriving at less-price-elastic sales. On the other hand, it may lead customers to compare prices, which would result in more-price-elastic sales. Bolton finds support for the first case; that is, sales are more price inelastic for categories and brands that are frequently displayed.

Feature activities are often used to provide information about the prices and price promotions in a retail outlet. Thus, frequent feature activity in a category is likely to make current consumers more aware of the prices and the occurrence of promotional activities in the category (Bolton 1989; Moriaty 1985). This suggests that brands in categories with frequent retailer advertising activity should have high immediate price elasticity and also a large drop in sales after the price promotion. A similar distinction can be made for brands that are more often promoted within a category.

Another theory is based on the notion that feature and display activity may increase brand salience and price
saliency (Shankar and Krishnamurthi 1996). The former may induce consumers to differentiate brands more, thus increasing their relative preferences and reducing the consideration set. This would lead to lower regular price elasticity (Mitra and Lynch 1995). Conversely, increased price salience may lead to more price comparison within the category.

**Brand-Specific Characteristics**

*Brand size.* Bolton (1989) documents that brands with a relatively high market share tend to operate on the flat part of their sales response functions. Thus, larger brands tend to be less own price elastic. Blattberg, Briesch, and Fox (1995) mention this relationship as one of the empirical generalizations for promotions. A possible explanation for this is that large brands tend to have higher quality; in turn, this could lead to lower price elasticities.

*Price segment of a brand.* A discount may attract several types of consumers: (1) those who usually purchase a competing brand, (2) those who would otherwise find the brand too expensive, and (3) those who are already loyal (Raju 1992). The promotion of a brand in an expensive price category may persuade all three types of consumers to buy the promoted product. However, the promotion of a lower-priced product is unlikely to attract consumers from the second category, suggesting that the immediate effect and the cumulative effect are lower (given that the regular consumer base is equal across the different price segments). In addition, lower-income (and, thus, more price-sensitive) shoppers may constitute a larger fraction of the consumer population of less expensive brands (Raju 1992). Such consumers are more inclined to buy the brand when it is discounted and save money by stockpiling. Furthermore, we expect that the immediate and long-term effects of regular price are higher for lower-priced brands within a category.

**ANALYZING IMMEDIATE AND DYNAMIC EFFECTS**

In this section, we present a modeling framework for estimating (the determinants of) the dynamic effects of price promotions on log sales when the logarithm of sales is unit-root stationary. We first consider a model for one product category. We focus on the discussion on price effects, but the methodology holds for any set of marketing instruments. Then, in the next section, we extend the model to capture multiple categories.

Recent literature on market structures has shown that marketing efforts, such as temporary price promotions, do not have permanent effects on sales. A prerequisite for permanent effects of temporary promotions is the nonstationarity of sales. Srinivasan, Popkowksi Leszczyc, and Bass (2000), Nijs and colleagues (2001), and Pauwels, Hanssens, and Siddarth (2002), among others, have shown that in the categories considered, almost all log sales series for fast-moving consumer goods are stationary. This result is not surprising because a unit root in log sales implies that all frequent temporary price promotions lead to permanent increases in sales, which seems to be an unrealistic assumption. Thus, to study dynamic effects of temporary price promotions, it is more important to examine the cumulative effect of a temporary price promotion on current and future log sales than to study the permanent effect.

For the regular price, changes are likely to be permanent. Even if sales are stationary, permanent changes in price may lead to permanent changes in the sales. Thus, for such variables, it is relevant to consider the effect of a permanent change on sales in the long run.

To describe the dynamic pattern in sales of brands in a product category, we begin with a vector autoregression (VAR) with explanatory variables (VARX). We denote the sales of brand $i$ at time $t$ by $S_{it}$, for $i = 1, \ldots, I$ and $t = 1, \ldots, T$, where $I$ is the number of brands in the market. To model the vector of sales $S_t = (S_{1t}, \ldots, S_{It})'$, we consider a VARX(1) model:

\[
\log S_t = \mu + \Gamma \log S_{t-1} + \sum_{k=1}^{K} (A_k \log X_{kt} + C_k \log X_{k,t-1}) + \epsilon_t,
\]

where $\epsilon_t \sim N(0, \Sigma)$ and $\mu$ denotes a vector of intercept parameters. The vector $X_{kt} = (X_{k1t}, \ldots, X_{klt})'$, $k = 1, \ldots, K$, denotes an $I$-dimensional vector of a potential explanatory variable. For example, $X_{kt}$ denotes the $k$th marketing-mix variable of brand $i$ at time $t$ (e.g., promotional price or regular price). The terms $A_k$ and $C_k$ are $I \times I$ parameter matrices. The diagonal elements of these matrices describe the own effect of the marketing variables, and the off-diagonal elements represent the cross effects.

It follows from Equation 1 that the immediate effect of a change in $X_k$ on the log sales is given by the elasticity

\[
\frac{\partial S_t}{\partial X_{kt}} \frac{\partial \log S_t}{\partial \log X_{kt}} = A_k.
\]

Thus, the immediate effect is equal to $A_k$ and does not depend on whether the change is permanent or temporary.

**Dynamic Effect of Permanent Changes**

To determine the dynamic effects of a marketing instrument $(X_{kt})$ on sales, we solve Equation 1 for $\log S_t$ by repeated substitution. This results in

\[
\log S_t = \Gamma^\tau \log S_{t-\tau} + \sum_{j=0}^{\tau-1} \Gamma^j \left[ \mu + \sum_{k=1}^{K} (A_k \log X_{k,t-j} + C_k \log X_{k,t-j-1}) + \epsilon_{t-j} \right].
\]

Under the stationarity condition (i.e., the eigenvalues of $\Gamma$ are within the unit circle), the influence of log sales at time $t - \tau$ on current log sales disappears for large $\tau$ as $\lim_{\tau \to \infty} \Gamma^\tau = 0$. Next, if we set the explanatory variables at fixed values (i.e., $X_{kt} = X_k$ for all $t$ and $k = 1, \ldots, K$), it holds for $\tau \to \infty$ that

\[
\log S_t = (I - \Gamma)^{-1} \mu + \sum_{k=1}^{K} (I - \Gamma)^{-1}(A_k + C_k) \log X_k
\]

\[
+ \sum_{j=0}^{\infty} \Gamma^j \epsilon_{t-j},
\]
where \( I \) denotes the identity matrix. As \( E[\epsilon_{t+j}] = 0 \) for all \( j \), the long-term expectation, if we assume a constant marketing mix, of the vector of log sales given \( X_t, ..., X_K \) equals

\[
E[\log S|X_t, ..., X_K] = (I - \Gamma)^{-1}\mu + \sum_{k=1}^{K} (I - \Gamma)^{-1}(A_k + C_k)\log X_k.
\]

This expectation denotes the long-term relationship between log sales and the explanatory variables. This relationship is especially useful to determine the long-term effect of a permanent change in one of the explanatory variables. The size of the absolute values of the eigenvalues of \( \Gamma \) translates into the speed of convergence to the long-term equilibrium. The long-term elasticity of \( X_k \) on \( S \) is given by

\[
\frac{\partial S}{\partial X_k} = \frac{\partial \log S}{\partial \log X_k} = (I - \Gamma)^{-1}(A_k + C_k) = B_k.
\]

The diagonal elements of \( B_k \) represent the elasticity of marketing-mix variable \( k \) of brand \( i \) on brand \( i \), and the off-diagonal elements represent the cross-elasticities. Using the terminology we introduced previously, if \( X_k \) gives the regular price, then \( B_k \) gives the effect of a permanent price change on sales in the long run.

**Dynamic Effect of Temporary Changes**

It follows immediately from Equation 3 that under stationarity, a temporary change in one of the \( X_{kt} \) variables at time \( t \) has no impact on the sales at time \( t + j \) in the long run. This is because the term \( \Gamma \) is zero for large \( j \). Only a permanent change in the value of a marketing instrument can have a permanent long-term effect on the sales, which then depends on the values of the relevant parameters. To summarize the dynamics for a temporary promotion, it is more important to measure the cumulative effect on future sales.

Under stationarity, it is straightforward to show that the cumulative effect of a temporary change in \( \log X_{kt} \) on current and future log sales is given by

\[
\sum_{j=0}^{\infty} \frac{\partial \log S_{t+j}}{\partial \log X_{kt}} = \sum_{j=0}^{\infty} \Gamma^j(A_k + C_k) = (I - \Gamma)^{-1}(A_k + C_k) = B_k.
\]

In summary, for temporary (price) promotions, we interpret \( B_k \) as the cumulative effect, and for permanent actions, such as permanent changes in the regular price, it measures the long-term effect. In the subsequent application, we observe that, in general, the immediate effects of price promotions are larger in size than the cumulative effects. In other words, some of the immediate increases in sales due to a promotion might be compensated by lower sales in future periods. For the regular price, the immediate effect is smaller than the long-term effect.

**Error Correction Specification**

To analyze the dynamic effects of (permanent) changes in \( X_k \), the VARX representation in Equation 1 is not directly suitable; in addition, \( X_{kt} \) and \( X_{k,t-1} \) can be highly correlated. Although we assume stationarity (eigenvalues of \( \Gamma \) within the unit circle), it is still difficult to interpret directly the parameters in a VAR model with current and lagged exogenous variables. Indeed, the parameters combine the immediate and dynamic effects of the explanatory variables on the dependent variables. Therefore, it is more useful to write the VARX model in an error correction format because this provides a direct link between the various effects of a marketing instrument on log sales and the relevant model parameters.

In the marketing literature, the ECM has been used by, for example, Franses (1994) and Paap and Franses (2000) to distinguish the immediate from the dynamic effects. This approach contrasts with studies by, for example, Mela, Gupta, and Jedidi (1998) and Jedidi, Mela, and Gupta (1999), in which the dynamics enter through the model parameters. In these studies, the preferences and marketing sensitivity of households may change as a consequence of (intensified) promotional activities. In this case, the dynamic effect is defined as the impact of a promotion on the future when the changes in individual behavior are taken into account. In this article, we take a different approach and view (aggregate) household behavior as constant. The dynamics in sales are directly caused by feedback loops in household behavior.

To disentangle the immediate effects of \( X_{kt} \) on the log sales from the dynamic effects—that is, to allow for direct estimation of these effects—it is convenient to rewrite in the format of an ECM (see Hendry, Pagan, and Sargan 1984):

\[
\Delta \log S_t = \mu + \sum_{k=1}^{K} A_k \Delta \log X_{kt} + \sum_{k=1}^{K} \Pi B_k \log X_{k,t-1} + \epsilon_t,
\]

where \( \Pi = (I - \Gamma) \), \( B_k = (I - \Gamma)^{-1}(A_k + C_k) \), and \( \Delta \) denotes the first-differencing operator, \( \Delta y_t = y_t - y_{t-1} \). Note that this transformation involves only a rearrangement of terms; that is, Equation 8 is exactly equivalent to Equation 1. The advantage of the ECM representation is that we can link explanatory variables directly to the immediate effects \( A_k \) and the dynamic effects \( B_k \), as we discuss subsequently. Under stationarity, parameter estimation in the ECM representation is not much more difficult than it is in the VARX specification in Equation 1, because it involves only a parameter transformation.

Although the ECM in Equation 8 describes only the relationship between two consecutive periods, ECMs are well suited to analyze the long run (for a discussion, see Granger 1993). The long-term relationship between log sales and the log \( X_{kt} \) variables is put in the error correction term \( \log S_{t-1} - \sum_{k=1}^{K} B_k \log X_{k,t-1} \), and thus the long-term effects are given by \( B_k \). That is, this parameter gives the marginal effect of a permanent change of log \( X_{kt} \) on the log sales in the long run. The parameter matrix \( \Pi \) contains the adjustment parameters and determines the speed of convergence to the long-term relationship. Note that the speed of
convergence is the same for explained shocks through marketing instruments as it is for unexplained shocks.

Finally, the autoregressive structure of our model does not automatically imply that changes in \( X_k \) have a dynamic effect on sales. A special case of the model is when \( A_k = B_k \) for all \( k \). The ECM in Equation 8 then simplifies to a common factor model (Hendry, Pagan, and Sargan 1984); that is, Equation 8 reduces to

\[
\begin{align*}
\log S_t - \sum_{k=1}^{K} A_k \log X_{kt} &= \mu \\
+ \Gamma \left( \log S_{t-1} - \sum_{k=1}^{K} A_k \log X_{k,t-1} \right) + \epsilon_t,
\end{align*}
\]

A temporary change in \( X_{kt} \) now has an effect only on current log sales and not on future sales. Thus, the immediate effects are equal to the cumulative effects.

**HB-ECM**

In this section, we discuss our model for the case of a large number of categories. The HB-ECM allows us to estimate the moderating effects of brand and category characteristics on price elasticities. In a separate subsection, we discuss the differences between our approach and the existing literature; we take the work of Nijs and colleagues (2001) as a representative article.

**HB Analysis**

Let \( S_{ct} \) denote the \( I_c \)-dimensional vector of sales for category \( c \) in week \( t \). Note that categories are allowed to have different numbers of brands \( I_c \). The \( I_c \)-dimensional vectors \( X_{ckt} \) contain the \( k \)th marketing-mix variables for the brands in category \( c \) in week \( t \). The ECM in Equation 8 for category \( c \) is given by

\[
\begin{align*}
\Delta \log S_{ct} &= \mu_c + \sum_{k=1}^{K} A_{ck} \Delta \log X_{ckt} \\
+ \Pi_c \left( \log S_{c,t-1} - \sum_{k=1}^{K} B_{ck} \log X_{k,t-1} \right) + \epsilon_{ct},
\end{align*}
\]

where \( \epsilon_{ct} \sim N(0, \Sigma_c) \) for \( c = 1, \ldots, C \) and \( t = 1, \ldots, T_c \). Note that we allow for different intercepts \( \mu_c \), immediate \( \Lambda_c \) and dynamic \( \lambda_c \) effects, and covariance matrices for the error terms \( \Sigma_c \) across categories. We also allow the adjustment parameters \( \Pi_c \) to be different across categories. The categories may even have a different number of brands and observations.

To relate the immediate and dynamic elasticity parameters to explanatory variables, we collect the parameters describing the effects of marketing-mix variables of brand \( i \) on the sales of brand \( i \) (because we focus on the own effects) in the \( I_c \)-dimensional vectors \( \alpha_{ic} = \text{diag}(A_{ic}) = (\alpha_{ic1}, \ldots, \alpha_{icK})' \) and \( \beta_{ic} = \text{diag}(B_{ic}) = (\beta_{ic1}, \ldots, \beta_{icK})' \) for \( k = 1, \ldots, K \). The immediate and dynamic elasticities will obviously differ across brands and across categories.

Some of these differences can be attributed to observable characteristics of the brand and/or category, such as depth and frequency of promotion or perishability of the product, as we discussed previously. Another part of the differences across elasticities cannot be explained. In summary, we propose to describe the immediate and dynamic elasticity parameters by

\[
\begin{align*}
a_{ick} &= \lambda_{1k} z_{ic} + \eta_{ick} \\
\beta_{ick} &= \lambda_{2k} z_{ic} + \nu_{ick},
\end{align*}
\]

where \( z_{ic} \) is an \( L \)-dimensional vector containing an intercept and \( L-1 \) explanatory variables for brand \( i \) in category \( c \), such as frequency and depth of promotion and category competitiveness. The \( L \)-dimensional vectors \( \lambda_{1k} \) and \( \lambda_{2k} \) describe the effects of the brand characteristics on the immediate and dynamic elasticities, respectively. The error terms \( \eta_{ick} \) and \( \nu_{ick} \) have zero mean and are assumed to be uncorrelated across brands and categories. However, we allow for correlation in the error terms across the \( k \) marketing-mix variables; that is, we assume that \( \eta_{ic} = (\eta_{ic1}, \ldots, \eta_{icK})' ~ N(0, \Sigma_{ic}) \) and \( \nu_{ic} = (\nu_{ic1}, \ldots, \nu_{icK})' ~ N(0, \Sigma_{ic}) \).

We call the preceding model the HB-ECM. Akçura, Göñül, and Petrova (2004) and Montgomery and colleagues (2004) use a VAR structure in a Bayesian setting for latent utility variables, but as far as we know, we are the first to use a VAR model in error correction format for sales in an HB setting. To estimate the parameters in the model in Equation 10 with Equations 11 and 12, we use a Bayesian approach. Bayesian estimation provides exact inference in finite samples. To obtain posterior results, we use the Markov chain Monte Carlo (MCMC) simulation method. In Appendix A, we derive the likelihood function of the model together with the full conditional posterior distributions, which are necessary in the Gibbs sampler.

Another estimation strategy that is often applied in practice is a two-step procedure in which individual market-level models are first estimated, and in a second stage regression, the parameters from the market-level models are related to brand and market characteristics (see, e.g., Nijs et al. 2001). However, this method is theoretically less elegant because the uncertainty in the first-level parameter estimates is not correctly accounted for in the second stage. In finite samples, this leads to underestimation of the uncertainty in the parameter estimates in the second stage.

**Comparison with Existing Literature**

Before we continue with a discussion of our empirical results, we list the main differences between our approach and the current standard approach in the literature. Again, we take the work of Nijs and colleagues (2001) as a representative study.

First, Nijs and colleagues (2001) obtain the dynamic effects from impulse response functions (IRFs) on the basis of estimated VAR models. This incorporates possible competitive and feedback effects of competitors. In contrast, our measure of the dynamic effect in the ECM excludes these effects. This enables us to focus better on the determinants of “dynamic demand reactions.” In our view, competitive reactions and demand (consumer) reactions to promotions are two topics that need to be addressed sepa-
rately. A proper analysis of competitive reactions requires a thorough understanding of consumer response to changes in marketing variables. In the end, the consumer will view a competitive reaction as another promotion. On the basis of the outcomes of our model and given a likely competitive reaction, the net effect of the promotion can be easily judged. The combination of the ECM with a model for competitive reactions remains an issue for further research.

Second, the HB-ECM allows us to disentangle the immediate and dynamic effects into separate estimable parameters. This enables us to relate these effects directly to store and brand characteristics. Thus, we do not need to rely on derivative measures, such as the IRF, or on a two-step approach to relate immediate and dynamic effects to moderating variables. Through this error correction specification, we also avoid one of the disadvantages of the two-step procedure. The two-step procedure does not appropriately account for the uncertainty in the first-level parameter estimates when estimating the second-stage model. In finite samples, this leads to the underestimation of the standard errors of the parameters in the second-stage regression. The ECM model allows us to judge the accuracy of our dynamic effects estimates because we can easily obtain standard errors for these estimates. For example, Nijs and colleagues (2001) compute the cumulative effects of promotions using accumulated IRFs. Because these accumulated IRFs are nonlinear functions of the model parameters, the uncertainty in the estimated effect is usually difficult to compute. More important, the uncertainty is often large compared with the uncertainty in model parameter estimates. Therefore, from an efficiency point of view, it is more reliable to estimate directly a parameter representing the long-term effects than to rely on the impulse responses.\(^1\)

Another difference is that we investigate category and brand characteristics that may affect price elasticities. Finally, in addition to focusing on the determinants of promotional price elasticities, we consider the effects of changes in the regular price. Much of the marketing literature to date mainly focuses on the determinants price promotion elasticities.

**EMPIRICAL RESULTS**

In this section, we use our HB-ECM to explain differences in immediate and dynamic effects of promotional price and regular price on sales across brands and product categories. First, we discuss the product categories we consider in our analysis and the available data. Next, we discuss the estimation results.

**Data and Variables**

The data we consider are weekly sales volumes of fast-moving consumer goods in 25 product categories. We obtained the data from the database of a large supermarket chain, Dominick’s Finer Foods; these data were collected in the Chicago area during the period from September 1989 to May 1997. Sales are aggregated from stockkeeping unit (SKU) to brand level using static weights, as described by Srinivasan and colleagues (2004), who use the same data set. In several cases, aggregation leads to biased results. For example, Christen and colleagues (1997) show that unbiased estimates of promotion elasticities cannot be obtained straightforwardly from data that are aggregated across stores. We use a similar aggregation—that is, from SKUs to brands—but because we are interested explicitly in brand-level elasticities, the aggregation does not pose problems here.\(^2\)

Our data comprise the following product categories: bottled juice, cereals, cheese, cookies, crackers, canned soup, dish detergent, “front-end” candies (i.e., those displayed by the checkout registers), frozen diners, frozen juice, fabric softener, laundry detergents, oatmeal, paper towels, refrigerated juice, soft drinks, shampoos, snack crackers, toothbrushes, canned tuna, toothpaste, and bathroom tissue. In each product category, we take only the top four brands; thus, we have \(4 \times 25 = 100\) different brands. We specify 25 ECMs, as in Equation 10. The dependent variable \(S_t\) consists of the total weekly sales of the brands in the separate product categories. As explanatory variables, we consider the marketing-mix variables, display, feature, regular price, and promotional price indexes.

The original database contains only the actual price. To decompose the actual price series into regular and promoted price, we smooth the actual price series using cubic splines with asymmetric weights. In the smoothing algorithm, positive errors are weighted ten times stronger than negative errors. In this way, we construct a series that follows the actual price in the case of no promotion, and it does not follow temporary drops in price. However, sustained drops are reflected in the regular price. A disadvantage of this method is that abrupt changes in the regular price may not be detected immediately, but it is beyond the scope of this article to consider alternative regular price algorithms and the possibly different results. For some categories, the actual price shows seasonal variation; we include seasonal dummies in the smoothing algorithm for these categories. To measure price discounts, we use a price index, that is, the actual price divided by the regular price. This price index is a natural measure for the size of a promotion, and it also allows for a comparison across categories. The immediate and dynamic own effects of the price index and the regular price, denoted by \(\alpha_{ck}\) and \(\beta_{ck}\), are explained by characteristics of the brand and product category in the second stage of the model, as explained in Equations 11 and 12.

In principle, the display and feature variables reflect the percentage of SKUs of the brand that are promoted in a given week. However, these variables turn out not to be consistently monitored in this database. In some cases, a display or feature may not have been recorded. Therefore, the variables are an imperfect measure for the actual promo-

\(^1\)Note that in a multiplicative model, which is the most frequently used model in marketing and is used by Nijs and colleagues (2001), the sum of the impulse responses to changes in the logs of variables over the dust-settling period has no straightforward interpretation (for a recent account, see Wieringa and Horváth 2005).

\(^2\)Although it is not possible to retrieve the average promotion effectiveness across all SKUs when using brand-level data, the brand-level estimate still provides useful information. This estimate can be interpreted as a promotion elasticity; it gives the percentage change in brand sales due to a percentage price change. However, the specification of the brand-level model is arguable because it does not match the aggregation of separate log-linear SKU-level models.
tional actions. However, to account for the effects of display and feature as much as possible, we include these variables in the first layer of our model, but we do not include the effects as dependent variables in the second layer. We do not discuss their estimates, because their interpretation can be unclear. For the cross-effects of marketing instruments, we allow only for cross-promotional price effects. To this end, we include all competitive price indexes as explanatory variables.

Finally, we control for seasonal variation in the sales series. To account for possible seasonality in the sales series, we include 13 seasonal dummies in the model, each covering four consecutive weeks. We chose the starting point of the period of four consecutive weeks to produce the best fit. If necessary, we also include dummies to capture special holidays (Easter, Memorial Day, Christmas) and lagged values of these dummies to deal with the dynamic effects of these events. We conducted this preanalysis for each category separately. Unit-root analysis shows that all sales series are (trend) stationary, after correcting for possible seasonality and possible breaks in the regular price series.

In summary, we include only the $\alpha_{i,c,k}$ and $\beta_{i,c,k}$ of the promotional price index and the regular price in the second layer of the HB-ECM. As explanatory variables of $\alpha_{i,c,k}$ and $\beta_{i,c,k}$, we use brand-level characteristics and category-level characteristics. Note that to construct these variables, we need to make use of the sales data. Strictly speaking, this may induce an endogeneity problem. However, this is not a problem here, because we use only summary statistics of sales, and we also use the model only for descriptive purposes.\footnote{Our model is a descriptive model. Following the work of Franses (2005), we apply diagnostics on residual autocorrelation for the ECM models before we estimate the HB two-level specification. There are no strong indications that we need to modify the dynamic structure of the models. For the sake of coherence and interpretation, we specify first-order dynamics for all equations.} Note also that the causal direction of the relationships that we find among elasticities and brand or category characteristics may not always be clear. In some cases, a “reverse causality” may also explain the effect.

Furthermore, although the recording of display and feature in the database is not perfect, we use these variables to obtain a measure of the relative frequency of these promotions. The underlying assumption here is that the reporting process is the same across brands and categories. A summary and the formal definition of these variables appear in Appendix B.

Estimation Results

We analyze the HB-ECM using Gibbs sampling (see Appendix A). Posterior results are based on 200,000 draws, of which we use the first 100,000 as burn-in. To remove correlation in the chain, we consider only every tenth draw to compute posterior results. Unreported plots of the draws of the model parameters of the second layer (i.e., Equations 11 and 12) show that the Markov chain has converged.

We begin by summarizing the posterior means of the effects of the (log) price index and the log regular price in graphs. Figure 1 presents the distribution of the posterior means of the immediate effect of price promotions and of regular price changes, the cumulative effect of price promotions, and the long-term effect of a regular price change. These histograms show the posterior means across all brands and all product categories. Overall, the dispersion in the dynamic effects tends to be smaller than that in the immediate effects. To gain insight into how much of this variation is explained in the second layer of our model, we calculate the posterior mean of the percentage of explained variance of the price effects. For the immediate effect, we explain 42% of the variance, and for the cumulative effect, we explain 67%.

For price promotions, the sign of all posterior means is in accordance with our expectations. Both the immediate and the cumulative effects are negative for all brands in all product categories. Overall, the cumulative effect tends to be smaller in absolute value than the immediate effect. In general, some of the positive effects of a price promotion are compensated in the periods following the promotion by, for example, the effects of stockpiling. We can quantify this finding by considering the posterior probability that the magnitude of the immediate effect is larger than the cumulative effect. Over all brands and categories, this posterior probability is .75.

For the regular price, the graphs in Figure 1 show much more dispersion. For the immediate and the long-term effect, there are brands with a positive regular price effect and brands with a negative effect. However, the mean immediate effect and the mean cumulative effect over all brands are negative. The graphs also seem to indicate that it is not possible a priori to indicate whether the immediate effect of regular price is larger or smaller than the long-term effect. The posterior probability that the immediate effect is larger in magnitude than the long-term effect is .57, which confirms our initial idea. The posterior mean of the percentage of explained variance indicates that 90% of the differences in the direct effect can be explained. For the long-term effect, it turns out to be much more difficult to explain the differences; we explain only 20% of the variance.

When we compare the promotional price elasticity with the regular price elasticity, we find that, overall, the regular price elasticity tends to be closer to 0. For the immediate effect, the posterior probability that the promotional price elasticity has a larger magnitude than the regular price elasticity is .72. The corresponding probability for the dynamic effects is .78.

Moderating factors of price promotion elasticities. We now turn to the second layer of our HB-ECM, in which we explain differences in the effects of price promotions and regular price on sales. Table 2 presents the posterior means and posterior standard deviations of the parameters in the second level of our model (i.e., Equations 11 and 12). This table gives the determinants of the immediate and cumulative effects of price promotions and the immediate and long-term effects of regular price changes. All brand and category characteristics have been standardized; that is, we have transformed them to have a mean of 0 and a variance of 1. Therefore, the parameter estimates for the intercept have the interpretation as the mean effect. From the intercept estimates for the promotional price, it is clear that the immediate effect is larger in magnitude than the cumulative effect.
We focus on the determinants of the promotional price effects because in the literature, this has received almost exclusive attention. We comment only on characteristics that significantly influence the effectiveness of price promotions. We find that more perishable product categories or categories with a low budget share tend to have a smaller immediate promotional price elasticity. The first conclusion is consistent with our previous conjectures, and the second is consistent with Macé and Neslin’s (2004) findings. In general, categories that are characterized by a large dispersion in prices have smaller immediate and cumulative promotional price effects. Such markets are likely to have several well-defined product segments. Therefore, switching among these segments in response to a promotion may be uncommon. Next, categories with a high market concentration tend to have stronger cumulative price elasticities.

An interesting finding is that though price promotion frequency for a category does not seem to influence the immediate or cumulative effectiveness of a price promotion, the relative frequency of price promotions of a brand within a category does influence the cumulative promotional price elasticity. Note that Nijs and colleagues (2001) find a significant effect across categories, whereas Bolton (1989) does not. In general, brands with a high frequency tend to have a small cumulative elasticity. However, the average depth of the price promotions in a category has a strong influence on the price promotion elasticity. Deeper price promotions correspond to stronger price effects. Even after we control for differences in the depth of promotions across categories, the relative depth of promotion for a particular brand enforces the elasticity. This holds for the immediate and the cumulative effect. These findings correspond to our hypothesis and to the findings of Raju (1992) and of Foekens, Leeiflang, and Wittink (1999) for their Brand B.

We hypothesized that brand sales are more elastic for categories and brands that are frequently featured in flyers and newspapers. This turns out to be true only for the category feature frequency and the immediate promotional price elasticity. Again, the results on the immediate effects coincide with Bolton’s (1989) findings. Consumers are

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4These conclusions should be taken with care because the display and feature measures in the Dominick’s data are rather noisy.
Table 2
POSTERIOR MEANS OF THE EFFECTS OF COVARIATES ON IMMEDIATE AND DYNAMIC EFFECTS OF PRICE PROMOTIONS AND REGULAR PRICE CHANGES
($\lambda_1$ AND $\lambda_2$ IN EQUATIONS 11 AND 12)

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Price Index</th>
<th></th>
<th></th>
<th>Regular Price</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Immediate Effect</td>
<td>Cumulative Effect</td>
<td>Immediate Effect</td>
<td>Long-Term Effect</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>-2.291 (.067) ***</td>
<td>-1.907 (.078) ***</td>
<td>-.542 (.369)</td>
<td>-826 (.128) ***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Category-Level Characteristics</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average budget share</td>
<td>-.232 (.088) ***</td>
<td>-.191 (.106) *</td>
<td>.291 (.669)</td>
<td>-.259 (.183) ***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Utilitarian</td>
<td>-.090 (.085)</td>
<td>.131 (.111)</td>
<td>-.017 (.488) **</td>
<td>.503 (.159) ***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Perishability</td>
<td>.145 (.085) *</td>
<td>-.006 (.097)</td>
<td>-.723 (.448) *</td>
<td>.273 (.157) *</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Market concentration</td>
<td>-.068 (.075)</td>
<td>-.193 (.099) *</td>
<td>.163 (.364)</td>
<td>-.173 (.143)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Price dispersion</td>
<td>.271 (.077) ***</td>
<td>.224 (.093) **</td>
<td>.760 (.426) *</td>
<td>-.308 (.139) **</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Price promotion frequency</td>
<td>-.051 (.094)</td>
<td>-.118 (.124)</td>
<td>.569 (.442)</td>
<td>-.374 (.172) **</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Depth of price promotions</td>
<td>-.234 (.076) ***</td>
<td>-.186 (.074) **</td>
<td>-.198 (.361)</td>
<td>-.167 (.143)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Feature frequency</td>
<td>-.317 (.100) ***</td>
<td>-.116 (.104)</td>
<td>-.007 (.439)</td>
<td>.047 (.177)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Display frequency</td>
<td>.094 (.085)</td>
<td>-.033 (.093)</td>
<td>.697 (.560)</td>
<td>.120 (.187)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Brand-Level Characteristics</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Relative price promotion frequency</td>
<td>.152 (.089)</td>
<td>.221 (.107) **</td>
<td>-.481 (.345)</td>
<td>.247 (.163)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Relative depth of price promotions</td>
<td>-.111 (.067) *</td>
<td>-.126 (.054) **</td>
<td>.125 (.230)</td>
<td>.117 (.121)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Relative feature frequency</td>
<td>.030 (.080)</td>
<td>.045 (.098)</td>
<td>.035 (.334)</td>
<td>-.058 (.160)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Relative display frequency</td>
<td>-.169 (.071) **</td>
<td>-.181 (.082) **</td>
<td>-.266 (.263)</td>
<td>-.105 (.130)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Brand size</td>
<td>-.162 (.076) **</td>
<td>-.178 (.091) **</td>
<td>-.319 (.245)</td>
<td>-.030 (.143)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Price segment</td>
<td>-.183 (.070) ***</td>
<td>-.082 (.084)</td>
<td>.121 (.298)</td>
<td>.029 (.132)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Characteristics are standardized to have mean 0 and variance 1.
Notes: Posterior standard deviations are in parentheses. *, **, and *** = zero not contained in 90%, 95%, and 99% highest posterior density region, respectively.
more aware of the prices in a category with frequent price-oriented advertising. Such a category attracts price-sensitive consumers, and households are more inclined to stockpile (the higher immediate effect is partly offset in the long run). Note that within a category, there is no effect of the price promotion frequency.

The display frequency of the product category as a whole turns out to be irrelevant in explaining the price effects. However, the relative display activity within a category partly explains the immediate and cumulative effect of a price promotion. Brand sales are more elastic for brands with a high relative display frequency (this contradicts Bolton’s [1989] findings).

The price segment has a negative influence on the immediate promotion price elasticity. This means that brands in a higher-price segment have stronger promotional price effects. The effect of brand size also corresponds with our conjecture. Larger brands tend to have smaller immediate and cumulative price effectiveness.

**Moderating factors of regular price elasticities.** For the relationship between brand and category characteristics and the regular price elasticity, we cannot rely on previous literature. Therefore, we list only some of the interesting findings.

Perhaps not unexpectedly, across all brands, the mean immediate effect of regular price is much smaller than the immediate effect of a price promotion; the found mean effect is close to zero. This finding is in accordance with our hypotheses. Changes in regular price do not attract new customers immediately. In the long run, customers become accustomed to the new price; therefore, the effects of regular price changes are expected to be relevant only in the long run. Furthermore, we find few characteristics that correlate with the immediate regular price effect. However, the magnitude of the influence of characteristics that are relevant is relatively large. This partly explains the large dispersion in the immediate effect of regular price that we found previously.

For utilitarian categories, we find a strong (negative) regular price elasticity. For this type of product, decreases in regular price lead to immediate increases in sales. Two other category characteristics are marginally important. The price dispersion in a category is positively correlated with the immediate regular price effect. Again, in such categories, short-term brand switching may be unlikely because these categories often consist of separate product segments. Finally, categories in which the products are highly perishable tend to have stronger immediate regular price elasticities.

For the long-term effect, we find a substantial mean effect; that is, the value of zero is not contained in the 99% highest posterior density region of the intercept. The absolute value of the long-term effect is greater than the absolute value of the immediate effect of a regular price change. Again, this corresponds to our previous hypotheses. A regular price change does not induce consumers to accelerate their purchases. At the same time, a regular price change may induce consumers to rethink the composition of their shopping basket in the long run. Compared with the immediate effect, we find more characteristics that explain the long-term effect. Surprisingly, again, none of the brand-level characteristics turn out to be relevant. Four of the category characteristics are relevant for the long-term regular price elasticity.

Utilitarian products tend to have smaller long-term regular price elasticities. Previously, we found a large immediate effect for these products. Although the immediate effect may be large, the long-term effect of decreases in regular price is much smaller. In categories that have a high price dispersion or have relatively frequent price promotions, the long-term effects of regular price tend to be stronger. The sign of the impact of price dispersion on the long-term effect is opposite to that on the immediate effect. Even with separate product segments, (permanent) switching between brands will occur in the long run. In this case, differences in regular price may reflect differences in product quality. An increase in regular price may be a signal for a higher-quality brand; as such, in the long run, a regular price increase may lead to higher sales. For the long-term regular price effect, we find that perishability is important. Contrary to the immediate effect, we find that in the long run, perishable products have smaller regular price effects.

**CONCLUSIONS**

In this article, we proposed an HB-ECM to explain the differences in immediate and dynamic effects of price on sales. We explicitly distinguished between the effects of temporary price promotions and permanent changes in regular price. We applied the model to weekly sales for 100 different brands in 25 product categories. In the second layer of the model, we related the immediate and dynamic effects of price promotions and changes in regular price to brand-level and category-level characteristics. We obtained parameter estimates using MCMC.

The HB approach allows us to analyze the dynamic effects of price in a statistically coherent way. Our results show that price elasticities can be explained by several brand-specific and category-specific factors. We find that many of the results for the immediate effect of price promotions are in line with previous literature and that though the influence of these factors on cumulative effects is somewhat lower in most cases, it is statistically significant and has the same sign as the effect on immediate elasticities. The dynamic effects are smaller, but these do not cancel out from the relationship between price promotion elasticity and category and brand characteristics.

We find mostly significant moderating effects on the elasticity of price promotions. We also find significant effects for the cumulative elasticity. Brands in categories characterized by high price differentiation and that constitute a lower share of budget are less sensitive to price discounts. Deep price discounts in a category or for a brand increase customers’ immediate price sensitivity. Another interesting finding is that though the relative display frequency within a category influences the promotional price elasticity, the feature frequency matters only across categories. A possible explanation for this is that whereas the use of feature mainly generates a general need for the product category, display attracts the attention of the buyer when he or she is making the actual brand choice.

For changes in regular price, we find few relevant explanatory variables. Perhaps as might be expected, the immediate effect of a regular price change is often close to zero. The long-term effect of a regular price increase usu-
ally leads to an increase in sales. This is especially true for categories that are characterized by a large price dispersion and frequent price promotions and for hedonic, nonperishable categories.

Our study can be extended in several ways. First, moderating factors of cross-brand elasticities could be considered. This would be especially interesting with data on brands of the same manufacturer or data at the SKU level. Second, as we pointed out previously, we do not consider competitive reactions or feedback effects in our ECM model, which allows us to focus purely on the determinants of dynamic demand reactions. However, it might be interesting to observe how the results would change if a model that considers such relationships were built.

**APPENDIX A: BAYES ESTIMATION**

To analyze the HB-ECM, we consider the exact likelihood function. We put the first observations in each category equal to the long-term equilibrium; that is,

\[
\log_S \tau_{c1} = -\Pi c^{-1} \mu_c + \sum_{k=1}^K B_{ck} \log X_{ckt1} + \epsilon_{c1},
\]

with \( \epsilon_{ic} \sim N(0, V_c) \), where \( V_c \) is the long-term variance.\(^5\)

To derive the likelihood function, we summarize the elements of \( A_k \) and \( B_k \), which we relate to explanatory variables, in the \( K \)-dimensional row vectors \( \alpha_{ic} = [\alpha_{ic1}, \ldots, \alpha_{ick}]^T \) and \( \beta_c = [\beta_{c1}, \ldots, \beta_{ck}]^T \). We can rewrite Equations 11 and 12 in matrix notation as

\[
\alpha_{ic} = \Lambda_1 z_{ic} + \eta_{ic}, \quad \beta_c = \Lambda_2 z_{ic} + v_c,
\]

for \( i = 1, \ldots, I_c \), where the \( L \times K \) matrices \( \Lambda_1 \) and \( \Lambda_2 \) contain the vectors \( \lambda_{1k} \) and \( \lambda_{2k} \), respectively. The likelihood function of the model is given by

\[
\prod_{c=1}^{C_c} \prod_{i=1}^{I_c} \phi(\epsilon_{ic}; 0, V_c) \prod_{t=2}^{T_c} \phi(\epsilon_{ic}; 0, \Sigma_c)^{I_c} \phi(\alpha_{ic}; \Lambda_1' z_{ic}, \Sigma_c) \phi(\beta_c; \Lambda_2' z_{ic}, \Sigma_v) d\alpha_{ic} d\beta_c,
\]

where \( \phi(x; \mu, \Sigma) \) is the density function of the multivariate normal distribution with mean \( \mu \) and variance \( \Sigma \) evaluated at \( x \) and where \( \alpha_c = (\alpha_{ic1}, \ldots, \alpha_{ick})' \) and \( \beta_c = (\beta_{c1}, \ldots, \beta_{ck})' \).

To obtain posterior results, we use the Gibbs sampling technique (Geman and Geman 1984) with data augmentation (see Tanner and Wong 1987). An introduction to the Gibbs sampler can be found in the work of Casella and George (1992; see also Smith and Roberts 1993; Tierney 1994). Thus, the latent variables \( \alpha_c \) and \( \beta_c \) are sampled alongside the model parameters \( \{ \{ A_{ck} \}_{k=1}^K, \{ B_{ck} \}_{k=1}^K, \mu_c, \Pi_c, \Sigma_c \}_{c=1}^{C_c}, \Lambda_1, \Lambda_2, \Sigma_{\eta}, \) and \( \Sigma_v \). The Bayesian analysis is based on uninformative priors for the model parameters. To improve convergence of the MCMC sampler, we impose inverted Wishart priors on the \( \Sigma_\eta \) and \( \Sigma_v \) parameter with scale parameter \( \kappa_1 I_K \) and degrees of freedom \( \kappa_2 \). We set the value of \( \kappa_1 \) to 1/1000 and \( \kappa_2 \) to 1 such that the influence of the prior on the posterior distribution is marginal (for a discussion, see Hobert and Casella 1996).

In the remainder of Appendix A, we derive the full conditional posterior distributions of the model parameters and the latent variables \( \alpha_c \) and \( \beta_c \). In deriving the sampling distributions, we build on Zellner’s (1971, Ch. 8) results.

### Sampling of \( \Pi_c \)

The full conditional posterior distribution of \( \Pi_c \) is not of a known family. To sample \( \Pi_c \), we use the Metropolis–Hastings (M–H) sampler of Metropolis and colleagues (1953) and Hastings (1970). To obtain a candidate sampling distribution, we use the fact that Equation 10 is just a multivariate regression model. We rewrite Equation 10 as

\[
\Delta \log S_{ct} - \mu_c - \sum_{k=1}^K A_{ck} \Delta \log X_{ckt} = \Pi_c \left( \log S_{ct-1} - \sum_{k=1}^K B_{ck} \Delta \log X_{ckt-1} \right) + \epsilon_{ct},
\]

This equation is a multivariate regression model with a normal distributed error term and regression parameter matrix \( \Pi_c \). Thus, if we neglect the model for the initial observation (given in Equation A1), the full conditional posterior distribution of \( \hat{\Pi}_c \) will be matrix normal with mean

\[
\hat{\Pi}_c = \left( \sum_{t=2}^{T_c} W_{ct} W_{ct}' \right)^{-1} \sum_{t=2}^{T_c} W_{ct} Y_{ct}',
\]

and variance

\[
\hat{\Sigma}_{\Pi_c} = \Sigma_{\Pi_c} \left( \sum_{t=2}^{T_c} W_{ct} W_{ct}' \right)^{-1},
\]

with \( Y_{ct} = \Delta \log S_{ct} - \mu_c - \sum_{k=1}^K A_{ck} \Delta \log X_{ckt} \) and \( \epsilon_{ct} = \log S_{ct-1} - \sum_{k=1}^K B_{ck} \Delta \log X_{ckt-1} \). We use this distribution as the candidate for the M–H sampler. We denote the sampled candidate by \( \Pi_c^{\text{cand}} \).

Because we cannot neglect the model for the first observation, the true full conditional posterior density of \( \Pi_c \) is proportional to the matrix normal candidate density and the density of the first observation (see Equation A1). This allows us to construct a particular form of the M–H sampler, which is known as the independent sampler. Because the candidate density is part of the target density (full conditional posterior density), the acceptance–rejection probability simplifies to

\[
\frac{\phi(\epsilon_{ic1}; 0, V_c)|_{\Pi_c = \Pi_c^{\text{cand}}}}{\phi(\epsilon_{ic1}; 0, V_c)|_{\Pi_c = \Pi_c^{\text{old}}}} \frac{\hat{\Sigma}_{\Pi_c}^{\text{cand}} \phi(\hat{\Pi}_c^{\text{cand}}; \hat{\Sigma}_{\Pi_c}^{\text{cand}}; \hat{\Sigma}_{\xi_c}^{\text{cand}})}{\phi(\epsilon_{ic1}; 0, V_c)|_{\Pi_c = \Pi_c^{\text{old}}}} \frac{\phi(\hat{\Pi}_c; \hat{\Sigma}_{\Pi_c}^{\text{old}}; \hat{\Sigma}_{\xi_c}^{\text{old}})}{\phi(\epsilon_{ic1}; 0, V_c)|_{\Pi_c = \Pi_c^{\text{old}}}},
\]

\(^5\)The long-term variance follows from Equation 3 and is given by \( \Sigma_c = \Sigma_c - \Omega^2 \Sigma_r \).
where $\Pi_{\text{old}}$ denotes the previous draw and $\varepsilon_{cl} = \log\Sigma_{cl} + \Pi_{\text{old}}^{-1} \mu_c - \sum_{k=1}^{K} B_{ck} \log X_{ckl}$ (for a similar approach in an exact likelihood analysis of an autoregressive model, see Chib and Greenberg 1995).

**Sampling of $\Sigma_c$**

The full conditional posterior distribution of $\Sigma_c$ is not of a known family. To sample $\Sigma_c$, again, we use an M–H sampler. To obtain a candidate sampling distribution, again, we use the fact that Equation 10 is just a multivariate regression model. Thus, if we neglect the model for the first observation, the true full conditional posterior distribution of $\Sigma_c$ is an inverted Wishart distribution with scale parameter $\sum_{k=1}^{K} B_{ck} \log X_{ckl}$ and $T_c - 1$ degrees of freedom, where $\varepsilon_{ct} = \log\Sigma_{ct} - \sum_{k=1}^{K} A_{ik} \log X_{ckl} - \Pi_c (\log S_{ct} - 1 - \sum_{k=1}^{K} B_{ck} \log X_{ckl})$. This will be the candidate distribution of the M–H step, which provides us with $\Sigma_{c\text{and}}$.

Because we cannot neglect the first observation, the true full conditional posterior density of $\Sigma_c$ is proportional to the inverted Wishart candidate density and the density of the first observation. Again, this leads to the independent sampler variant of the M–H sampler. Because the candidate density is part of the target density (full conditional posterior density), the acceptance–rejection probability simplifies to

$$
\phi(\varepsilon_{ic}; 0, V_c) | \Sigma = \Sigma_{c\text{and}}
\phi(\varepsilon_{ic}; 0, V_c) | \Sigma = \Sigma_{c\text{old}}.
$$

where $\Sigma_{c\text{old}}$ denotes the previous draw of $\Sigma_c$.

**Sampling of $\Lambda_1$ and $\Lambda_2$**

To sample $\Lambda_1$, we can write Equation A2 as

$$
\Lambda'_{ic} = \Lambda_{ic} + \Lambda'_{ic},
$$

and thus it is a multivariate regression model with regression matrix $A_1$. Therefore, the full conditional posterior distribution of $\Lambda_1$ is a matrix normal distribution with mean

$$
\Lambda_{ic} \sim \mathcal{N}\left(\Pi_{\text{old}}^{-1} \mu_c - \sum_{k=1}^{K} B_{ck} \log X_{ckl}, \sum_{c=1}^{C} \sum_{i=1}^{T_c} \sum_{k=1}^{K} B_{ck} \log X_{ckl} \right)
$$

and covariance matrix

$$
\Sigma_{\Lambda_1} = \sum_{c=1}^{C} \sum_{i=1}^{T_c} \sum_{k=1}^{K} B_{ck} \log X_{ckl}^{-1}
$$

The derivation of the sampling distribution of $\Lambda_2$ proceeds in the same manner. The full conditional posterior distribution of $\Lambda_2$ is a matrix normal distribution with mean

$$
\Lambda_{ic} \sim \mathcal{N}\left(\Pi_{\text{old}}^{-1} \mu_c - \sum_{k=1}^{K} B_{ck} \log X_{ckl}, \sum_{c=1}^{C} \sum_{i=1}^{T_c} \sum_{k=1}^{K} B_{ck} \log X_{ckl}^{-1}
$$

and covariance matrix

$$
\Sigma_{\Lambda_2} = \sum_{c=1}^{C} \sum_{i=1}^{T_c} \sum_{k=1}^{K} B_{ck} \log X_{ckl}^{-1}
$$

**Sampling of $\Sigma_\eta$ and $\Sigma_\nu$**

To sample $\Sigma_\eta$, note that Equation A2 is a multivariate regression model. Thus, the full conditional posterior distribution of $\Sigma_\eta$ is an inverted Wishart distribution with scale parameter $\Sigma_\eta^{-1} = \sum_{c=1}^{C} \sum_{k=1}^{K} B_{ck} \log X_{ckl} - \Pi_c (\log S_{ct} - 1 - \sum_{k=1}^{K} B_{ck} \log X_{ckl})$ and degrees of freedom $\kappa_2 + \sum_{c=1}^{C} \sum_{k=1}^{K} B_{ck} \log X_{ckl}$. The $\kappa$ terms result from the inverted Wishart prior on $\Sigma_\eta$, which is used to improve convergence of our Gibbs sampler (for a discussion, see Hobert and Casella 1996). We can do the sampling of $\Sigma_\nu$ in exactly the same manner. The parameter $\Sigma_\nu$ is sampled from an inverted Wishart distribution with scale parameter $\kappa_2 I_c + \sum_{c=1}^{C} \sum_{k=1}^{K} (B_{ic} - \Lambda_{ic}^2 z_{ic})(B_{ic} - \Lambda_{ic}^2 z_{ic})'$ and degrees of freedom $\kappa_2 + \sum_{c=1}^{C} \sum_{k=1}^{K} I_c$.

**Sampling of $\mu_c$ and Cross-Effects in $A_{ck}$ and $B_{ck}$**

To sample $\mu_c$ and the parameters measuring the cross-effect in $A_{ck}$ and $B_{ck}$, we first split up $X_{ckl} = (X_{ck1l}, ..., X_{ckKl})'$ for $k = 1, ..., K$ into two parts—$X_{ckl}^{\text{own}} = [X_{ck1l}, ..., X_{ckkl}]'$, and $X_{ckl}^{\text{cross}} = [[X_{ck1l}]_{j=1}^{k-1} (I_c \otimes \Pi_c)]_{j=k+1}^{K}$. This will be the candidate distribution of the M–H step, which provides us with $\Lambda_{c\text{and}}$. We define $X_{ct}^{\text{own}} = \text{diag}(X_{ct1}^{\text{own}}, ..., X_{ctKL}^{\text{own}})'$ and $X_{ct}^{\text{cross}} = \text{diag}(X_{ct1}^{\text{cross}}, ..., X_{ctKL}^{\text{cross}})'$. We can now rewrite Equation A1 and Equation 10 as

$$
\log S_{cl} - \log X_{ctl}^{\text{own}} b_{cl} = -\Pi_c^{-1} \mu_c + \log X_{ctl}^{\text{cross}} b_{cl} + \varepsilon_{cl}
$$

$$
\log S_{ct} - \log X_{ct}^{\text{own}} a_{ct} - \Pi_c (\log S_{ct} - 1 - \log X_{ct}^{\text{own}} b_{ct}) = \mu_c + \log X_{ct}^{\text{cross}} a_{ct} - \Pi_c \log X_{ctl}^{\text{cross}} b_{ct} + \varepsilon_{ct}
$$

where $a_c$ and $b_c$ capture the cross-effects in the matrices $A_{ck}$ and $B_{ck}$ for $k = 1, ..., K$. We can rewrite this system in a multivariate regression model as

$$
Y_{ct} = W_{ct} + \varepsilon_{ct},
$$

where $Y_{ct}$ contains the left-hand side of Equation A15, $W_{ct}$ contains $(-\Pi_c^{-1} 0 1: \log X_{ctl}^{\text{cross}})$ for the first observation and $(I_c: \log X_{ctl}^{\text{cross}}: -\Pi_c, \log X_{ctl}^{\text{cross}})$ for the remaining observations, and $\gamma = (\mu_c, a_{ct}', b_{ct}')'$. The error term is normal distributed with mean 0 and covariance matrix $\Sigma_c (V_c$ for the first observation). Thus, the full conditional distribution of $\gamma$ is normal with mean

$$
\Bigg[ \sum_{i=1}^{T_c} W_{ci} V_{ci}^{-1} W_{ci} + \sum_{i=1}^{T_c} W_{ci} \Sigma_{ci}^{-1} W_{ci} \Bigg]^{-1}
$$

$$
\sum_{i=1}^{T_c} W_{ci} V_{ci}^{-1} Y_{ci} + \sum_{i=1}^{T_c} W_{ci} \Sigma_{ci}^{-1} Y_{ci}
$$

and covariance matrix

$$
\sum_{i=1}^{T_c} W_{ci} V_{ci}^{-1} W_{ci} + \sum_{i=1}^{T_c} W_{ci} \Sigma_{ci}^{-1} W_{ci}
$$

and covariance matrix

$$
\sum_{i=1}^{T_c} W_{ci} V_{ci}^{-1} Y_{ci} + \sum_{i=1}^{T_c} W_{ci} \Sigma_{ci}^{-1} Y_{ci}
$$
Sampling of \( \alpha_c \)

To sample \( \alpha_c \), we rewrite the second equation in Equation A15 as

\[
\Delta \log S_{ct} - \mu_c - \Delta \log X_{ct}^{\text{cross}} = -\Pi_c (\log S_{ct} - 1) - \sum_{k=1}^K \beta_{ck} \log X_{ct}^{(k-1)}
\]

which can be written in matrix notation as

\[
Y_{ct} = W_{ct} \alpha_c + \epsilon_{ct}
\]

where \( Y_{ct} = \Delta \log S_{ct} - \mu_c - \Delta \log X_{ct}^{\text{cross}} \), \( \alpha_c \) is a \((K_\text{Ic})\)-dimensional vector containing the terms \( \alpha_{c1}, \ldots, \alpha_{cK_\text{Ic}} \). To sample \( \alpha_c \), we combine Equations A20 and A21:

\[
-U_c = -I_{K_\text{Ic}} \alpha_c + \eta_c,
\]

where \( U_c \) is a \((K_\text{Ic})\)-dimensional vector containing the terms \( \Delta \log S_{ct}, \mu_c, \Delta \log X_{ct}^{\text{cross}} \), \( \alpha_c \) is a \((K_\text{Ic})\)-dimensional vector containing the terms \( \alpha_{c1}, \ldots, \alpha_{cK_\text{Ic}} \). The error term \( \eta_c \) is normal distributed with mean 0 and covariance matrix \((I_{K_\text{Ic}})\). To sample \( \alpha_c \), we use the following notation:

- \( \Delta \log S_{ct} \) is the change in sales in category \( c \) at time \( t \),
- \( \mu_c \) is the mean of \( \Delta \log S_{ct} \)
- \( \Delta \log X_{ct}^{\text{cross}} \) is the change in cross-category advertising
- \( \alpha_c \) is a \((K_\text{Ic})\)-dimensional vector containing the terms \( \alpha_{c1}, \ldots, \alpha_{cK_\text{Ic}} \)
- \( \eta_c \) is the error term

Thus, the full conditional posterior distribution of \( \alpha_c \) is normal with mean

\[
\alpha_c \sim \mathcal{N}(\Pi_c \log S_{ct} - 1, \Sigma_\alpha)
\]

and covariance matrix

\[
\Sigma_\alpha = \begin{bmatrix} (I_{K_\text{Ic}}) \end{bmatrix}^{-1}
\]

Sampling of \( \beta_c \)

To sample \( \beta_c \), we rewrite Equation A15 as

\[
\log S_{ct} - \log X_{ct}^{\text{cross}} - \Pi_c \mu_c = \log X_{ct}^{\text{own}} + \epsilon_{ct}
\]

which we can write in matrix notation as

\[
V_c^{-1/2} Y_{ct} = V_c^{-1/2} W_{ct} \beta_c + V_c^{-1/2} \epsilon_{ct}
\]

for \( t = 1, \ldots, T_c \), where \( Y_{ct} \) denotes the left-hand side of Equation A25 and \( W_{ct} \) denotes the right-hand side. Again, we write the \( I_c \) equations of Equation A3 as

\[
-(I_c \otimes \Sigma_{\nu}^{-1/2}) U_c = -(I_c \otimes \Sigma_{\nu}^{-1/2}) \beta_c + (I_c \otimes \Sigma_{\nu}^{-1/2}) \nu_c
\]

where \( U_c \) is a \((K_\text{Ic})\)-dimensional vector containing the terms \( \Delta \log S_{ct}, \mu_c, \Delta \log X_{ct}^{\text{cross}} \), \( \alpha_c \) is a \((K_\text{Ic})\)-dimensional vector containing the terms \( \alpha_{c1}, \ldots, \alpha_{cK_\text{Ic}} \). The distribution of the error term \( \nu_c \) is normal with mean 0 and covariance matrix \((I_{K_\text{Ic}})\). If we combine Equation A26 and Equation A27, we can observe that the full conditional posterior distribution of \( \beta_c \) is normal with mean

\[
\beta_c \sim \mathcal{N}(\Pi_c \log S_{ct} - 1, \Sigma_{\beta_c})
\]

and covariance matrix

\[
\Sigma_{\beta_c} = \begin{bmatrix} (I_{K_\text{Ic}}) \end{bmatrix}^{-1}
\]

APPENDIX A: THE DEFINITION OF EXPLANATORY VARIABLES (Z_{IC})

In Appendix B, we list the category and brand characteristics that we used in our empirical section to explain the dynamic effects of promotions. We give a description of each variable and, if necessary, a formal, mathematical definition. We organize the characteristics on the basis of the level at which they are defined (category level, brand level, or both) and the concept they measure (e.g., competitive intensity). We use the following notation:

- \( \text{S}_{ict} \) is the sales volume of brand \( i \) in category \( c \) at time \( t \),
- \( \text{M}_{ict} = \frac{1}{T_c} \sum_{t=1}^{T_c} \text{S}_{ict} \) is the market share of brand \( i \) in category \( c \) at time \( t \),
- \( \bar{\text{M}}_{ic} = \frac{1}{T} \sum_{t=1}^{T_c} \text{M}_{ict} \) is the (average) market share,
- \( \text{P}_{ipt} \) is the (actual) price of brand \( i \) in category \( c \) at time \( t \),
- \( \text{RP}_{ict} = \frac{\text{P}_{ipt}}{\text{P}_{it}} \) is the regular price of brand \( i \) in category \( c \) at time \( t \),
- \( \text{P}_{ict} = \frac{\text{P}_{ipt}}{\text{P}_{it}} \) is the (average) price in category \( c \) and
- \( \text{P}_{ict} \) is the (promotional) price index.

Average Budget Share

To measure the budget share of a category, we use the average total expenditures in the category over time; that is,

\[
\text{Average Budget Share} = \frac{1}{T_c} \sum_{t=1}^{T_c} \text{S}_{ict} \cdot \text{P}_{ict}
\]

Utilitarian

To characterize the type of a product category, we measure the utilitarian nature of the category. We operationalize
this characteristic with three levels (low, middle, and high), and we obtain the measures through the use of several experts. We coded the three levels with the numbers 0, .5, and 1. The team of experts consisted of a food-marketing expert and two marketing research experts. They had no relationship to this research project. The product categories involved in this study were easy to classify, and there was little disagreement among the experts. When there was a disagreement, we voted on the final classification. (Further details on the classification can be obtained on request.)

Perishability

An important component of the storability of a category is the perishability. Again, we define this characteristic through the opinions of several experts. This variable has three levels (low, middle, and high).

Market Concentration

We measure the market concentration in category c using 

\[ \sum_{i=1}^{1} \bar{M}_{ic} \log \bar{M}_{ic} \]  

(Raju 1992).

Price Dispersion

We can obtain the price dispersion in a category by comparing the highest regular price in a category with the lowest regular price; for category c,

\[ \sum_{t=1}^{T_c} \left[ \max_{i} (RP_{ict}) - \min_{i} (RP_{ict}) \right] / (T_c \times \bar{RP}_c). \]  

Price Promotion Frequency

We measure the price promotion frequency of a brand by counting the number of times that the price index within the category is below .95 (i.e., when there is at least a 5% discount). (See, for example, the work of Mulherin, Williams, and Leone [1998], who use a similar measure.) We obtained the price promotion frequency of the category by counting the number of times that at least one brand’s price index is below .95 (see Raju 1992). As explanatory variables, we use the frequency of the category and the relative brand-level frequency (i.e., brand frequency/category frequency).

Depth of Price Promotions

For a brand, we define the average depth of a promotion as

\[ \sum_{t=1}^{T_c} \log(P_{ict}) / \text{FREQ}_{ic}. \]  

where FREQ_{ic} denotes the price promotion frequency of brand i in category c. At the category level, we use the mean of the average brand-level depth of promotion. Raju (1992) uses similar measures; however, he uses the difference between the regular and the actual price rather than a price index. Again, we include the category-specific measure and a relative measure of depth of price promotion as explanatory variables.

Feature/Display Frequency

We can obtain the frequency of display or feature of brand i by taking the average of the percentage of SKUs promoted by the brand over time. Denoting the percentage of SKUs promoted by brand i in category c at time t as x_{ict}, we define the category-level frequency of promotion in category c by

\[ \sum_{t=1}^{T_c} \frac{1}{\prod_{t=1}^{T_c} (1 - x_{ict})}. \]  

The term (1 - x_{ict}) can be interpreted as the probability that an SKU of brand i is not on promotion in week t. In line with this reasoning (and if we assume independence in the timing of promotions across brands), the probability that no SKU is on promotion is \( \prod_{c=1}^{T_c} (1 - x_{ict}) \). Therefore, our measure for the frequency of promotion can be viewed as the average probability that at least one SKU is on promotion.

Brand Size

To measure the relative size of a brand in the category, we use the average market share; we define market shares using the units sold.

Price Segment

As an indication of the price segment to which a brand belongs in a certain category, we construct an index based on the regular price:

\[ \sum_{t=1}^{T_c} \frac{RP_{ict}}{T_c \sum_{t=1}^{T_c} \frac{RP_{ict}}{T_c}}. \]  

A high value of the index corresponds to a relatively expensive brand within the category.

REFERENCES


Explaining Dynamic Effects of Price Changes


