INTEREST-GROUP POLITICS
UNDER MAJORITY RULE

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1. Introduction

Models of political processes may be broadly classified as either majority-rule voting models or pressure-group models. The former represent some of the earliest work done in the formal analysis of democratic procedures and by now they constitute a large literature.\footnote{1} Pressure-group models of policy determination are a comparatively recent development, but they are the basis for a large proportion of empirical studies in public choice.\footnote{2} In their current states of development, however, neither of these approaches offers a fully satisfactory analysis. Majority-rule models appear to predict substantially less stability of political outcomes than is commonly observed. Moreover, the median-voter result that is predicted in those cases in which a stable equilibrium exists fails to account for some important aspects of public policy such as the use of the regulatory powers of the government to benefit what appear to be well organized and occasionally quite small associations of individuals. On the other hand, the pressure-group approach does not explain why the types of equilibria it predicts are expected to be stable or, perhaps even more tellingly, why it is that special-interest legislation usually entails indirect methods of wealth transfer through market intervention instead of lower-cost direct methods such as cash grants.\footnote{3}

The variety and importance of market interventions is impressive. Tariffs, quotas, agricultural price supports, occupational licensing, and other forms of entry restrictions all achieve income redistribution by means which cause prices to deviate from social marginal costs. The net deadweight cost of all such interventions is not known, of course, presumably because of the inherent difficulties of performing a general equilibrium calculation of their effects. However, the numerous
partial equilibrium studies that have been done suggest that these costs are not insignificant.  

Becker (1983) has argued that the deadweight costs inherent in market interventions play an important role in limiting the size of transfers and in determining who receives them, but he obtains that result by assuming that the political process is necessarily characterized by collective lobbying efforts by members of well-defined groups seeking costly redistributive policies. Left unspecified is why politicians maximizing the support they receive from rational citizens don’t replace these costly policies with more direct methods of transferring resources.

In this paper we demonstrate the theoretical link between majority-rule voting processes and interest-group politics. Unlike previous efforts (McCormick and Tollison, 1981, Denzau and Munger, 1984), we do not take the existence of pressure groups or the nature of the policies they seek as given. Instead, we begin with a simple model of a majority-rule voting game with costly coalition formation and show that special-interest proposals of the type commonly observed are likely to satisfy a necessary condition for the set of majority-rule outcomes to include transfers of resources among individuals of equal initial wealth in a way that more direct transfers cannot.

We are not the first to offer an explanation for the indirect nature of most redistributive government policies. Tullock (1967) has argued that such policies are less visible to the general electorate, thereby allowing politicians to confer benefits to an informed minority at the expense of the uninformed majority. Given the limited incentives for voters to acquire information about the effects of such policies, Tullock’s
hypothesis may well explain some of them. Nevertheless, many costly programs are both fairly well publicized and enduring.

Rubin (1975) postulates that forms of redistribution involving higher deadweight costs tend to be longer-lived than more direct payment schemes, so that politicians facing uncertain tenure in office can receive greater benefits by offering long-lived programs to interest groups. Underlying this result is the assumption that the periodic value of the long-lived transfers does not fall to or below the certainty equivalent of a stream of cash payments received only when a particular party is in power. Thus, Rubin shows that if costly redistribution yields greater benefits to a group than lump-sum transfers, it will be preferred by that group, but he does not show the circumstances under which such a case would arise. The natural presumption is that in the typical case the opposite is true precisely because of deadweight costs.

With the exception of Klingaman (1969), those who have considered the problem have not included the bargaining costs of forming a winning coalition as part of their analysis. We find that when the problems inherent in coalition formation are treated explicitly, they have important implications for the kinds of coalitions that will succeed and the types of redistributive policies that will be enacted. Our results suggest that in a democracy:

(1) Redistribution intended to reduce income inequality is more likely to be carried out through direct means than is nonequalizing redistribution.

(2) Nonequalizing transfers are unlikely to be enacted unless there are particular types of asymmetries among participants in the political process.
(3) Transfers that generate deadweight costs often induce asymmetries among participants. Such transfers will tend therefore to be relatively more successful policies for nonequalizing redistribution than will more direct means of reallocating income.

In the subsequent sections we discuss each of these propositions.

2. A Simple Model of Redistributive Politics

A useful and by now classic paradigm for modelling redistributinal politics in a democracy is the majority-rule "divide-the-dollar" game. In general, the core of this game, and of similar majority-voting games, is empty. Klingaman (1969), Weingast (1979), Niou and Ordeshook (1985), and others have argued that in this case we might expect to see the voters strike some kind of "universalistic" bargain. The argument is as follows. In the absence of a core, the outcome of the majority-rule game is uncertain, as \textit{ex ante} there are many majority coalitions that might form. Given some reasonable assumptions about the probabilities of the various possible outcomes of the game, each player can calculate his \textit{ex ante} expected payoff to playing the game. An alternative arrangement, which may or may not be available depending on the ability of players to make binding commitments, is for everyone to agree not to play the game and to accept instead some particular division of the dollar (which they would receive for certain). Presumably each player would prefer any division of the dollar which gives him more than his expected payoff under the majority-rule game. A division of the dollar which gives every player more than his expected payoff under the majority-rule game might then be a "stable" outcome; no one would be willing to switch to the majority-rule
game but would prefer to accept the given division. We call such outcomes
unanimous agreements.

We interpret a unanimous agreement as a constitutional provision
in the broad sense. It may involve an explicit set of "rules of the game",
as in the case of a written constitution, or it may refer to the widespread
acceptance of certain redistributive norms. Since we assume that all
citizens have full information, the agreement upon an explicit or implicit
constitution derives solely from agreement upon the distribution of wealth
that is known to result from that constitution.

In order for unanimous agreements to exist, there must be some
costs associated with playing the majority-rule divide-the-dollar game. In
Klingaman, these costs are the result of risk aversion. If players are risk
averse, they will strictly prefer to receive a known payoff x over a lottery
whose expected payoff is x. Since the majority-rule divide-the-dollar game
has no core, players probably cannot anticipate the outcome beforehand and
thus view the game as a lottery (this might be true even if the game has a
core). Suppose players believe that in playing the game: (i) only minimal
winning coalitions (MWCs) will form; (ii) all MWCs are equally likely; and
(iii) the MWC that does form will divide the dollar equally among all of its
members. Then, for example, if there are three players, for each player the
game represents a lottery that pays 1/2 with probability 2/3 and 0 with
probability 1/3. Since the expected payoff of this lottery is 1/3, each
player would prefer to receive 1/3 with certainty than to play the game.
Thus, the outcome (1/3, 1/3, 1/3) is a unanimous agreement.5

As a simple framework for studying the problem, consider the
following generalization of the majority-rule divide-the-dollar game, which
we call a costly redistributive voting game (CRVG). There is a set N -
of players, with \( n \) odd. Each player \( i \) is endowed with \( e_i \) dollars; normalize so that \( \sum_{i \in N} e_i = 1 \). Any coalition \( C \) of more than half of the players (i.e., any subset \( C \) of \( N \) with \( \#C > n/2 \), where \( \# \) denotes cardinality) is "winning". Each winning coalition \( C \) has the power to tax all players outside \( C \), and transfer the revenue to the members of \( C \). These taxes and transfers will not be costless to enact, however. There may be substantial costs in organizing the coalition, bargaining to reach an agreement among the coalition's members about what taxes and transfers to enact, and effectively communicating the desired outcome to policymakers (and, if necessary, persuading policymakers that in fact the outcome is desirable).

Perhaps more importantly, there may be significant deadweight costs in transferring the resources. Thus, associated with each winning coalition \( C \) is a cost \( g(C) \). Let \( W \) denote the set of winning coalitions. The characteristic function of a CRVG is then

\[
v(C) = \begin{cases} 
1 - g(C), & C \in W \\ 
0, & C \notin W 
\end{cases}
\]

Note that \( v \) may or may not be monotonic -- i.e., \( C \subseteq D \) may or may not imply that \( v(C) \leq v(D) \) -- depending on the shape of \( g \). For example, if \( g \) is increasing in coalition size (i.e., \( \#C \leq \#D \) implies that \( g(C) \leq g(D) \)), which is reasonable if \( g \) represents organizing and bargaining costs, then \( v \) will not be monotonic. Also, since monotonicity is necessary for superadditivity, \( v \) may or may not be superadditive (i.e., \( C \cap D = \emptyset \) may or may not imply that \( v(C)+v(D) \leq v(C \cup D) \)). Note also that we have assumed transferable utility. One might prefer to define the game with nontransferable utility, especially if deadweight costs are important.
(deadweight costs might easily produce nonlinear "payoff possibility frontiers" for some coalitions), but as this is not essential for the purposes of the present paper, we focus on the simpler case of transferable utility.

3. Equalizing Redistribution

When the citizens of a democracy start with unequal endowments, redistributive policies are likely outcomes of simple majority rule. The equal distribution of votes leads to a more equal distribution of consumption. We demonstrate this point for the case of an economy in which output is perfectly inelastically supplied by agents with differing endowments. Extension to the case in which factor suppliers also value untaxable leisure would modify our conclusion without altering the thrust of the argument.

Consider the following simple example with three players: endowments are $e_1 = 1/2$, $e_2 = e_3 = 1/4$, and costs are $c > 0$ for all winning coalitions. Then $v(i,j) = v(N) = 1-c$, for all $i \neq j$. Assume that players' expectations about the outcome of the voting game are as in (i)-(iii) above. Then the expected payoff to each player is $(1-c)/3$. As long as $c < 1/4$, the no-transfer allocation $(1/2, 1/4, 1/4)$ is not a unanimous agreement. The equal distribution of votes means that unless coalition costs are very high only rather egalitarian outcomes can be unanimous agreements; thus if endowments are unequal this means that all unanimous agreements may involve some income transfers.\(^6\)

Surely some of the redistribution that actually occurs in modern democracies is designed to help equalize endowments -- welfare programs and progressive taxes seem to be examples of this. For many redistributive
policies, however, this does not appear to be the case; at least, it is not obvious how such policies help to equalize incomes. Many taxes are arguably regressive (e.g., sales taxes and commodity taxes), agricultural subsidies often tend to go disproportionately to higher-income farmers, tariffs and import restrictions probably involve largely horizontal or upward transfers from the general public to the owners of industry-specific capital and skills, and occupational licensing may confer monopoly rents to higher-income professionals, to give but a few examples. It is doubtful that we can explain such transfers as efforts to equalize incomes. Rather, it would seem that such policies are designed to transfer income to (or from) particular groups irrespective of initial endowments.

Another way in which groups could be unequal is that they are of different sizes, and therefore have different endowments of votes. If we were to begin our analysis with this assumption, then redistribution toward groups with more votes would follow in a manner analogous to our result for unequal endowments of income. We wish to base our analysis at the level of the individual, however, and treat the formation of interest groups as endogenous. Accordingly, from now on we consider only voting games in which the players' endowments are equal. In this case, if dividing the dollar represents redistributing income, then the no-transfer allocation is (1/n, ..., 1/n) -- an equal division of the dollar means that no transfers are made, since each player returns home with his endowment.

4. Nonequalizing Redistribution

In Klingaman's model, the allocation in which each player gets an equal share of the dollar is a unanimous agreement. In Weingast's model, each player has a proposed "project," which gives benefits only to himself
and is to be funded by a uniform tax on all players. Each project requires the approval of a majority of the players to be funded. While this game is not exactly divide-the-dollar, we can reformulate it in these terms if we add the restriction that every coalition must divide its total income equally among its members. In this reformulation, the costs involved in the game are essentially that each majority coalition, except the coalition of the whole, must throw away some fraction of the dollar before paying its members, this fraction being inversely related to coalition size. Thus, neither of these models can be used to account for transfers if endowments are equal. The reason is that in both cases players are identical \textit{ex ante}.

We wish to characterize the conditions under which the no-transfer allocation is \textit{not} a unanimous agreement to a CRVG. This is not a necessary condition for the existence of a unanimous agreement that involves nonequalizing transfers, of course, since many possible agreements may exist. However, if it is significantly more costly to enforce a policy of redistribution than it is to enforce a no-transfers policy, then if the allocation \((1/n,\ldots,1/n)\) is in the set of unanimous agreements when endowments are equal, then it seems plausible that the no-transfers UA is the likeliest of all possible unanimous agreements. The lower are the enforcement costs of the no-transfers UA relative to those of the other possible UAs to any given CRVG, the more likely it is that the no-transfers UA will be chosen if it is in the set of possible unanimous agreements.

Of course, to determine the set of unanimous agreements, we must know the \textit{ex ante} expected payoffs to all players, and hence the likelihood of each of the different possible outcomes of the CRVG. Since there is no single best solution concept for cooperative games, we consider several
alternative assumptions about the way in which players calculate their expected payoffs in such games.

Our main finding in this section is that once we take into account the costs of bargaining and transferring resources, and the ability of players to reduce or possibly eliminate these costs if some unanimous agreement can be reached (especially a relatively low-cost no-transfers rule), majority-rule voting does not lead readily to nonequalizing income transfers. In particular, the underlying redistributive voting game cannot be too symmetric in order for us to be certain that transfers will occur. The necessary asymmetries are that some majority coalitions must suffer higher costs of obtaining transfers than other coalitions of the same size, and players must expect either that some coalitions are more likely to form or that there will be differential treatment of members in some coalitions.

4.1. CRVC's with Symmetric Costs

Consider the special case where the cost function treats players symmetrically. That is, suppose the cost of each coalition C depends only on the size of C, not on the identities of the players in C -- i.e., g(C) = g(C') for all C and C' such that |C| = |C'|. In such cases the characteristic function is also symmetric. It is reasonable in this case to assume that all players expect the same payoff ex ante, and, in fact, most of the commonly used cooperative solution concepts do treat all players the same ex ante. That is, if the solution yields a unique allocation (e.g., the Shapley value and the nucleolus) then this allocation gives all players the same payoff; or, if the solution yields a set of possible allocations (e.g., the bargaining set and the competitive solution) then if all points in the solution are equally likely, all players' expected payoffs are the same.
Also, as noted in the discussion of the games in Klingaman and in Weingast, the beliefs described by (i)-(iii) above yield equal expected payoffs.

It is clear that, since the sum of the players' expected payoffs is less than or equal to one, if all players receive the same expected payoff then this payoff is less than or equal to the endowment 1/n, and strictly less than the endowment if the game has significant costs (e.g., if all coalitions have positive cost). Thus, if coalition costs are symmetric, the no-transfer allocation is always a unanimous agreement.

Since we want to know when the no-transfer allocation is not a unanimous agreement, we now turn to CRVC's with asymmetric cost structures. We postpone until section 5 a discussion of the reasons asymmetric costs might arise.

4.2. Asymmetric CRVC's and "Symmetric" Expectations

When the game is not symmetric, the problem of making reasonable assumptions about players' expected payoffs is much more difficult. Game theory does not provide us with a solution, but rather with a menu of solution concepts to choose from, each with some appealing and some unappealing features.

One possibility is that players expect the distribution of outcomes of the game to satisfy conditions (i)-(iii) above. (Recall these assumptions: (i) only MWC's will form, (ii) all MWC's are equally likely, and (iii) the MWC that does form will divide the dollar equally among all of its members.) While these assumptions are probably more reasonable applied to symmetric games, they have also been used in nonsymmetric games. For example, Deegan and Packel (1978) use them to define a power index for the
class of simple games, which includes both symmetric and nonsymmetric games (e.g., weighted voting games with unequal weights may be nonsymmetric).

Somewhat surprisingly (at least at first glance), if players' expectations about the game are given by (i)-(iii) then the no-transfer allocation \((1/n,\ldots,1/n)\) of a CRVG is always a unanimous agreement, regardless of the shape of the coalition cost function. In fact, this is true under even more general assumptions about players' expectations, in which players may believe that winning coalitions other than minimal winning coalitions have a positive probability of forming, and in which the same winning coalition may divide its total payoff differently among its members at different times.

**Proposition 1.** Let \((N,v)\) be a CRVG and suppose that players' expectation are as follows: (i)' all coalitions of a given size have the same probability of forming (although this probability may vary with size), and (ii)' each winning coalition divides its total payoff equally among its members on average. Then \((1/n,\ldots,1/n)\) is a unanimous agreement.

**Proof.** Let \(S(k) = (C|\#C=k)\), let \(S_i(k) = (C|\#C=k, i\in C)\), and let \(p(k)\) be the probability that a coalition of size \(k\) forms. Then the expected payoff to each player \(i\) is

\[
E_i = \sum_{k=(n+1)/2}^{n} \frac{p(k)}{#S(k)} \sum_{C\in S_i(k)} \frac{[1-g(C)]}{k} = \sum_{k=(n+1)/2}^{n} \frac{p(k)}{#S(k)} \left[ \frac{#S_i(k)}{#S(k)} - \frac{1}{#S(k)} \sum_{C\in S_i(k)} g(C) \right].
\]

Now, \(\frac{#S_i(k)}{#S(k)} = \binom{n-1}{k-1}/\binom{n}{k}\), so

\[
k/n, \text{ so } E_i = \frac{1}{n} \sum_{k=(n+1)/2}^{n} p(k) - \sum_{k=(n+1)/2}^{n} \frac{p(k)}{k\#S(k)} \sum_{C\in S_i(k)} g(C).
\]

Evidently, \(\sum_{k=(n+1)/2}^{n} p(k) \leq 1\), and the second term is strictly positive unless
\[ \sum g(C) = 0 \text{ (or } p(k) = 0 \text{ for all } k = (n+1)/2, \ldots, n, \text{ in which case } E_i = 0), \text{ so } \]

\[ E_i \leq 1/n, \text{ and } E_i = 1/n \text{ only if } g(C) = 0 \text{ for all } C \text{ containing } i \text{ (i.e., only if } \text{the game imposes no costs on player } i). \text{ Q.E.D.} \]

The contrapositive of the proposition gives us conditions that are necessary for the no-transfer allocation not to be a unanimous agreement of a CRVG: either (1) some winning coalitions of the same size must have different probabilities of forming, or (2) some coalitions do not divide their total income equally among their members on average.

4.3. Asymmetric Expectations

a. The Core of CRVGs

The core, when it exists, is perhaps the most compelling solution concept for most cooperative games.\(^8\) Thus, if a CRVG has a core, it may be reasonable to assume that players always expect the outcome of the game to be in the core. Of course, if there are multiple core allocations, we must also make some assumption about players' expectations about the probability that each such allocation occurs, for example, that players expect all core allocations to be equally likely.

To define the core of a CRVG, we must modify slightly the usual definition. Normally, the core is defined on the set of imputations, that is, the set of payoff vectors \( x = (x_1, \ldots, x_n) \) that satisfy \( x_i \geq v((i)) \) for all \( i \), and \( \sum_{i \in N} x_i = v(N) \). The latter requirement makes sense for superadditive games, because no coalitions have greater value that the coalition of the whole. This is not true, however, for many interesting CRVG's (e.g., for CRVG's in which a coalition's costs increase with its
size). Thus, rather than restrict attention to imputations, we expand the set of allowable payoff vectors in a natural fashion. Say that coalition $C$ supports the payoff vector $\mathbf{x} = (x_1, \ldots, x_n)$ if and only if $\sum_{i \in N} x_i \leq v(C)$. A payoff vector $\mathbf{x}$ is allowable if and only if it there is some coalition $C$ that supports it. The core of a CRVG is then any allowable payoff vector $\mathbf{x}$ such that $\sum_{i \in C} x_i \geq v(C)$ for all $C \subseteq N$.

Some CRVG's have cores. For example, in a three-player game with $v((1,2)) = 1$, $v((1,3)) = 2/3$, $v((2,3)) = 1/3$, and $v(N) < 1$, $(2/3, 1/3, 0)$ is the unique core allocation. However, the core of a CRVG exists only under rather strong conditions on the cost function $g$. In particular, the game must entail substantial costs, and certain coalitions must have very high costs relative to others, as the following comment illustrates.

**Comment 1.** If $(N,v)$ is a CRVG with nonempty core, then for all $k = 1, \ldots, n$, the average value of all coalitions of size $k$ is less than or equal to $k/n$.

**Proof.** If $\mathbf{x} = (x_1, \ldots, x_n)$ is in the core, then $v(C) \leq \sum_{i \in C} x_i$ for all $C \subseteq W$. Thus, the average value of all coalitions of size $k$ is $V(k) = \frac{1}{\#S(k)} \sum_{C \subseteq S(k)} v(C)$, where $S(k) = \{C : |C| = k\}$. Now, letting $S_i(k) = \{C : |C| = k, i \in C\}$, we have $\sum_{C \subseteq S(k)} \sum_{i \in C} x_i = \sum_{i = 1}^{n} S_i(k) \sum_{i = 1}^{n} x_i$. Thus,

$$V(k) \leq \frac{\#S_i(k)}{\#S(k)} \sum_{i = 1}^{n} x_i.$$  

As noted in the proof of Proposition 1, $\frac{\#S_i(k)}{\#S(k)} = \frac{k}{n}$.

Thus, $V(k) \leq \frac{k}{n} \sum_{i = 1}^{n} x_i \leq \frac{k}{n}$ Q.E.D.

Thus, for example, the core of a CRVG exists only if the average value of all MWC's is less than or equal to $(n+1)/2n$. For games with many players,
this is approximately 1/2, meaning that the average cost of all MWC's is at least (approximately) 1/2. Comment 1 simply says that if the core of a CRVG exists, then for any size $k$, the average coalition of size $k$ cannot pay all of its members more than their endowments. Of course, this does not mean that there is no coalition of size $k$ that can pay all of its members more than their endowments.

It is interesting to note that core allocations do not necessarily give some MWC (i.e., some set of $(n+1)/2$ players) the "whole pie." That is, some core allocations give strictly positive payoffs to more than $(n+1)/2$ players, and other core allocations give zero payoffs to more than $(n-1)/2$ players. Consider the following two examples.

**Example 1.** Assume that $n=5$ and the characteristic function $v$ satisfies $v(N) \leq 1$, $v(\{1,2,3,4\}) = 1$, $v(C) = 2/3$ for all $C$ such that $#C=4$ and $C \neq \{1,2,3,4\}$, and $v(C) = 1/2$ for all $C$ such that $#C=3$. Then, for example, $(1/4,1/4,1/4,1/4,0)$ is a core allocation; moreover, no allocation that gives 2 or more players a payoff of 0 is in the core (such allocations are always dominated, via some three-player coalition).

**Example 2.** Assume that $n=3$ and $v$ satisfies $v(N) \leq 1$, $v(\{1,2\}) = v(\{1,3\}) = 1$, and $v(\{2,3\}) = 0$. Then $(1,0,0)$ is the unique core point. Here, due to the large costs of coalition $\{2,3\}$, player 1 essentially has a veto.

If the core of a CRVG is nonempty and players expect that if the CRVG is played then only core allocations will occur and all such core allocations are equally likely, then the no-transfer allocation is generally not a unanimous agreement. This is because either or both of the two
conditions described in section 4.2 may be satisfied. For example, in example 1 above, if \( v(N) < 1 \) then the only coalition that supports any of the core allocations is \( \{1,2,3,4\} \). Thus, \( \{1,2,3,4\} \) has a positive probability of forming, while all other coalitions of size 4 have no probability of forming, so condition (1) is satisfied. (Condition (2) is violated, however, since the average core payment to each member of \( \{1,2,3,4\} \) is \( \frac{1}{4} \) -- this is easily shown, using the assumption that all core allocations are equally likely). In example 2 above, both (1) and (2) are satisfied.

b. The Shapley Value

One way to generate the Shapley values of a game is to assume that the coalition of the whole always forms, but may form in any order, that all orderings of the players are equally likely, and that given an ordering each player receives his marginal contribution to the coalition consisting of all players that come before him in the ordering plus himself. Under this interpretation, condition (1) above is violated, since all coalitions of a given size are equally likely to form (for all coalitions of size \( k < N \) the probability of forming is zero), but condition (2) may be satisfied, since players' average marginal contributions to a coalition (where the average is taken over all possible orderings of the coalition's members) need not be equal. Thus, if players in a CRVG expect to receive their Shapley values, it is possible that the no-transfer allocation is not a unanimous agreement. As we now show, however, the no-transfer allocation is a unanimous agreement provided that a coalition's costs always increase as the coalition grows larger.
Proposition 2. Let \((N,v)\) be a CRVG with monotonically increasing costs -- i.e., \(g(C) \leq g(D)\) for all \(C\) and \(D\) such that \(C \subseteq D\). If each player's expected payoff is equal to his Shapley values, then \((1/n, \ldots, 1/n)\) is a unanimous agreement.

Proof. Let \(S_k(i) = \{C | C \setminus k, i \in C\}\), and let \(h(k) = 1/(n\cdot{k-1}) = 1/\#S_k(i)\). Then player \(i\)'s Shapley value in \((N,v)\) is

\[
\psi_i = \frac{1}{n} \sum_{k=1}^{n} h(k) \sum_{C \subseteq S_k(i)} [v(C) - v(C \setminus i)]
\]

\[
= \frac{1}{n} h\left(\frac{n+1}{2}\right) \sum_{C \subseteq S_k(i)}^{\frac{n+1}{2}} v(C) + \frac{1}{n} \sum_{k=(n+3)/2}^{n} h(k) \sum_{C \subseteq S_k(i)} [v(C) - v(C \setminus i)]
\]

\[
= \frac{1}{n} h\left(\frac{n+1}{2}\right) \sum_{C \subseteq S_k(i)}^{\frac{n+1}{2}} [1 - g(C)] + \frac{1}{n} \sum_{k=(n+3)/2}^{n} h(k) \sum_{C \subseteq S_k(i)} [g(C \setminus i) - g(C)].
\]

By monotonicity of costs, \(g(C \setminus i) - g(C) \leq 0\) for all \(C\), so

\[
\psi_i \leq \frac{1}{n} h\left(\frac{n+1}{2}\right) \sum_{C \subseteq S_k(i)}^{\frac{n+1}{2}} [1 - g(C)] = \frac{1}{n} \cdot \frac{1}{n} h\left(\frac{n+1}{2}\right) \sum_{C \subseteq S_k(i)}^{\frac{n+1}{2}} g(C) \leq \frac{1}{n}.
\]

Equality holds only if \(g(C) = 0\) for all \(C \in S_k(i) \frac{n+1}{2}\). Q.E.D.

If organizing and internal bargaining costs are important, we might reasonably expect costs to be monotonically increasing, as in the proposition. On the other hand, we should note that if costs are not monotonic (for example, if some players are especially good at mediating agreements), then some players' Shapley values may be higher than their endowment income, in which case the no-transfer allocation would not be a unanimous agreement.
c. The Bargaining Set and the Equal-Division Core

In contrast to Proposition 2 above, if players expect the bargaining set or equal-division core to determine the distribution of outcomes of the game, then the no-transfer allocation will not in general be a unanimous agreement, even if coalition costs are monotonic. This is best illustrated by a simple example.

**Example 3.** \( n = 3 \) and the characteristic function satisfies \( v(\{1,2\}) = .9, v(\{1,3\}) = .9, v(\{2,3\}) = .7, \) and \( v(\{1,2,3\}) = .5 \) (so coalition costs are monotonic, with \( g(\{1,2\}) - g(\{1,3\}) = .1, g(\{2,3\}) = .3, \) and \( g(\{1,2,3\}) = .5 \)). Here, \( \{1,2\} \) and \( \{1,3\} \) are in a sense "natural" coalitions, while \( \{2,3\} \) is not. (Also, note that the core is empty.)

The equal-division core and the bargaining set are both defined in terms of payoff configurations, that is, pairs \((S,\bar{x})\), where \( S \) is a coalition structure (i.e., a partition of \( N \)) and \( \bar{x} \) is an allocation that is feasible given \( S \) (i.e., \( \bar{x} \) satisfies \( \sum_{i \in C} x_i \leq v(C) \) for all \( C \in S \)). The equal-division core consists of all payoff configurations \((S,\bar{x})\) such that no coalition \( C \) can pay each of its members more than they receive under \( \bar{x} \), if it is forced to divide its total payoff equally among its members (i.e., for no coalition \( C \) is \( v(C)/\#C > x_i \) for all \( i \in C \); see Selten (1972) for a formal definition). Thus, in Example 3 above, the equal-division core consists only of payoff configurations of the form

\[
\{1,2\},\{3\} : (x_1,1-x_1,0), \text{ where } .45 < x_1 < .55
\]

\[
\{1,3\},\{2\} : (x_1,0,1-x_1), \text{ where } .45 < x_1 < .55
\]
No coalitions other than \((1,2)\) and \((1,3)\) can afford to pay their members enough to avoid being "invaded" by some other coalition. If players assume that all elements of the equal-division core are equally likely to occur, then their expected payoffs are \(u_1 = .5\) and \(u_2 = u_3 = .2\). Since \(.5 > 1/3\), player 1 prefers his expected payoff in the CRVG to his endowment, so the no-transfer allocation is not a unanimous agreement. Note that unanimous agreements of course exist, such as the allocation \((.54, .23, .23)\), but these agreements will always involve transfers from players 2 and 3 to player 1.

The bargaining set produces a similar outcome. The bargaining set consists of payoff configurations such that for any objection one player might have against another, the second player has a counterobjection. Thus, in Example 3, the bargaining set consists of the following payoff configurations:

\[
egin{align*}
(1,2),(3) & : (.55, .35, 0) \\
(1,3),(2) & : (.55, 0, .35) \\
(2,3),(1) & : (0, .35, .35) \\
(1,2,3) & : (.3, .1, 1) \\
(1),(2),(3) & : (0, 0, 0)
\end{align*}
\]

Note that for each coalition structure \(S\) there is a unique allocation \(x\) such that \((S,x)\) is in the bargaining set. If players assume that each of the first four coalition structures is equally likely, and the last structure \((1),(2),(3)\) (in which no winning coalition "forms") never occurs, then their expected payoffs in the CRVG are \(u_1 = .35\), and \(u_2 = u_3 = .2\). Again, since \(.35 > 1/3\), player 1 would prefer to play the CRVG rather that accept their endowments, so the no-transfer allocation is not a unanimous agreement.
Notice that in Example 3, only one of the three players receives more than his endowment under any unanimous agreement. This need not be the case, as the following example shows.

**Example 4.** \(n=3\) and the characteristic function satisfies \(v((1,2)) = .9, v((1,3)) = v((2,3)) = .7,\) and \(v((1,2,3)) = .5\) (so coalition costs are again monotonic, with \(g((1,2)) = .1, g((1,3)) = g((2,3)) = .3,\) and \(g((1,2,3)) = .5\)). Here, player 3 is "cursed."

In Example 4, the equal-division core consists of the following payoff configurations:

\[
(1,2),(3) : (x_1,1-x_1,0), \text{ where } .35 < x_1 < .55
\]

In this case, no coalitions other than \((1,2)\) can avoid invasion. Players expected payoffs are \(u_1 - u_2 = .45,\) and \(u_3 = 0,\) so both 1 and 2 prefer to play the CRVG rather than accept their endowments, so any unanimous agreement must pay them more than their endowments.\(^{12}\)

What these examples suggest is that if coalition costs are sufficiently low and sufficiently nonsymmetric, and if players expect the outcomes of the game to be determined by a solution concept such as the bargaining set or the equal-division core, then the no-transfer allocation will often not be a unanimous agreement. The redistributive unanimous agreement that results will offer each member of the losing group a smaller loss than he could expect under any CRVG. However, it is apparent that the nonexistence of a no-transfers unanimous agreement is not guaranteed by the mere existence of any type of bargaining costs. If we are to be sure that
nonequalizing transfers will occur under majority rule, the payoffs that
players expect to obtain under the redistributive voting game must be
sufficiently asymmetric.

Up to this point we have taken the asymmetries in coalition costs
as exogenously given. In the next section we consider some possible sources
of such asymmetries, especially those that may themselves be induced by
restrictions on the set of allowable redistributive policies.

5. **Asymmetries and Interest-Group Politics**

In this section we argue that the sorts of transfers typically
associated with interest-group politics -- price-increasing policies -- are
more likely to induce the asymmetries necessary for nonequalizing transfers
to occur than are policies of direct cash transfers from losing to winning
coalition members. When this is so, seemingly wasteful redistributive
policies may fulfill the conditions for the existence of nonequalizing
transfers while the more "efficient" policy of direct cash payments may not.

5.1. **The Organization of Groups**

We begin by considering the basis for the clustering of
individuals into groups as a preliminary stage in the search for a winning
coalition. If the minimal winning coalition is of size $#C$, the net payoff
to the members of that coalition is maximized if they minimize their
coalition costs. **Groups** will form if it is less costly for certain subsets
of potential coalition members to delegate an agent to act in their joint
behalf in negotiating with other (agents of) potential members of C than to
bargain individually. Any group that is smaller than $#C$ in size faces two
types of costs. **Internal costs** are the costs of organizing the individual
members into a cohesive political entity. Standard theories of collective action suggest that these costs are an increasing function of the size of the group. **External costs** are the costs of coalescing with other groups. These will increase with the number of groups in a coalition and, if search is required to seek out the best coalition for a group to join, they will also increase with the number of potential coalition members (i.e., the total number of groups). A group is "small" if its members treat their group's external costs as parametric to their choice of group size.

Groups are formed by people who find it less costly to associate with some people than with others, so that over some range of group size marginal internal costs are less than marginal external costs. There are innumerable characteristics that could conceivably be used to sort people into groups, but they are not all equally advantageous. Consider, for example, a group that forms to pursue a policy that grants a specific sum of money to each person with brown eyes, to be financed by a tax on people with eyes of a different color. Despite the fact that such a group would constitute a majority coalition on its own, it has no cost advantage over an equal-sized coalition of unrelated individuals unless brown-eyed people prefer to deal with other brown-eyed people as a general rule. Without such a preference, there is no reason for brown-eyed people to expect to do better in a CRVG than non-brown-eyed people. The same is true of groups based on hair color, height\(^{13}\), weight, dates of birth, and other such criteria. On the other hand, in societies where a majority of people prefer to associate with others of the same religion, nationality, or race, the costs of some winning coalitions may be substantially below those of other coalitions, so that a no-transfers unanimous agreement may not exist. In
such situations, we may observe discriminatory taxation or outright confiscation of property from the disfavored minority.

When all groups are small, however, and when all group members share equally in the net proceeds from a CRVG, then the no-transfers allocation will always be in the set of possible unanimous agreements. The proof of this is straightforward: Any small group will expand until its marginal internal cost of group formation equals its constant external cost of coalition formation. Since all small groups face the same external cost, in equilibrium they all incur the same marginal internal cost. Since all members of each group receive equal payoffs, every member of every group faces the same expected return from any such CRVG. It follows that the no-transfers allocation is in the set of possible universal agreements, since expected payoffs are symmetric. If we impose the additional assumption that all policies involving direct cash payments to the members of a winning coalition require equal payments to all members of any particular group in the winning coalition, then we can conclude that all cash-payment transfer policies in a society of small groups have the no-transfers rule as a unanimous agreement.

If we drop the assumption of equal payoffs to all members of a given group, then it might be possible to generate enough asymmetries in players' expected payoffs to eliminate the no-transfers allocation as a possible outcome, but in doing so we admit the possibility that direct payments to the members of the groups in a winning coalition will lead to costly negotiations over the intragroup division of the payoff. If any winning coalition receives direct payments, any losing coalition can try to bid away enough of the members of the old winning coalition to attain a majority by offering them a better payoff than they receive from the
incumbent winners. The ensuing negotiations among the members of the incumbent coalitions will reduce their expected net payoffs from the CRVG.

5.2. Expected-Payoff Asymmetries and Market Interventions

Nonequalizing redistribution through government policies to alter market prices, which we refer to generally as "price-support games", differ from the cash-payment games analyzed above in one crucial aspect. For an industry with a rising supply curve, the payoffs to different members of any group need not be equal in an equilibrium assignment of individuals to groups. Zero expected returns from changing groups does not imply equal returns to all players because new entrants to a group (newly employed factors of production) face higher costs of joining that group than do its previous members. A policy of taxing one group to finance the purchase of the output of another group at a higher price can therefore create a stable difference in the expected returns of the members of the two groups.

It is also likely that price-support games involve lower bargaining costs than do cash-payment games. The payoffs to each group member in a price-support game are uniquely determined by the support price and the member's (factor supplier's) opportunity cost. Each member seeks the same objective -- a higher support price -- and the division of the total group payoff is not a subject of negotiation. While we have previously assumed that agreement on equal division of cash payments among all members of a group could be attained costlessly, a fuller treatment of the bargaining problem would recognize that there may well be advantages to groups seeking price supports relative to otherwise identical groups seeking cash payments. Perhaps the most important cost advantage of price-support games is that by their very nature they tend to induce natural affinities of
individuals for an easily recognized and relatively small subset of all possible groups. In a cash-transfer game, the number of possible dimensions along which "closeness" of individuals can be defined may be arbitrarily large, in which case any proposed dimension for organizing groups (such as eye color) can be easily met by a counter proposal (e.g., hair color). This problem is not absent from price-support games, since the definition of the supported commodity can be broadened or narrowed, but it is surely a less severe problem than it is for cash-payment games.

Perhaps the most important aspect of price-support games is that, just as they induce natural affinities of individuals for groups, so too can they induce natural affinities of groups for other groups. This creates an important difference from our previous model of group costs, in which there was no dimension along which group's representatives differed. While there is no particular reason to imagine that any group cares what other groups are in its coalition in a cash-transfer game, there are persuasive reasons to expect groups seeking price supports to prefer some coalition partners to others. Indeed, even if the internal costs of all groups were identical, the preference of groups for particular coalition partners in a price-support game may still induce asymmetries in expected payoffs that are sufficient to rule out a no-transfers unanimous agreement. We present two simple examples to illustrate how this effect operates.

In each example we assume a three-good economy where all three goods are in perfectly inelastic supply. The price-support policy we consider operates in the following way: Three groups form, each representing one of the three industries. The two groups that form a majority levy a tax on the third industry and use the proceeds of the tax to finance government purchases of their output at above-equilibrium prices.14
These purchases are then disposed of costlessly. Under our assumption of perfectly inelastic supply, the optimal tax rate on the minority's product is 100%, and it leaves the tax-inclusive price of the commodity unchanged. Unique equilibria at positive prices are assumed to exist in each of the three markets. We also assume that in the unregulated equilibrium total spending on each good is identical. Normalizing, so that the unregulated equilibrium prices are all unity, the unregulated equilibrium quantities (which are of course the fixed quantities supplied) are \( x_i^0 = x_j^0 = x_k^0 \). For each commodity \( i \), the demand function is \( x_i = \alpha_i - \beta_i p_i + \sum_{j \neq i} \gamma_{ij} p_j \). Every two-group coalition incurs bargaining costs equal to 2c.

**Example 1.** (Demand for good 3 is more price-elastic at initial prices):

\[ \alpha_1 - \alpha_2 < \alpha_3, \beta_1 - \beta_2 < \beta_3, \text{ and } \gamma_{ij} = 0 \text{ for all } i, j. \]

Consider first the payoff to the coalition \((1,3)\). For simplicity we assume equal price supports for both groups in the winning coalition, so the payoff to each group \( i \) is \((1+s_{13})x_i - c\), where \( s_{13} \) represents the height of the price support. The government's budget constraint is \( \Delta x_2 \geq (1+s_{13})(-\Delta x_1 - \Delta x_3) = (1+s_{13})(\beta_1 s_{13} + \beta_3 s_{13}) \), so the maximum subsidy that \((1,3)\) can obtain is implicitly defined by \( s_{13}(1+s_{13}) = x_2^0 / (\beta_1 + \beta_3) \). Obviously the payoff vector for the coalition \((2,3)\) is identical to that for \((1,3)\).

Analogous calculations verify that the support price received by each member of the coalition \((1,2)\) satisfies \( s_{12}(1+s_{12}) = x_2^0 / (\beta_1 + \beta_2) \). Since \( x_2^0 = x_3^0 \) and \( \beta_2 < \beta_3 \), it is evident that \( s_{12} > s_{13} \). Thus, under the price-support game, groups 1 and 2 both prefer to ally with each other rather than with group 3, and a no-transfers rule is not preferred to price supports by a majority despite the costs of the price-support program.
Example 2. (Goods 1 and 2 are substitutes): \( \alpha_1 = \alpha_2 = \alpha_3 = \alpha, \beta_1 = \beta_2 = \beta_3 = \beta, \) and \( \gamma_{12} > 0 = \gamma_{13} = \gamma_{23}. \)

Again we assume equal payoffs to each group in the winning coalition. Then the support price received by coalition \((1,2)\) satisfies \( s_{12}/(1+s_{12}) = X^0_3/2(\beta - \gamma_{12}), \) while the one received by coalitions \((1,3)\) or \((2,3)\) satisfies \( s_{13}(1+s_{13}) = X^0_2/2\beta. \) Clearly, \( s_{12} > s_{13}, \) so groups 1 and 2 prefer to ally with each other rather than with group 3. Thus, as in the previous example, transfers via price supports are preferred to no-transfers by a majority.

The point of these examples is simply that the asymmetries necessary to induce a majority of informed voters to prefer price supports to no transfers may easily arise from market demand and supply conditions. Their usefulness in inducing asymmetric payoffs is enhanced by the fact that these types of redistributive policies also allow differences in the organizational costs of interest groups to persist. Since the expected payoff to any group is inversely related to the demand for its output (cf. Example 1), our analysis supports Becker's conclusion that the deadweight costs that would result from a particular price-support policy tend to reduce the likelihood that it will be imposed.

5.3. The Durability of Prohibitions Against Cash Transfers

We have shown that restrictions on the forms of redistributive policy may be necessary in many instances to rule out the possibility of a no-transfers agreement. A final question that arises at this point is whether it is possible to improve on the distribution of wealth resulting
from wasteful unanimous agreements such as those described in section 5.2, once they have been agreed upon.

In a single-period framework, the answer to this question is unambiguously affirmative. A Pareto-superior set of cash payments could be substituted for any given set of price supports by eliminating the deadweight losses that are an unavoidable consequence of such market distortions. In a static economy, it is conceivable that cash payments which mimic a set of price supports could form the basis of a unanimous agreement even in a multi-period setting. If none of the initial conditions of an asymmetric redistributive game will ever change, then every player would prefer a nonequalizing redistributive arrangement that imposes no efficiency losses to one that does impose such costs.

In an economy in which the future outcomes of an asymmetric redistributive game are uncertain, however, it may be more costly to impose a particular income distribution, rather than a set of procedural rules, in the initial unanimous agreement. If the underlying payoff structure changes substantially, then a regime rigidly committed to the outdated distribution of income will require an entirely new "constitution," whereas a polity whose "constitution" specifies rules rather than outcomes can adapt relatively smoothly to the change in circumstances, merely by revising statutes. Assuming, as seems plausible, that the former involves considerably higher negotiation costs than the latter, a dynamic society may be better-off with a constitution that specifies rules for making transfers, even if the allowed transfers sometimes involve deadweight costs.

A further problem with any attempt to use cash payments to mimic the distribution of income that would result from a price-supports unanimous agreement is that the expectation of such an attempt would undermine the
basis for asymmetries in expected payoffs that are necessary to rule out a no-transfers unanimous agreement in the first place. Asymmetric expected payoffs are consistent with the resulting unanimous agreement only if the polity can commit credibly to abide by the rules which induce those asymmetries. An implication of this is that, in societies where such a commitment has been made, there must be legal or moral sanctions against direct nonequalizing payments by the government. Thus, it is not at all inconsistent with our model that direct government payments to individuals are shrouded in secrecy or that they provoke a public outcry when they are uncovered. The fact that such schemes must be concealed does not imply that a well-understood price-support policy would fail politically. Our analysis suggests that quite the opposite is true. Opposition to direct-payment policies may be a necessary part of a unanimous agreement that allows other forms of nonequalizing redistributive policies.

6. Conclusion

The continued use of highly distortionary price- and quantity-regulating schemes to alter the distribution of wealth is not necessarily evidence of voter ignorance of the effects of those schemes. Nor is the simultaneous use of both direct payments (or general income taxes) and price-regulatory policies to redistribute income evidence of inconsistency on the part of legislators. Both types of policies may be outcomes of voting games played by fully informed, rational people. The important difference between the two policies is that unrestricted (or uncommitted) direct payments are perfectly compatible with equalizing redistribution but they do not dominate rules against nonequalizing transfers. No-transfer rules dominate in games in which players are symmetric, but they may be
dominated by certain nonequalizing transfers if there is enough asymmetry. Asymmetric sufficient to cause price-support schemes to dominate unrestricted direct payments can easily arise out of market conditions for the goods for which supports are sought.

An important caveat that applies to our results is that they derive from models that neglect differences in the basic political endowments of the participants. Those differences that ultimately exist arise out of the form of the voting game chosen by the players. A possible extension of the analysis would be to assume greater initial influence of certain players due to their wealth or other personal characteristics.
NOTES

1. See Ordeshook (1986) for a summary of this work.

2. The foremost example of such models is Becker (1983). Antecedents include Olson (1965), Stigler (1971), and Peltzman (1976).

3. We should point out here that although our focus is on redistributive politics, we do not wish to imply that all government policies are primarily redistributive in nature. Governments supply public goods, and sometimes regulate or tax in response to externalities.

4. A sense of the relative importance of distorting transfers is conveyed by Rubin (1975), who notes that in 1970 federal direct cash subsidy payments were comparable in size to the costs imposed by two regulatory schemes (the Interstate Commerce Commission and federal oil regulation.)

5. A unanimous agreement not to redistribute can also be derived as the equilibrium of a multi-period noncooperative game in which players choose whether or not to lobby for redistribution in each period, lobbying being costly. The unanimous agreement not to lobby (hence, not to redistribute) is analogous to cooperative play in a repeated prisoners' dilemma.

6. Of course, one might argue that the assumption of an equal distribution of votes, where the term "voting" refers to a citizen's overall ability to influence political decisions, is unrealistic when there is an unequal distribution of income. Also, there is some experimental evidence indicating that the outcome of a game is sensitive to the initial endowments, so a characteristic function cannot describe all of the relevant information (see Roth, 1975). In such cases, (i)-(iii) may not be reasonable assumptions when endowments are unequal.
7. This use of "symmetric" follows Shubik (1982, p. 130).

8. See Maschler (1976), however, for cases in which the core seems to make unreasonable predictions.

9. In fact, the no-transfer allocation is only in the core of "inessential" CRVG's, that is, CRVG's in which the value of every coalition is no greater than the sum of its members' endowments.

10. This is true for other solution concepts as well, such as the competitive solution (McKelvey, Ordeshook and Winer, 1978) and solutions involving aspirations (Bennet, 1983). We limit discussion here to two solution concepts in the interests of space.

11. An objection by player i against player j is a proposal to replace x with a new allocation y that is preferred to x by all players in some coalition C that includes i but not j. A counterobjection by j is another allocation z that is preferred to x by all players in some coalition D that includes j but not i, and is also preferred to y by all players in C ∩ D. Note that i may make an objection against j only if i and j are in the same element of the coalition structure. See Davis and Maschler (1967) for formal definitions.

12. It is interesting to note that if players expect the bargaining set to determine outcomes in Example 4, then the no-transfers allocation is a unanimous agreement.

13. Consider the case of the "Club of Tall People in Germany," with recently announced that it was "abandoning the appeal for tax breaks it has made for the last 35 years. [The club's chairman] said he saw no purpose in adhering to the demand, which has been rejected several times" (Chicago Tribune, May 1989?).
14. If there is a world market for the protected commodities, tariffs or quotas will be necessary adjuncts to the domestic price supports.

15. Alternatively, we could assume that the government sells its purchases on world markets at the initial price. This would enable the government to afford an even higher domestic support price than our present formulation.
REFERENCES


Ordeshook, P.C. *Game Theory and Political Theory*, ...


