Bidder Discounts and Target Premia in Takeovers*

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Abstract

When a takeover is announced, the sum of the stock-market values of the firms involved often falls, and the value of the acquirer almost always does. Does this mean that takeovers do not raise the values of the firms involved? Not necessarily. We set up a model in which the equilibrium number of takeovers is constrained efficient. Yet, upon news of a takeover, a target’s price rises, the bidder’s price falls, and, most of the time the joint value of the target and acquirer also falls.

1 Introduction

On news of a takeover, the share price of the target firm usually rises sharply, while that of the acquiring firm usually falls. The joint value may or may not rise. Surveying the field, Andrade, Mitchell and Stafford (2001) report that since 1973 target premia were 20 or 30 percent, acquirer discounts were minus 3 or 4 percent, and that the joint values show no clear pattern. They conclude (p. 118) that “the fact that mergers do not seem to benefit acquirers provides a reason to worry ...[that mergers do not raise value].” To explain such evidence (Shleifer and Vishny 2001) have assumed that investors are irrational and (Roll 1986) has assumed that managers use takeovers to extend their empires at the expense of the shareholder. The evidence about the bidder discount and joint discount has been taken to imply that takeovers often just redistribute rents from acquirers to their targets or that they even destroy rents.

We show, however, that even when investors are rational, when agency problems are absent, and when mergers do create value, the acquirer’s value and even the joint value of the bidder and target may fall when a takeover is announced. The bidding firm’s value falls because its bid reveals that its internal investment opportunities are

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poor. The target’s value rises because an acquisition signals that the target’s internal investment opportunities are good. A takeover benefits both parties, and the number of takeovers is, in a sense, constrained-efficient. And yet the bidder discount usually outweighs the target premium so that the joint value drops.

We discuss other related work later on in the paper; the highlights there are Holmes and Schmitz (1990, 1995), Campa and Kedia (forthcoming), Villalonga (2001), Gort, Grabowski, and McGuckin (1985), and Myers and Majluf (1984). But first we present the model and then discuss its robustness.

2 Model

In the model that we are about to describe, takeovers are the mechanism that moves good projects from bad managers to good managers. At the outset firms differ in the quality, $x$, of their management.\footnote{Andrade et al (2001) and Jovanovic and Rousseau (2002) report evidence that acquisitions are disproportionately made by high-$Q$ firms.} Each firm then draws a project and the quality, $z$, of projects, too, differs over firms. Some good managers end up with bad projects and vice versa. Takeovers then serve to shift the good projects from bad managers to good managers. A firm’s output is

$$xz. \quad (1)$$

Thus the quality of a project, $z$, and the firm’s ability to implement a project, $x$, are complements. Among firms, $x \geq 0$ is distributed according to the cumulative distribution function $F(x)$. Projects are either good or useless: $z \in \{0, 1\}$. A fraction $\lambda$ of projects is good, and the fraction $1 - \lambda$ is useless.

A firm cannot change the quality of its management. It can, however, acquire another firm and manage its project. A manager can handle only one project. If a firm $(x, z)$ buys firm $(x', z')$, it then uses its own management, $x$, and the project, $z'$, of the firm that it has acquired. It drops its own, useless project, and lets go the manager of the target. The output of the merged entity will be\footnote{This partially reflects findings by McGuckin and Nguyen (1995) and Schoar (2000) that the productivity of the target’s plants rises (in this case from $x'$ to $x$) while that of the acquirer’s plants falls (in this case from $z$ to zero as the plant is ‘abandoned’). We re-visit this issue in Section 4. The merged firm produces more output than the combined stand-alone outputs of the two firms. Hence, productivity rises after a merger. Lichtenberg and Siegel (1987) find that plants changing owners had lower initial levels of productivity and higher subsequent productivity growth than plants that did not change hands.}$

$$xz'. \quad (2)$$

To an acquirer, then, only the target’s $z'$ matters. The departing manager receives no severance payment and does not stand in the way of the takeover as long as it
benefits the shareholders of his firm. We assume that all mergers are driven only by
the prospect of real gain, all investors are rational and all managers act only in the
long-term interests of shareholders. This is not to suggest that, empirically, mergers
are always exactly like that. Rather, we assume that all mergers create value, that
the stock market is efficient, and that there is no agency problem, in order to see
exactly where these extreme assumptions lead.

2.1 A sketch of equilibrium
We begin with an intuitive derivation of the equilibrium takeover activity and of the
price, \( q \), of targets.

The supply of targets.—Let us start off by assuming as a constraint something
that will later emerge as an equilibrium action; namely, that, before it can be taken
over, a firm must certify that its \( z = 1 \). The total mass of potential targets is \( \lambda \). The
act of certifying the quality of its project costs the firm \( c \). If the firm does not wish
to be a target and if, instead, it were to manage its own project, its payoff would be
\( x \). Thus the direct plus the foregone-earnings costs of being a target are \( c + x \). Thus,
at a price of \( q \), the number of willing targets would be all the \( z = 1 \) firms for which
\( c + x \) is less than \( q \). That number is

\[
\lambda F(q - c) \equiv S(q).
\]

Evidently, \( S(q) \) is an upward sloping supply curve that is continuous if \( F \) is.

The demand for targets.—Any firm that drew a project \( z = 0 \) is a potential
acquirer. Unless it manages to acquire another firm, this firm faces a revenue of zero.
The number of such firms is \( 1 - \lambda \). If firm \( x \) manages to buy another firm with a
certified \( z = 1 \) project, its revenue will be \( x \). Thus, at a price of \( q \), the number of
willing acquirers would be the \( z = 0 \) firms for which \( x - q \) is positive. That number is

\[
(1 - \lambda)[1 - F(q)] \equiv D(q),
\]
a downward sloping demand curve.

Figure 1 shows the equilibrium price of targets \( q^E \) and number of takeovers \( T^E \),
where the two curves intersect. At \( q = 0 \), all the \( 1 - \lambda \) firms with \( z = 0 \) projects are
willing to buy, hence the demand curve cuts the horizontal axis at the point \( 1 - \lambda \). As
\( q \) gets large, all the \( \lambda \) firms with \( z = 1 \) projects are willing to sell, and so the supply
curve approaches \( \lambda \). Rents are split between the targets and the acquirers, with the
largest rents in each group going to the highest-\( x \) acquirers and the lowest-\( x \) targets.

Figure 1 relates to a stage of the game at which most of the uncertainty has been
resolved. It says nothing about bidder discounts and target premia. Here now is a
more precise account of the game.
2.2 The five stages of the game

We shall treat all takeovers as cash sales although this is done purely for expositional reasons. The dividends of the combined entity are then paid to the shareholders of the acquiring firm. Shareholders are risk neutral and they hold on to their shares until the firm pays its dividend and liquidates or until it is bought by another firm. We assume that a manager acts in the shareholder’s interest. That is, he puts the firm up for sale if the cash payment, \( q \), exceeds his firm’s stand-alone dividend. Alternatively, the manager buys another firm if the dividend he can secure his original shareholders net of the cash paid for the acquired firm, exceeds his firm’s stand-alone dividend.

Events occur in five stages:

1. Firms form. Based on its \( x \) (which is public knowledge) each firm is sold at its Stage-1 price \( p(x) \).

2. The firm privately observes \( z \). It may (truthfully) disclose its \( z \) to everyone at a cost \( c \).\(^3\)

\(^3\)In reality there is a whole set of indicators of project quality, and each of them probably carries a different \( c \). Gray et al (1990, Table 1) report 33 of them. Firms seem to favor disclosing of their major new products and major capital expenditure projects in progress, but do not favor disclosing their major patents.
3. Firm may enter the takeover market as a buyer or a seller.⁴

4. Based on its Stage-2 and Stage-3 choices, the firm’s price takes on its “Stage-4” level. (It is to this stage that Figure 1 relates)

5. The firm pays its dividend and liquidates.

2.3 Stage-3 actions and Stage-4 prices

Equilibrium.—Key to equilibrium is a pair of real numbers, \( x_0 \) and \( x_1 \), where \( x_0 < x_1 \). These two numbers divide the set of \( x \)’s into three regions – top, middle, and bottom. Targets come from the bottom region, acquirers from the top region. Firms from the middle region stay out of the takeover market. We start describing the equilibrium with an account of the Stage-3 actions and Stage-4 prices of the firms in each region.

The bottom region – \( x \leq x_0 \).—If such a firm draws \( z = 1 \), it discloses that fact. It becomes a takeover target and sells at the price \( q \). All targets sell at the same price.⁵ Firm \( x_0 \) is indifferent between disclosing \( z \) (and getting its shareholders a payoff of \( q - c \)), and not disclosing and managing its own project (and getting its shareholders a payoff of \( x_0 \)). That is,

\[
q - c = x_0. \tag{3}
\]

If such a firm does not disclose its \( z \), the market rationally infers that any firm with \( x < x_0 \) that has not disclosed is a \( z = 0 \) firm. The Stage-4 price of such a firm is zero. To sum up, then, in the bottom region, a firm’s Stage-4 price is \( q \) if \( z = 1 \), and it is zero if \( z = 0 \).

The middle region – \( x \in (x_0, x_1) \).—Such a firm does not disclose its \( z \) and it does not bid for other firms. The market infers nothing from its inaction. If such a firm has \( z = 1 \), it can guarantee its shareholders more than \( q - c \), and it would refuse (and successfully repel) any takeover bid at the price \( q \). If, on the other hand, such a firm has \( z = 0 \), buying another firm at the price \( q \) would leave it with a negative net payoff. Thus, if a firm from this region did not refuse a takeover bid, it would reveal itself to be a “lemon”. Thus no one bids for firms in this region and their prices remain unchanged at \( \lambda x \).

⁴I.e., a firm can repel an unwanted bid. Of all takeover bids, only 8.3% of all bids are hostile and only 4.4% eventually succeed (Andrade et al, 2001). We do not explain such mergers.

⁵Strictly speaking, disclosure should occur before the takeovers announcement. Certainly the price of the target-to-be does rise a bit before the takeover announcement (Pound and Zeckhouser 1990). Firms must, in fact, disclose business plans and other trade secrets at the IPO stage, and when taking out patents. Many firms are taken over during the several months’ waiting period between an IPO filing and SEC approval. Some are taken over precisely for their intellectual property, i.e., their patents. The direct costs of such disclosures may be small, but they may help the firm’s competitors and any profits thus foregone should also be a part of \( c \). Firms seem to regard ‘competitive disadvantage’ as the largest part of \( c \) (Gray et al 1990, Table 4).
The top region – \( x \geq x_1 \).—A firm that has drawn \( z = 0 \) has spare capacity in its organization capital \( x \) that it will seek to employ. Such a firm buys a discloser from the first region thereby raising its own output and dividend from zero to \( x \); and its Stage-4 price is \( x - q \). The lowest-quality bidder \( x_1 \) is indifferent between bidding (and getting a payoff of \( x_1 - q \)), and managing its own project (and getting zero), so that

\[
q = x_1. \tag{4}
\]

If it does not bid, this signals to the market that the firm’s \( z = 1 \), and its Stage-4 price is \( x \).

Why don’t the \( z = 0 \) firms refrain from bidding and thereby secure a jump in their price? Because it then would deliver a zero dividend to its shareholders who are following a “buy and hold” strategy.\(^6\) To sum up, then, in the top region, a firm’s Stage-4 price is \( x \) if \( z = 1 \), and \( x - q \) if \( z = 0 \). These Stage-4 prices are depicted by heavy lines in Figure 2. In the middle region there is just one heavy line because, at stage-4, the market cannot distinguish the good firms from the lemons. In the other two regions, the actions of the firms have fully revealed their type.

The discontinuities in the price-functions of Figure 2 are misleading; the shareholders of the good firms receive a continuous payoff as we raise \( x \), and so do the shareholders of the bad firms. The situation is illustrated in Figure 3. Now the gains

\(^6\)If enough shareholders were to sell their shares at Stage 4, and if the manager wanted to maximize interim shareholder utility, he would refrain from bidding and masquerade as \( z = 1 \) firms. Other shareholder holding strategies may induce \( z = 0 \) firms to not bid.
to trade are apparent; in the absence of takeovers, the $z = 1$ firms would be paying their shareholders a dividend of $x$ (i.e., the 45° line), and the $z = 0$ firms would be paying their shareholders a dividend of zero (i.e., the horizontal axis). But what we end up with is a Pareto improvement, with the targets and the bidders doing strictly better than they would if there were no trade.

\[ \lambda F (q - c) = (1 - \lambda) (1 - F[q]) \]  

so that we are at an intersection of the two curves in Figure 1.

2.4 Stage-1 prices, $p(x)$

For firms in the middle region the Stage-1 prices are the same as the Stage-4 prices, namely, $\lambda x$. In the two other regions, a firm’s Stage-1 price is a weighted sum of the prices it will fetch at stage 4, the weights being the probabilities of $z$ being zero and one. The stage-1 prices are:

\[ p(x) = \begin{cases} 
\lambda (q - c) & \text{if } x \leq x_0, \\
\lambda x & \text{if } x \in (x_0, x_1), \\
\lambda x + (1 - \lambda) (x - q) & \text{if } x \geq x_1.
\end{cases} \]  

Since there are no aggregate shocks, the value of the stock market as a whole is the same at Stage 4 as it is at Stage 1. But the dispersion of prices is higher at Stage 4.
2.5 Existence of a positive-takeover equilibrium

Trade in firms stems from differences in comparative and absolute advantage in management, i.e., from the dispersion in $x$’s. Let $x_{\text{min}}$ be the smallest and $x_{\text{max}}$ the largest value of $x$ in the support of $F$. If the range of $x$ is larger than $c$, takeovers will take place in equilibrium.

**Proposition 1** (Existence) If $F$ is continuous, if $0 < \lambda < 1$, and if

$$c < x_{\text{max}} - x_{\text{min}},$$

then takeovers do occur in equilibrium,

$$x_1 = x_0 + c$$

and $q \in (c, x_{\text{max}})$ uniquely solves

$$\lambda F (q - c) = (1 - \lambda) (1 - F [q]).$$

**Proof.** Solving (3) for $q$ and substituting into (4) implies that if equilibrium exists, (8) must hold. Eq’s (3), (4), and (5) imply (9). It remains to be shown that, for each $\lambda$ and $c$, (9) has a unique solution for $q$. This follows in 2 steps: (i) at $q = x_{\text{max}}$ the RHS of (9) is zero whereas, by (7), the left-hand side of (9) is strictly positive and (ii) at $q = c$ the opposite is true. Since the LHS of (9) is continuous and increasing in $q$ whereas the RHS is continuous and decreasing, we are done. ■

2.6 Comparative statics

The parameters of the model are $c$, $\lambda$, and $F$. The conditions of the existence theorem provide clues to how the solution changes when the parameters change. Condition (7) emphasized that takeovers are driven by the dispersion of $x$’s, summarized by their range – no takeovers can occur if (7) fails. But the theorem also requires that there be dispersion in the $z$’s; if all firms had the same-quality projects (which would happen if $\lambda = 0$ or $\lambda = 1$), again there would be no takeovers. The number of equilibrium takeovers is therefore non-monotonic in $\lambda$. Finally, takeover activity declines with $c$. Formally, differentiation of (9) reveals that

$$\frac{\partial q}{\partial \lambda} < 0 \quad \text{and} \quad \frac{\partial q}{\partial c} > 0.$$ \hspace{1cm} (10)

When $\lambda$ is high, there are more good projects in total and the demand for targets falls relative to their supply and, hence, so does $q$. On the other hand, when it costs more to disclose quality, the price of targets will rise so as to reflect that fact. Eq’s (4) and (10) imply that

$$\frac{\partial x_1}{\partial \lambda} < 0 \quad \text{and} \quad \frac{\partial x_1}{\partial c} > 0,$$ \hspace{1cm} (11)
so that the number of takeovers, \((1 - \lambda) [1 - F(x_1)]\) decreases with \(c\). Eq’s (3) and (10) imply that
\[
\frac{\partial x_0}{\partial \lambda} < 0, \quad \text{and} \quad \frac{\partial x_0}{\partial c} < 0,
\]
this latter because we just established that the number of takeovers, which also equals \(\lambda F(x_0)\), decreases with \(c\).

### 2.7 Discounts and premia

At Stage 4, all targets trade at a premium over their Stage-1 prices, and all bidders trade at a discount. We measure the premia and discounts as percentages of Stage-1 prices \(p(x)\).

**The target premium.**—From (6), the premium is
\[
\left(\frac{q - c}{p(x)}\right) - \frac{p(x)}{\lambda (q - c)} = \frac{1 - \lambda}{\lambda},
\]
and it is the same for all targets. The target premium is high when good projects are scarce and when, as a result, a disclosure that \(z = 1\) is especially good news. Since target premia are on average about 0.2, the relevant value seems to be \(\lambda \approx 0.83\).

**The bidder discount.**—Conversely, the bidder discount is high when good projects are plentiful and when, as a result, the revelation of \(z = 0\) that is implicit in a firm’s decision to acquire another, is especially bad news. The discount is smaller for the high-\(x\) bidders because all bidders pay the same price, \(q\), but the high-\(x\) bidders benefit more. The absolute value of bidder’s discount, i.e., the fraction of value lost upon announcement, is
\[
\delta(x) \equiv \frac{\lambda q}{x - (1 - \lambda)q},
\]
From (4) \(x_1 = q\), and so \(\delta(x_1) = 1\); the marginal bidder loses all of his value. As \(x\) rises, the discount steadily shrinks and converges to zero as \(x\) gets large.

**The values combined.**—The target’s \(x\)’s do not affect their prices at any stage. Therefore only the acquirer’s \(x\) affects the sum of the two firms’ Stage-1 and Stage-4 prices. Relative to the sum of the two firms’ ex-ante values, the ex-post “joint” value of the merged firm is
\[
J(x) = \frac{x - c}{q(2\lambda - 1) + (x - \lambda c)},
\]
an expression that is relevant for \(x \geq x_1\) only. The next proposition reports a result that we shall use later: For \(\lambda \geq 1/2\), the joint values drop.\(^7\)

\(^7\)The model also implies a rise in the price of the non-bidders with \(x > x_1\). This “non-bidder premium” is
\[
\frac{x - p(x)}{p(x)} = \frac{(1 - \lambda)x_1}{\lambda x + (1 - \lambda)(x - x_1)}.
\]
At \(x = x_1\), this is the same as the target premium, and it goes to zero as \(x\) gets large.
Proposition 2 *(Joint values).* For any \(c\), \(J(x)\) is strictly increasing in \(x\) if \(\lambda \geq 1/2\) and
\[
J(x) < 1 \text{ for all } (c,x) \text{ and all } \lambda \geq 1/2.
\] (14)

**Proof.** The first claim follows immediately from (13). As for (14), the right-hand side of (13) is less than unity when \(\lambda = 1/2\) and it is even smaller when \(\lambda > 1/2\) because \(q > 0\). ■

Since it does not involve \(x\), (14) applies to the joint value of all takeovers. The drop is smaller for the high-\(x\) firms.

In all cases, including those in which the joint value drops, the merger raises the joint output of the two firms by an amount \(x_A - x_T\) where \(x_A\) is the acquirer’s \(x\) and \(x_T\) is the target’s \(x\).

### 2.8 Takeovers and exits

Takeover activity is often measured by the value of the targets as a fraction of stock-market capitalization.\(^8\) The total spent on takeovers is \(q\lambda F(x_0)\), and so the capitalization of targets relative to total capitalization is
\[
m = \frac{q\lambda F(x_0)}{\int_0^\infty p(x) dF(x)}.
\] (15)

Another interesting statistic is the fraction of exits—firms that end up not producing output for reasons other than that they are taken over. In the model, such firms are those whose \(z\) is zero and whose \(x\) is below \(x_1\). The number of such firms is \((1 - \lambda) F(x_1)\), but their Stage-4 value is zero. Relative to the stock market, the Stage-1 value of these firms combined is
\[
\varepsilon = \frac{(1 - \lambda) \left[ (q - c) \lambda F(x_0) + \lambda \int_{x_0}^{x_1} x dF(x) \right]}{\int_0^\infty p(x) dF(x)}.
\] (16)

We shall calculate \(m\) and \(\varepsilon\) in the following example.

### 2.9 Example: Uniformly distributed \(x\)

We briefly show the kinds of statistics that a model of this general type can generate. We choose \(\lambda\) so as to fit the target premium of 0.2, and then ask what other quantitative implications this has.

Assume that \(F(x) = x\); i.e., \(x\) uniformly distributed on \([0, 1]\). Then (9) reads
\[
\lambda (q - c) = (1 - \lambda) (1 - q)
\]

\(^8\)E.g., Jovanovic and Rousseau (2002), Figures 1 and 8.
so that
\[ q = x_1 = 1 - \lambda (1 - c) \quad \text{and} \quad x_0 = (1 - \lambda) (1 - c) \]
The target premium is \((1 - \lambda) / \lambda\). When \(c = 0\), \(q = x_0 = x_1 = 1 - \lambda\), and the bidder’s discount is
\[ \delta (x) = \frac{\lambda (1 - \lambda)}{x - (1 - \lambda)^2}. \]

In the rest of this section we assume that \(\lambda = 0.83\) so that our model fits the target premium or 0.20. Using this value of \(\lambda\), we plot \(\delta (x)\) in Figure 4.

The bidder discount is an order of magnitude too large, especially at the lower values of \(x\). In section 4 we show how this quantitative error can be fixed by allowing a larger span of control for managers. The joint premium \(J\), as \(x\) ranges over the set of possible bidder-types \(x \in [1 - \lambda, 1]\), is
\[ J (x) = \frac{x}{x + (1 - \lambda) (2\lambda - 1)}. \]
Taking the value \(\lambda = 0.83\) under which the model fits the target premium, we plot \(J (x)\) in Figure 5.

The model also overpredicts \(J (x)\) in the negative direction. In Section 4.1, we also show that raising managerial span of control can push \(J (x)\) towards, and even above unity. The non-bidder premium, \(\frac{x - p (x)}{p (x)}\) is 0.20 at \(x = x_1\), but at \(x = 1\) it is only 0.03.
\[ \frac{x - p (x)}{p (x)} = \frac{(1 - \lambda) x_1}{\lambda x + (1 - \lambda) (x - x_1)}. \]
Figure 5: The ratio of post-bid joint values to the pre-bid joint values when $\lambda = 0.83$ and $c = 0$.

Still assuming $c = 0$, the value of each target is $q = 1 - \lambda = 0.17$, whereas the capitalization of the average acquirer is $-q + E(x \mid x \geq 0.17) = 0.42$. Therefore the capitalization of the average acquirer is 2.5 times that of the average target. In fact, this ratio is somewhere between 5 and 10 (Andrade et al Table 1). This the model can easily handle once we raise the managerial span of control as we explain in Section 4.

On the other hand, when $\lambda = 0.83$, $m = 0.049$, which slightly overpredicts the relative capitalization of targets (0.025) during the period 1970-2000, and $\varepsilon = 0.008$, which is about 2.3 times lower than the relative capitalization of non-acquired firms (0.019) that de-listed from NYSE and Nasdaq since 1970.\footnote{Jovanovic and Rousseau (2002).}

How much value do takeovers add? In other words, how much more output do we have compared to the case in which takeovers are not allowed? Without takeovers, aggregate output would be

$$\lambda \mu_x = \frac{1}{2} \lambda = 0.415.$$ 

Maximal output with takeovers occurs when $c = 0$, and, from (17) below, it is

$$\lambda \mu_x + (0.17) \int_{0.17}^{1} xdx - (0.83) \int_{0}^{0.17} xdx = 0.486.$$ 

As a fraction of the no-takeover output, the maximal gain is $\frac{0.486 - 0.415}{0.415} = 0.17$.

3 Welfare

Our welfare measure is net aggregate output, $Y$. If agents could not recontract from the Stage-2 random assignment of $z$ to $x$, aggregate output would be $\lambda \mu_x \equiv$
With takeovers, however, output net of disclosure costs becomes

\[ Y = \lambda \mu_x + (1 - \lambda) \int_{x_1}^{\infty} x \, dF - \lambda \int_{0}^{x_0} (c + x) \, dF \] (17)

This is how much could be produced if, at a cost \( c \), the planner could truthfully elicit all the \( z = 1 \) from firms with \( x < x_0 \), and reassign them to firms with \( x > x_1 \). It turns out that the equilibrium maximizes \( Y \) with respect to \( x_0 \) and \( x_1 \), subject to the resource constraint

\[ \lambda F(x_0) = (1 - \lambda) [1 - F(x_1)] \]

**Proposition 3 (Equilibrium Maximizes \( Y \)).** The equilibrium allocations maximize \( Y \). Moreover, when \( c \in (0, x_{\text{max}} - x_{\text{min}}) \),

\[ \frac{dY}{dc} = -\lambda F(x_0) < 0. \]

**Proof.** The Lagrangian is

\[ L = Y + \theta \{ \lambda F(x_0) - (1 - \lambda) [1 - F(x_1)] \} \]

The first-order conditions are

\[ -(c + x_0) + \theta = 0 \]

and

\[ -x_1 + \theta = 0 \]

The second-order derivatives with respect to \( x_0 \) and \( x_1 \) are negative and the cross partials are zero. Therefore \( L \) is globally strictly concave in the vector \((x_0, x_1)\). Combining the two conditions and observing that the constraint must hold proves the first claim. The second claim then follows from the envelope theorem. The strict inequality follows from Proposition 1 (eq. [9]) by which \( F(x_0) > 0 \).

So, if the planner must pay \( c \) for every discovery of a \( z = 1 \) firm, then the equilibrium also maximizes aggregate output net of disclosure costs, much as one would expect based on Figure 1. In this sense, then, equilibrium is constrained efficient.

As \( c \to 0 \), \( x_0 \) and \( x_1 \) tend to the same value, call it \( x^* \) which solves the equation

\[ \lambda F(x) = (1 - \lambda) (1 - F[x]), \]

Letting \( x^* \) denote the optimum and simplifying,

\[ F(x^*) = 1 - \lambda. \] (18)
so that the number of projects reassigned, $\lambda F(x^*)$, is just $\lambda(1 - \lambda)$. This is also the first-best level of takeovers because this is what a planner could attain if he had knowledge of the $z$’s without having to bear the disclosure costs.

The welfare properties of equilibrium seem to be unrelated to the change in the joint total value of the bidder and the target (Proposition 2) – takeovers are always associated with a level of output that exceeds $\lambda \mu_x$ regardless of what $\delta(x)$ and $J(x)$ happen to be. This is because without aggregate risk, all future welfare gains from reassignment are already included in $p(x)$.

4 Extensions

In this section we relax some of the assumptions and look into some other implications of the model.

4.1 Larger span of control

Suppose that a manager can handle up to $n$ projects, and that the production function of his firm is

$$y = x \sum_{i=1}^{n} z_i$$

To keep things simple, each manager is still endowed with just one project to begin with. Take the case where $c = 0$. Then there are just 2 regions, i.e., $x_0 = x_1 \equiv \hat{x}$. Bidders in the region where $x \geq \hat{x}$ wish to acquire $n$ firms if their endowed project is bad, and $n - 1$ firms if their endowed project is good. (5) reads $\lambda F(\hat{x}) = n(1 - \lambda)(1 - F[\hat{x}]) + (n - 1)\lambda(1 - F[\hat{x}])$, or simply

$$\lambda = n(1 - F[\hat{x}])$$

Conditions (3) and (4) are unchanged and therefore

$$q = \hat{x} = F^{-1}\left(\frac{n - \lambda}{n}\right)$$

Note that $\lim_{n \to \infty} \hat{x} = x_{\text{max}}$. The number of takeovers,

$$\lambda F(\hat{x}) = \lambda\left(\frac{n - \lambda}{n}\right),$$

rises with $n$, because the best managers now have a wider span of control.

Thus modified, the model now yields a smaller bidder’s discount — roughly by a factor of $1/n$. That is because only $1/n^{th}$ of a firm’s eventual operations have a quality that is in doubt at Stage 1. The remaining $n - 1$ projects will, after the
takeover market has cleared, all be of quality \( z = 1 \), and the firm’s Stage-1 price, \( p(x) \) will reflect that fact. The target premium is not affected, however, because a firm for which \( x < \hat{x} \) still becomes a target with probability 1 and so for such a firm, \( p(x) = \lambda x \) as before.

### 4.2 Pooling equilibria

Pooling equilibria exist when \( c \) is high. In such an equilibrium, targets do not disclose anything, and the acquirer is not sure what \( z \) he is getting. Acquirers come from the top region, as before, but the logic behind (4) now implies that

\[
\lambda x_1 - q = 0
\]

The equilibrium still has three regions, firms with \( x < x_0 \) disclose nothing and sell for the price of \( q \). Again a middle set \( (x_0, x_1) \) exists where firms do not disclose and no one bids for them. Firms with \( x < q \) must not want to repel, and therefore

\[
x_0 = q
\]

which, of course, is the same as (3). It is the \( x_{\text{max}} \) firm that cares the most about having a \( z = 1 \) project to manage for sure. Its expected gain from being sure of having a good project would be \( (1 - \lambda) x_{\text{max}} \). A firm that disclosed that it had \( z = 1 \) could therefore extract from the bidder at most \( (1 - \lambda) x_{\text{max}} \). Therefore, under no circumstance would any firm disclose if the following condition held:

\[
c > (1 - \lambda) x_{\text{max}}
\]

Thus we have

**Proposition 4 (Pooling Equilibrium)** If (21) holds, a pooling equilibrium exists in which \( x_0, x_1, \) and \( q \) satisfy (19), (20), and (5). Moreover, when

\[
c > x_{\text{max}} - x_{\text{min}},
\]

the pooling equilibrium is unique.

When \( x \) is bounded and \( \lambda \) is relatively close to unity, there are values of \( c \) for which both equilibria exist.\(^{10}\) Such an equilibrium does not entail any spending on disclosure, and projects do flow towards better managers, though at a rate smaller than the first-best level \( x^* \). Hence, takeovers raise total output. The pooling equilibrium entails bidder discounts, but no target premia.

\(^{10}\)As in Jovanovic (1982), a coexistence of disclosure and pooling regions appears to be a possible equilibrium for some parameter values, but these, too, disappear when \( x_{\text{max}} \to \infty \).
This equilibrium is of little interest because (21) is not likely to hold practice: payments made to investment banks and other certifiers of quality at the takeover stage amount to at most a percent or two of target value. Even if we add to $c$ the cost of leaking trade secrets to competitors, $c$ should still be far smaller than the RHS of (21). To see this, consider once more the case in which $x$ is uniformly distributed, as in Section 2.9. The RHS of (21) is $1 - \lambda$, which also happens to be the gross value of the target. Hence, the disclosure costs have to be comparable with the total value of the target in order for the pooling equilibrium to exist.

4.3 Correlated $x$ and $z$

For takeovers to arise at Stage 4, some good managers must, at stage 2, have drawn bad projects. If, instead of being constant, the probability of drawing a good projects were to rise with $x$, the fraction of bad matches would fall. Let

$$\Pr \{z = 1 | x\} = \lambda(x),$$

where $\lambda'(x) > 0$. Holding constant the overall number of good projects

$$\bar{\lambda} = \int_0^\infty \lambda(x) dF(x),$$

a more positive slope of $\lambda(x)$ would (a) raise the Stage-2 correlation between $x$ and $z$ and, hence, reduce the number of takeovers, activity, and, (b) raise the target premia and the bidder discounts. Intuitively, this would be because $z = 1$ would now be a bigger positive surprise for a low-$x$ firm, and $z = 0$ would be an even bigger negative surprise for a high-$x$ firm.

4.4 Market for $x$

Since $x$ is that part of a firm that is common knowledge, why would there be no market for $x$? Indeed, if we do assume that $x$ is some attribute of the firm’s human capital, there in fact is a market for $x$ – the labor market – that our model assumes does not function. Suppose, for the moment, that $x$ is something that the firm actually owns and not, as is the case with human capital, something that it rents. A low-$x$ firm with a $z = 1$ could buy a higher $x$ from a firm that has $z = 0$. Since $x$ is public knowledge, no disclosure would be needed, and the equilibrium would have $x_0 = x_1$, with $(1 - \lambda) (1 - F(x_1))$ units of $x$ moving from the high-$x$ firms with $z = 0$ projects to the $\lambda F(x_0)$ low-$x$ firms with $z = 1$ projects. Finally, total output in (17) would correspond to its level when $x_0 = x_1$ and $c = 0$.

Why, then, is this a model of the takeover market, and not a model of a market for one of the firms’ components, namely $x$? The answer must be that human capital is costly to move from firm to firm, specially when it is specific to a team. Most of
what we call “firm-specific” human capital is probably tied to the group of workers employed by that firm. If the whole team were to move to another firm – this sometimes happens on Wall Street when an entire team of analysts moves from one investment bank to another – this capital would be just as useful in the new firm as it was in the old one. And yet we would call this human capital firm specific precisely because it is virtually impossible for a large team to move to another firm and stay intact unless the new firm literally is right across the street. Prescott and Visscher (1980) call such capital organization capital, and they describe the costs of creating it.

To sum up, we intend \( x \) to stand for an asset that firms have used for a while, the expertise of a team that has managed projects in the past and that have learned to function well as a unit. When the team finds itself with spare capacity, instead of dispersing, it will look for takeover opportunities. Gort, Grabowski, and McGuckin (1985) also argued this.

### 4.5 The \( Q \)-theory of takeover investment

The model is a version of the \( Q \)-theory of takeover investment, in that projects tend to move from low-\( Q \) firms to high-\( Q \) firms. By a firm’s \( Q \) we mean the ratio of the Stage-1 market value of the firm, \( p(x) \), to the ‘replacement’ value of its ‘capital’. Recall that a firm cannot replace its \( x \), only its \( z \), and so we think of the firm’s tangible capital as its \( z \) – this is what a firm can ‘replace’.

Let \( p_z \) denote the Stage-1 replacement cost of an unscreened \( z \). Then the Stage-1 \( Q \) is defined as

\[
Q(x) = \frac{p(x)}{p_z}.
\]

Since \( p_z \) is common to all firms, we have

**Proposition 5** Acquirers have higher \( Q \)'s than do the targets

**Proof.** From (6), for \( x < x_0 \), \( Q(x) = \frac{1}{p_z} \lambda (q - c) \). But (6) and (4) imply that \( Q(x_1) = \frac{1}{p_z} \lambda q \). That is, the lowest-\( Q \) acquirer has at least as high a \( Q \) as does any target. And, since \( Q \) is strictly increasing in \( x \) for \( x > x_1 \), the same is true for any acquirer. \( \blacksquare \)

A merger raises the joint output of the two firms by an amount \( x_A - x_T \) where \( x_A \) is the acquirer’s \( x \) and \( x_T \) is the target’s \( x \). Since \( Q \) is increasing in \( x \), this translates into the statement that joint gains should be higher and joint losses smaller for mergers in which \( Q_A - Q_T \) is high. Lang, Stultz, and Walkling (1989) and Servaes (1991) find that the mergers that create the most value are those between high-\( Q \) bidders and low-\( Q \) targets, which is consistent with Proposition 5.
5 Relation to the literature

Holmes and Schmitz (1990, 1995) study the transfer of businesses through sale rather than takeover. In their 1990 model, good managers acquire firms from good developers of new ideas. And in their 1995 model, whether a firm stands alone, sells its business or exits depends on the quality of the firm, and on the quality of the match between the entrepreneur (manager) and his firm. A manager of a high quality business to which he is poorly matched will sell that business, while a manager who is matched badly to a low-quality firm will exit. In our model, by contrast, a good manager is better at managing any project and, if his firm does not have a good project, he will look to acquire one that does. In the Holmes and Schmitz model, low-quality managers sell their businesses to new entrants. So, in their models, as in ours, who manages what firm depends entirely on fundamentals.

McCardle and Viswanathan (1994) study a firm that contemplates entering a new market. It can do this on its own, or, instead it can bid for one of the incumbents. A takeover bid reveals weak internal investment opportunities and leads to a reduction in the bidder’s stock price even though the takeover has a positive value for the bidder. The target’s price rises because a takeover removes the threat of direct entry, thus raising the bargaining power of the targeted firm. Because it is the entrant that bids for an incumbent, their model is better suited to conglomerate diversification into an oligopolistic market. Our model, by contrast, is better suited to horizontal mergers because we assume that any manager can manage any firm’s project equally well.

The bidder discount in our model arises for reasons similar to the discount that arises upon an equity issue in Myers and Majluf (1984). Campa and Kedia (forthcoming), Villalonga (2000) have argued that the so-called diversification discount arises purely because of selection bias. These notions go back to simultaneous equations bias involving endogenous discrete variables as in Heckman (1978).

6 Conclusion

Initially we wanted to build an equilibrium model in which takeover targets experienced jumps in price and acquirers experienced price declines. In the model, the sole purpose of a takeover is to transfer a business project to a better manager. Although quality of a project is assumed to be known only to the firm itself, to our surprise the equilibrium turned out to be constrained-efficient and, for reasonable parameter values the joint value of the target and acquirer falls.

As Proposition 5 shows, this is a version of the $Q$-theory of investment. The pre-announcement $Q$’s of the acquirers exceed those of the targets. Jovanovic and Rousseau (2002) find that such a model helps explains some of the takeover investment, although it cannot explain all of it. We noted a few other explanations in the introduction and there may, indeed, be several reasons for takeovers. We have shown
that the facts on $Q$ can be explained by a model in which takeovers improve efficiency, even though stock prices react negatively to their announcements.

References


