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PUBLIC EMPLOYMENT, TAXES
AND THE WELFARE STATE IN SWEDEN

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Abstract

All employment growth in Sweden since the early 1960's is attributable to labor market entry of women, working in local public sector jobs that implement the Welfare State. Sweden has "monetized" or "nationalized" the family. Women are paid at public expense to provide household services for other families. Subsidizing purchased household services encourages labor force participation of women through substitution of market- for self-provided services. It also reduces the marginal cost prices of household goods and encourages substitution of household goods for material goods. A kind of social cross-hauling occurs: when subsidies are increased and taxes raised to finance them, production of material goods declines and production of household goods increases. Women enter the market and work more in each other's households and less in the material goods sector. Efficiency distortions of current child policies in Sweden may be as large as half of total expenditures on childcare. The current 90% subsidies to public childcare are probably too large. A one percent decline in the rate of subsidy accompanied by balanced budget tax decreases would reduce the deadweight losses by one percent, at current policy.
PUBLIC EMPLOYMENT, TAXES AND THE WELFARE STATE IN SWEDEN

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I. The Issues

Public employment accounts for about one-third of employment in Sweden today. Its rapid growth reflects growth in the Welfare State. Beginning in the early 1960’s, virtually all employment growth in Sweden has been by the entry of women into the labor force working in local government jobs that service the welfare system. Fertility in Sweden is among the highest in Europe, especially considering the high female labor force participation rate.

Rising labor force participation of women and the increasing role of the state in social insurance are world-wide trends of the twentieth century. But in few other countries has the public sector grown so fast, nor achieved such a large scale relative to the economy as in Sweden and other Scandinavian countries. Public employment and public outlays are from 50 to 100 percent larger than in most developed countries. The standard of living is high in Sweden. However, the causal linkages from the welfare state to economic fortunes are tenuous. Sweden had achieved one of the highest per capita incomes in the world well before the Swedish Model was implemented. Perhaps it was the great wealth generated by the Swedish economy that allowed this model to grow and flourish, for living standards, while still high and generally growing, have eroded relative to other wealthy nations in the past two or three decades. Economic growth in Sweden has not kept pace with that of Europe generally, even

¹I am most indebted to my Swedish coworker, Henry Ohlsson, for valuable assistance and support, and regret that in the end he elected not to be coauthor. I am also indebted to Robert Lucas and Nancy Stokey for helpful discussions and to Peter Diamond, Stanley Engerman, Victor Fuchs, Assar Lindbeck, Stephan Lundgren and Birgitta Swedenborg for comments and criticism of an initial draft, though they do not necessarily agree with what remains. I am solely responsible for errors and interpretations.
excluding the severe macro economic slump of the last few years (Lindbeck, et. al., 1994)

The economics of the welfare state gives cause for concern about these trends. Government expenditures account for more than 60 percent of output in Sweden today, much larger than every other (non-Scandinavian) rich country. See table 1. By itself, there is nothing to suggest that the size of government expenditures per se affects either living standards or growth rates one way or another. What is important is that government expenditures must be financed by taxation. All taxes distort economic behavior and blunt the information content of the price system that guides individual behavior. Taxes cause private valuations of taxed goods and services to differ from their true social costs. They introduce potential inefficiencies in an economic system. The size of the public sector has to be considered from both expenditure and tax sides simultaneously to understand this point. Marginal effective tax rates for the average citizen were 70% or more a few years ago and though somewhat smaller today, remain extremely large relative to other rich countries.²

This paper analyzes how the Welfare State interacts with the economics of household. The most important finding is that the Welfare State encourages excessive production of household goods and discourages production of material goods. Too many people provide paid household (family) services for other people and not enough are employed in the production of material goods. This is what explains the growth of local government employment of women and growth of the Welfare State. A rough quantitative assessment of the distorting effects of the distorting effects of financing childcare suggests that the losses may be substantial. Direct childcare subsidies in Sweden today are approximately SEK 60,000 about ($8000) per child per year. Unless Swedish women desire to purchase substantially more childcare services than current rules allow, the estimates imply that these subsidies are too large and result

² In recent years there has been much excellent discussion and assessment of tax wedges in Sweden. See especially Hansson (1984) for a sketch of the general calculation for Sweden and Lindbeck (1993) for various analysis of the components of the welfare system. This work is closely related to those studies.
in hidden costs—shortfalls of actual from potential output in the overall Swedish economy. The estimated costs of these distortions cover a broad range, the magnitude of which depends on professional judgements of economic parameters, especially the elasticity of labor supply of women with children. These judgements differ among economists. Nonetheless, the estimates presented below imply that social costs would fall if childcare subsidies were reduced to some extent.

The role of the household is crucial in any economic analysis of the Welfare State in Sweden because that is where most State activities are centered, and it is well known that the household sector is a large component of total economic activity in all countries (Quah, 1993, Thomas, 1992). The government is not involved in public production of ordinary goods and services in Sweden. The production sector largely is in private hands and most commercial transactions are organized through private markets. Sweden maintains strong private property institutions, free markets in consumer and producer goods, and personal and political freedom that probably have insured that resources supplied to the private sector flow to their highest socially valued uses. And though private business is subject to substantial regulation, it is about on the same scale and magnitude as in other developed market economies. Where Sweden and other Scandinavian States especially differ from modern western economies is in a greatly enlarged government role in household and family activities. In essence, Sweden has “monetized” the household sector of its economy by substituting public for privately produced household services on a grand scale in the past three decades.3

The increasing market value of women’s time is the primary cause of the growth of both private and State provided household services throughout the world. Rising wages and work opportunities for women have increased the cost of staying home to produce household services oneself, and have decreased demand for it. Fertility has declined at the same time that labor force participation of women

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3Lindbeck (1988) has put it in a more dramatic way, saying that Sweden has “nationalized the family.” This view has been very influential on my thinking.
has increased in most countries. In addition, technological improvements have made market production more efficient than self-production of many household services. For instance, changing medical technology and longer life spans have increased the productivity and demand for formal medical and old-age services. The great value of skilled labor in modern technology requires that the fewer children we have be educated (by others) much more intensively than in the past. But it is exceptional that all employment growth in the Swedish Economy has been confined to the local public sector, that nearly all of it has been accounted for by women, and that female labor force participation is so large relative to fertility.

In most other countries a larger share of household activities are provided privately within the informal household sector, often in transactions that never appear in national accounts. In Sweden a large fraction of women work in the public sector to take care of the children of other women who work in the public sector to care for the parents of the women who are looking after their children. If Swedish women take care of each other’s parents in exchange for taking care of each other’s children, how much additional real output does it? In order for the State to provide services socially that otherwise would be privately produced in the family or in the private sector, many ordinary, inherently personal activities must be reckoned in explicit monetary terms, tax revenues must be raised to finance them, and complex rules and conditions must be imposed to limit undesirable side effects. At the same time that Swedish family policy encourages high fertility and large families, other aspects of the Welfare State encourage women to participate in the labor force and shift some of the costs of raising their children to others.

The next section presents some basic facts about the growth of public employment in Sweden and shows some details of how it has affected the female labor market. Section III summarizes family policies in Sweden. Section IV sketches the economics of the household and how taxes and subsidies affect behavior, while Section V presents some illustrative calculations of deadweight losses of these
policies under various assumptions. Conclusions are found in Section VI.

Before getting into the details, it is useful to state the main ideas up front. Given that the labor supply activities of women generally are thought to be sensitive to financial considerations and that Sweden has chosen the high-tax road to social welfare, the theory of the second best suggests an efficiency case for subsidizing child care and other complementary costs of labor force participation of women. Subsidies encourage market work that income taxes inefficiently discourage. High marginal income tax rates inefficiently subsidize self-production of household services because the use of one’s own time in the household is tax exempt. Women (and men) spend too much time in self-production of household services that would be more efficiently rendered by buying them in the market. For example, if the marginal tax rate is 50 percent, a woman who could earn SEK 120,000 in the labor market and has to pay SEK 60,000 for child care gets very little net monetary return from the transaction. Many would forego the market opportunity and stay at home, even though their gross earnings and social contribution to aggregate production might exceed social costs. This inefficiently suppresses what otherwise might be an active and viable private market in day care and related services. Subsidizing child care lowers the cost of female labor force participation, eliminates this distortion, and improves social welfare. But the analysis in Section IV shows something more. Such subsidies introduce other distortions because they require increased taxes on other goods to finance them. They decrease the price and excessively increase the social demand for state-provided (i.e., subsidized) household services. Women are encouraged to work too much in the state subsidized household sector, taking care of other families’ household needs, and not enough in the material goods sector. There is excessive consumption of childcare-related services. The optimality of in-kind work subsidies to women therefore comes down to balancing one distortion in household production against another in material goods consumption.

II. Trends in Public Sector Wages and Employment

Labor force surveys depict the main developments in the Swedish labor market during the period
1963-1992 for people 16-64 years of age. Labor force participation has steadily increased (Fig. 1) and is now at a very high level. Population grew at an annual average rate of 0.3 percent but the labor force increased at the rate of 0.8 percent. Employment increased on average by 0.6 percent, while the number of people working increased by only 0.4 percent per year, similar to the rate of growth of population. Temporary leaves (vacations, sick leave, parental leave, study leave) account for the difference between employment and working.

Figure 2 shows that local government jobs account for almost all employment growth in Sweden. They expanded at the rate of 4.4 percent per year. Private sector and central government employment remained essentially flat, growing at only 0.1 percent per year. Local government employment growth is, however, slowing down, averaging 8.3 percent in 1964-72, 4.9 percent during 1972-1982, and 0.9 percent during 1982-1992.

Figure 3 and Figure 4 show how the gender composition of employment has changed. Total employment of men was essentially the same in 1992 as in 1963, and the number of men in different sectors also remained constant. Two-thirds of the men have been employed in the private sector. Male central government employment has been very stable, whereas male local government employment has increased slightly. All aggregate employment growth is attributable to women. Their annual employment growth rate was 1.5 percent, and by the end of the period the number of employed women was almost the same as the number of employed men. Female employment in the private sector and in central government has been constant, so almost all employment growth in Sweden is due to the entry of women working in local government jobs. Employment growth for women was mainly in part-time jobs during the 1960s and 1970s. However, full-time employment grew and part-time employment remained constant during the 1980s (Figure 5), when annual hours worked started to increase. These trends in hours worked are the same for men and women and for the private and public sectors. Note that average

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4 The source in most cases is the Swedish Labor Force Surveys that started in 1963. The definitions of the surveys were altered somewhat in 1986.
hours worked in central government is the same as the overall market average throughout the period, and that average hours worked in local government are substantially smaller than elsewhere. This is one of the reasons why women are more frequently found in local government employment. However, women work fewer hours than men in all sectors. The difference on average is 600 hours per year (Figure 6).

Average hourly wage rates in central and local government have changed substantially relative to the private sector over the period (Fig. 7). There is a downward trend in relative public sector wages, even though employment increased substantially. The 20 percent public sector wage premium of the mid-1960s was almost extinguished by 1976. The premium increased between 1976-82 and fell during most of the 1980s. Wages of central and local government follow each other closely even though local government employment grew much faster.

Some of these movements are attributable to changes in the demographic composition of employees in the public and private sectors. Average years of schooling of workers increased in both sectors over the past 20 years. Educational attainment of public sector workers in Sweden is substantially larger than that of private sector workers, but the gap is narrowing. In 1972 public employees averaged 11.04 years of schooling and private sector workers averaged 9.35 years. By 1992 the corresponding numbers were 12.12 and 10.91, respectively. The initial 18 percent difference in educational attainment fell smoothly and uniformly over the years to 11 percent today.

Narrowing of educational differences explains some of the trend in Figure 7. Differential hiring rates, rapid growth of local government employees in the 1970s, and change in relative age structures contribute much to the rest. New hires tend to be younger workers who earn less than more experienced workers. Decreasing average age of workers is closely associated with relative employment expansions and increasing age with relative declines in employment. Average age of central government workers

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5 Average hourly wage rates are computed using data on paid out wages and salaries and total hours worked among employees. The sources are the SNEP-W data base (1965-1969) and Statistics Sweden (unpublished tables, 1970-). SNEP-Q is a quarterly econometric model of the Swedish economy developed at Uppsala University and FIEF (Trade Union Institute for Economic Research).
grew slightly over the period and the average age of private sector employees was unchanged. But in local government the average fell during 1966-80, when employment was expanding so rapidly. Average age of local government workers increased thereafter, as employment growth slowed and the day care sector expanded.

Figure 8 shows how the industrial composition of public sector employment changed. During the 1960s and 1970s, employment in medical care and education increased rapidly. Since 1980, employment in education has been constant and employment growth in medical care slowed down. Publicly provided child care for preschool children at day care centers was 2 percent of public employment in the mid-1970s, but has grown explosively ever since. Presently, employment in public day care is almost half as large as the education sector and a third of the medical care sector. It now accounts for 16 percent of public employment, not including those employed in public after-school-hour care for school children.

The enormous growth in day care employment has occurred without any increase in the relative pay of day care workers. Average monthly pay of preschool teachers is 70 percent as large as white collar manufacturing workers and 90 percent as large as blue collar manufacturing workers. There are no noticeable trends here. The pay of preschool teachers compared with female blue collar workers in manufacturing has actually decreased. How was it possible to recruit women to the local government sector? If it is not the pay, what has made the benefits exceed the costs in the labor supply calculations of women? There is no doubt that family policy programs in Sweden were crucial to these reallocations.

III. Family Policy Programs in Sweden

The increasing price of women's time is the main cause of increasing female labor force participation, in Sweden and elsewhere. However, the apparent concentration of women in local government is pronounced in Sweden and participation is large relative to fertility. The Swedish Welfare State family policies—publicly provided child care, parental leave and parental insurance, child allowances
and housing allowances, as well as the design of the income tax—have contributed to this. Sweden experienced a baby boom during the 1980s. In 1989, Sweden had the second highest fertility rate in Europe, next to Ireland.

**Personal income taxes.** Sweden changed its income tax accounting system from families to individuals. In 1966 separate individual income taxation was made optional. It was made individual in 1971, with no exemptions or deductions for dependents. This had a large effect on after-tax wages of "secondary" wage earners in families. For example, for married couples earning the average manufacturing wage, the marginal tax rate on earnings of a half-time working spouse fell from 55 percent in 1970 to 32 percent in 1971. A highly progressive individual income tax system contains strong incentives for spouses to equalize their earning, labor force participation, and hours of work.

**Publicly provided child care.** The expansion of subsidized, publicly provided child care has decreased the personal costs of labor force participation of Swedish women. Figure 9 shows how the number of preschool children and the number of them in publicly provided child care changed. Until recently virtually all day care was publicly produced. In 1983, 52 percent of preschool children were in publicly provided day care; either at day care centers, kindergartens, or in private day care homes, with "day mothers" employed by the local government. Despite the 1980's baby boom, the share of preschool children in public day care had increased to 57 percent in 1992. Many of the remaining preschool children were with parents on paid parental leave.

The central government used to pay day care subsidies to local governments, depending on the number of children enrolled. Local governments also subsidize day care. Total public sector expenditure in 1991/92 on day care subsidies was SEK 26 billion. Since 1975, families on average have paid 10 percent of the cost while the public sector has paid 90 percent. Of the latter, an increasing proportion

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6 The source is Statistiska Meddelanden, Serie S. Gustafsson and Stafford (1994) present an illuminating comparison of publicly provided day-care services across three countries.
was paid by the central government over time (Gustafsson & Stafford, 1992). Recently the system of matching central government grants to local governments has been replaced with lump sum grants. This has doubled the marginal costs of day care for local governments. The annual per-child cost was SEK 62,000, or $7,500-$10,000, using exchange rates of the past few years. These large per-child fees reflect the fact that care of small children is extremely labor intensive and that very high care quality is provided in Sweden. There are four children per server, much smaller than the student teacher ratio in elementary schools.

*Parental leave and parental cash benefits.* Paid maternity leave was introduced in 1955 when 3 months were paid. Presently, 15 months are paid. The system encourages women to establish an earnings history before having children, because the parental cash benefit depends on previous earnings. It also encourages women to postpone bearing children if earnings are increasing, and to space children more closely. Compensation is at least as large as for the previous child if the next child is born within 30 months. Otherwise it is lower. The compensation can be obtained until the child is 8 years old, so almost all expenditure concerns preschool children. Total expenditure in 1991/92 was SEK 18 billion. The compensation is taxable. Assuming that everyone has the lowest marginal tax rate, the net expenditure for the public sector is SEK 13 billion.

*Child allowances.* Beginning in 1948, the central government has paid fixed monthly child allowances to children under 16 years of age. The allowance was roughly SEK 800 per month or SEK 10,000 per year in 1991/92. Total expenditure was SEK 17 billion. The per-child allowance is increased by 50 percent from the third child and on. It is not taxed.

*Housing allowances.* Housing allowances are means tested; depending on family income, number of children, and housing costs. For all practical purposes these are equivalent to a means-tested child allowance. Central and local government pay 50 percent each. Total expenditure in 1991/92 was SEK 5 billion. The allowance is not taxed.
Summary. An approximate estimate of total public expenditure on programs for preschool children is summarized in table 2. Total annual public sector tax expenditure on preschool children was SEK 48 billion, corresponding to SEK 60,000 per preschool child per year ($8,000). In Spring, 1994 the majority in Parliament decided to introduce a child care allowance. Parents with a child 1-3 years of age will get SEK 2,000 per month provided that the child is not in publically provided day care. The allowance will be taxed. Estimated annual expenditure is SEK 3.5 billion.

IV. Household Welfare Economics

Broadly speaking the programs described above have two main components. One is payment from general tax revenues for childbirth and parental home care of infants and very young children. The other is subsidized care of preschool children outside the home. These policies were designed to increase the fertility of Swedish women and to tilt the allocation of their time toward market rather than nonmarket uses (Sundstrom and Stafford, 1992). Apparently they have achieved their goals. Some of the economic consequences on the allocation of time are analyzed in this section. Fertility aspects have been more extensively analyzed by others (Aronsson and Walker, 1994; Walker 1992).

The point of departure is a well-known result from the theory of the second best. Subsidizing purchased inputs in household production to reduce the costs of labor force participation improves social efficiency when substantial income tax distortions inefficiently deter market work incentives. What has been missed in the prior discussion is that they also reduce the relative cost of household goods and encourage socially excessive market production of household goods at the expense of material goods. Too many people are involved in the household production of other families, and too few are in the production of nonhousehold goods and services. This second effect does not necessarily mean that household subsidies are inappropriate. Rather, assessing the consequences of policy requires balancing one

\[7\] Child support advances, another program affecting families, are not included. The central government serves as an intermediary between divorced parents. If a parent does not pay child support or the (income based) support is below a certain threshold, the central government advances basic support. The expenditure of this program was SEK 3 billion in 1991/92.
distortion against another. These issues are examined in more detail below, building upon household production theory (Becker 1965, Gronau 1977, Lindbeck 1982) and the economics of the second best (Sandmo 1990).

A. The Allocation of Time.

This section sets the basic model and notation (see appendix for complete notation and other details). Consider an economy with two classes of goods: \(x\) represents "material" goods and services that are produced in firms and transacted in markets; and \(z\) is household goods that are self-produced by combining own time with purchased inputs. Consumer preferences over goods \(x\) and \(z\), are represented by the utility function \(u = u(x,z)\). The material good \(x\) is produced by labor services hired in a market (along with capital and other inputs, suppressed here) under constant returns. The self-production function for household goods is \(z = f(h,M)\), where \(h\) is own time devoted to the household and \(M\) is a market good, best interpreted as the hired time of others. Household production is also assumed to exhibit constant returns.

This specification of tastes is restrictive in assigning purely instrumental roles for time used in \(x\) and \(z\) production. Time spent in direct contact with one's own children, for example, is just treated as an imperfect substitute for purchased inputs and has no utility value in and of itself. This specification biases the case in favor of work-cost subsidies because parental love of children naturally acts to "subsidize" household production; its full implicit price includes the opportunity cost of time minus the value of the direct marginal utility of \(h\).

Let \(t\) be the amount of time supplied to the labor market, \(w\) the market price of time, and \(p\) the price of purchased \(M\) services. Taking \(x\) as numeraire and normalizing the total amount of time at unity, the time-budget constraint is \(t + h = 1\). The financial-budget constraint defining income available for taxation is \(wt = x + pM\). Combining these gives \(w = x + wh + pM\); full income \((w)\) can be spent to purchase material goods in the market, own time for use in the household and the market services of household inputs.
1. Structure of Demand

It is useful to solve the consumer's problem in two steps. First fix \( z \) and combine \( h \) and \( M \) to minimize production costs. Second, given the cost of \( z \), the consumer chooses \( x \) and \( z \) to maximize \( u(x,z) \).

The household rationally charges itself the market opportunity costs of time in assessing the true cost of \( z \). With constant returns and homogeneity the cost function is:

\[
q(w,p,z) = \min \{wh + pM + \lambda(z - f(h,M))\}
\]

where \( q(w,p) = \lambda \) is both marginal and average cost of \( z \), increasing in both \( w \) and \( p \). Differentiating total cost with respect to \( w \) and \( p \) gives input demand functions that are separable in output and factor prices: \( h = zq_h(w,p) \) and \( M = zq_p(w,p) \). The constraint for the second problem is \( I = x + q(w,p)z \), where \( I = w \) is full income in this case. The consumer chooses \( x \) and \( z \) to maximize utility. The indirect utility function is defined by:

\[
G(w,p) = \max \{u(x,z) + \mu[I - x - q(w,p)z]\}
\]

from which ordinary consumer demand functions \( x = x(I,q) \) and \( z = z(I,q) \) follow.

Taxes and subsidies affect net wages and prices seen by consumers. The virtue of this roundabout construction lies in decomposing the effects of tax-distorted price changes into two kinds of substitution and income effects, one for production and the other for consumption. Substitute household good demand \( z(I,q) \) into the input demands for \( h \) and \( M \) and note that \( I \) and \( q \) depend on \( w \) and \( p \) from cost minimization. Then repeated application of the scale and substitution decomposition in the derived

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\(^8\) I have chosen to formulate the problem in the traditional way, examining choices and distortions at the intensive margin in a representative agent model. Bergstrom and Blomquist (1993) outline the approach for studying choices at the extensive margin among heterogeneous agents for this problem.
demands for h and M and the adding-up rule yields the following elasticities (see appendix):

\[ \eta_{hp} = (1-\theta)(\sigma_p + \eta_{zq}) \quad \eta_{hp} = -\theta \sigma_p + (1-\theta)\eta_{zq} \]

\[ \eta_{hw} = -(1-\theta)\sigma_p + \theta \eta_{zq} + \eta_{zq} \quad \eta_{hw} = \theta \sigma_p + \theta \eta_{zq} + \eta_{zq} \]

(3)

where \( \eta_j \) is the uncompensated demand elasticity of variable \( j \) with respect to \( j \), \( \theta = \text{wh}/\text{qz} \) is the cost share of own time in the production of \( z \), and \( \sigma_p \) is the elasticity of substitution between \( h \) and \( M \) in \( f(h,M) = z \) production. The first and second terms in each of the expressions in (3) represent the direct and indirect effects of factor price changes. The terms in \( \sigma_p \) reflect direct substitution between \( h \) and \( M \) in \( z \) production when relative factor prices change. The terms in \( \eta_{zq} \) reflect indirect changes in factor demand induced by scale effects, because factor-price changes alter the shadow price of \( z \) and change the consumption demand for \( z \) relative to \( x \). The third terms in the wage elasticities reflect an additional income effect on the individual demand for \( z \), because changes in \( w \) change full income.

2. Production and supply.

Assume that \( x \) and \( M \) production are linear in their (time) inputs. Write \( t = m + \ell \), where \( m \) is time supplied to produce good \( M \) and \( \ell \) is time supplied to produce good \( x \). Choose units so that \( x = \ell \). Then \( M = \alpha m \), where \( \alpha \) is a constant reflecting the number of children per day-care mother (\( \alpha = 4 \) in Sweden). The model should be extended to consider substitution of quality for quantity of purchased services in the household, but that is not pursued here. To a first order approximation, the total quantity responses in this model can be interpreted as the combined effect of quantity and quality.

Since time spent in \( M \) or \( x \) production are assumed to be effort-equivalent, they must pay the same hourly wage \( w \) in a competitive market. The competitive supply price of \( M \) is its marginal cost of production, or \( p = w/\alpha \), about one-quarter the market wage in Sweden (ignoring the 14 percent share of capital costs in Swedish Day Care Centers (Schwartz and Weinber, 1993)). The marginal product of
labor in material goods production is 1.0 and x is the numeraire, so \( w = 1 \) in a competitive equilibrium. At these prices the first-order conditions associated with (1) and (2) are feasible and their solution describes the competitive equilibrium. Think about it as follows. Imagine an economy with a large number (a continuum) of identical households. Then all make the same choice of \( x, z \) and \( h \) in equilibrium. Aggregate markets for \( x \) and \( M \) are cleared when the required fraction of workers supply all their market work time to \( x \) production, and the remainder supply all of their market work time to \( M \) production.

B. The Effects of Taxes and Subsidies

Following Sandmo (1990), the model is modified to include government expenditure and taxes. Suppose the government must raise revenue of amount \( g \) and that nondistorting poll taxes are not available. Instead, \( g \) must be financed by taxing market income (income taxes), material goods production (VAT or sales taxes), or the value of market inputs in home production (generally a subsidy). With three market goods—labor, material goods and purchased household services—and the requirement that the government balance its budget, there are only two independent tax instruments: VAT taxes are treated as redundant here. As usual, \( g \) is treated as exogenously determined to isolate the pure efficiency aspects of taxes. It is redistributed to consumers as lump-sum transfers of \( x \).

Let \( \tau \) be the rate of income tax per unit of market-supplied labor and let \( \rho \) be the unit tax (if positive) or subsidy (if negative) on \( M \), the purchased input in household production. The government collects revenues from two sources, \( \tau(1-h) \) from income taxation, where \( 1-h = t \) is total time supplied to the market sector, and \( \rho M \) from taxing or subsidizing marketed household inputs. The government budget constraint is

\[
(4) \quad g = \tau(1-h) + \rho M.
\]

The consumer's budget constraint becomes
\[(5) \quad (w - \tau)(1-h) = x + (p + \rho)M,\]

from which the social budget constraint follows:

\[(6) \quad w = x + pM + wh + g.\]

In the competitive equilibrium with taxes \(w\) and \(p\) remain fixed at \(w=1\) and \(p=1/\alpha\) from the linear cost assumptions.

There are inefficient tax wedges between private and social valuations. An interesting positive question is, given \(g\), what happens when the subsidy is increased slightly and the income tax simultaneously increased to finance it? If the subsidy is increased, taxes must be raised by just enough to balance the budget after consumers have made all behavioral adjustments to the new situation, satisfying their personal budgets in (5). However, to the first order all of these secondary repercussions cancel out along the social budget in (6). What remains is the condition that socially feasible changes in taxes and subsidies must satisfy the Slutsky-like condition

\[(7) \quad (1-h)\frac{d\tau}{\rho} + Md\rho = 0\]

In fact, all tax and subsidy variations satisfying equation (7) imply constant utility, so that income effects on \(x\) and \(z\) in consumption are washed out in this experiment (see appendix).

The behavioral effects of this experiment are found by recomputing the elasticities in (3) under the additional constraint that when the subsidy changes the price of \(M\), the income tax changes to satisfy (7). For example, the differential \(dh\) in the comparative statics now has two terms instead of one: \(dh = [(\partial h/\partial w)(dw/d\tau) + (\partial h/\partial \rho)(\partial p/\partial \rho)]d\rho\). Making all the substitutions, repeatedly applying the Slutsky decomposition, exploiting the constant supply price technology, and converting to elasticities
ultimately yields

$$(d\log M/d\log \rho)_{\text{budget balance}} = -(1-\phi)[\theta \sigma_p + (1-\theta)\sigma_z]/(1-h)$$

(8)

$$(d\log h/d\log \rho)_{\text{budget balance}} = (1-\theta)(1-\phi)\sigma_p \sigma_z]/(1-h)$$

Here $\phi = qz/l$ is the budget share of $z$ in total consumption and $\sigma_z$ is the elasticity of substitution between $x$ and $z$ in consumption in $u(x,z)$. Increasing the subsidy on purchased household inputs and financing it by increased income taxation reduces the price of $M$ seen by households and increases demand. Family subsidies encourage households to substitute $M$ for $h$ in household production and to substitute $z$ for $x$ in consumption. Both work in the same direction to increase the derived demand for $M$. They work in opposite directions on the demand for $h$: consumption substitution effects increase the demand for own time in the household, but production substitution effects reduce it. The net change in $h$ can go either way, depending on which kind of substitution is greater.

Cost minimization implies that $d\log z = \theta d\log h + (1-\theta)d\log M$. Substituting from (8) gives

$$(d\log z/d\log \rho)_{\text{budget balance}} = -(1-\theta)\sigma_z[1-\phi(1-\theta)]/(1-h)$$

(9)

$$(d\log x/d\log \rho)_{\text{budget balance}} = d\log t/d\log \rho)_{\text{budget balance}} = -(d\log (m+h)/d\log \rho)_{\text{budget balance}}$$

$$= -[\phi/(1-\phi)](d\log z/d\log \rho)_{\text{budget balance}}$$

The first expression in (9) proves that $z$ must increase in this budget balancing experiment. The second equation indicates how the composition of market output and the allocation of time are altered. Material goods production and the time allocated to it must decrease. Cross-hauling is a necessary outcome: total
time allocated to household production in the economy unambiguously increases. Output of material goods falls.

The change in the composition of household time is slightly more complicated. From (8) the amount of market-purchased household time (m) increases. Because the effects of substitution in production and substitution in consumption work in opposite directions on the derived demand for own household time, h can either rise or fall, but even if it falls the amount of hired household time must increase by more. Certainly, subsidies encourage work outside the home. But there is a sense in which all of it is work in someone else's home, not in the material goods sector. Parents work for each other for taxable pay needed to help finance the subsidies that induce them to work for each other in the first place, rather than remain working for themselves, "self-employed," in the tax-sheltered nonmarket household sector. Growth in public employment in the Welfare State is a predictable economic consequence of substitution of State subsidized services for own-provided services.

This experiment has been constructed so that economic welfare remains constant along the way, and the resulting reallocations have no incremental social economic value. Nevertheless, measured national income changes. In this economy, real national income at constant prices is \( x + pM = wt = 1-h \), so the sign of the change in NI is the negative of the sign of dh. From (8), measured national income increases or decreases as \( \sigma_p \) is greater or less than \( \sigma_c \). When \( \sigma_c > \sigma_p \), measured national income is actually reduced by family subsidies.

C. Optimal Taxes and Subsidies

Consider next the "optimal" tax-subsidy scheme, where the government raises the given revenue \( g \) at the least social efficiency cost. We seek tax rates \( \tau \) and \( \rho \) that maximize utility subject to the government's budget constraint, i.e., that maximize

\[
G(w-\tau, p+\rho) + \psi[g - (1-h)\tau - M\rho]
\]
where $G$ is the indirect utility function defined in (2) subject now to the constraint in (5) and $u$ is a Lagrange multiplier. It is understood that $w$ and $p$ in (10) are fixed at their general equilibrium supply prices in the economy. First order conditions are

$$-G_u - \nu[(1-h) - \tau \cdot \partial(1-h)/\partial w - \rho \cdot \partial M/\partial w] = 0$$

(11)

$$G_p - \nu[M + \tau \cdot \partial(1-h)/\partial p + \rho \cdot \partial M/\partial p] = 0$$

Convert the derivatives in (11) to elasticities, substitute from (3) and solve the two linear equations for $\tau$ and $\rho$. Recalling that $\mu$ is the marginal utility of money in the consumer’s problem, the result is

$$\tau = \nu^i(\mu + \nu)[\theta \sigma_y + (1 - \theta)\sigma_p] / \theta \phi \sigma_p(\sigma_c - \eta_a)$$

(12)

$$\rho = -\nu^i(\mu + \nu)(\sigma_p - \sigma_c) / \alpha \phi \sigma_p(\sigma_c - \eta_a)$$

Assume that the M-sector is small relative to $x$ and $g$ so that $\tau > 0$ is necessary for government finances. The expression for $\tau$ in (12) shows that the optimal income tax approximately is a weighted average of the inverses of the two substitution elasticities, consistent with standard economic intuition that optimal tax rates are smaller when substitution is greater.

The expression for $\rho$ in (12) is much different than the expression for $\tau$. It depends on the difference between the two kinds of substitution effects. If $\sigma_p = \sigma_c$, it is best not to subsidize (or tax) market inputs in household production at all. A subsidy is warranted only when $\sigma_p > \sigma_c$, that is, when

---

9 As usual for this problem taxes are written in absolute rather than percentage terms without loss of generality. Units always can be chosen to normalize equilibrium prices at unity, so taxes and subsidies have an ad valorem interpretation. On this see Atkinson and Stiglitz (1980) and Harberger (1971).
the ability to substitute own time for purchased time in household production is greater than the ability to substitute material goods for household goods in consumption. If \( \sigma_o > \sigma_p \), hired substitutes for self-production in the home should be taxed extra, to discourage their use, not subsidized and nationalized.

D. Deadweight Losses

The formulas in (12) above illustrate the main point of this analysis; that the second-best optimality of household subsidies depend on a delicate comparison of substitution effects. Since the childcare sector (M) is a small component of the economy and so many other factors are involved in the setting of taxes and social welfare policy, we present them only to make the analytical point as sharply as possible. Harberger Triangles (1964) are the best available tool for assessing the empirical magnitude of the resulting distortions.

Define the expenditure function \( S(w, p; u) \) as the minimum expenditure \( x + qz \) necessary to achieve a given level of utility. The compensating variation is found by expanding \( S(w, p) \) in Taylor's series up to second order, ignoring remainder terms, and using duality theory to express the first and second derivatives of \( S \) as Hicksian demand functions and their derivatives (substitution effects only). Converting to elasticities using the relations in (3) yields

\[
(13) \quad \text{Absolute Deadweight Loss} = \frac{qz\{\theta(1-\theta)\sigma_p[\tau + \rho]^2 + (1-\phi)\sigma_q[\theta \tau - (1-\theta)\rho]^2\}}{2}
\]

where \( \tau \) and \( \rho \) are interpreted as percentage rates of tax or subsidy.

(13) captures the efficiency tradeoff in a very direct way, depicted in Figure 10. Differential taxes and subsidies cause distortions in household production. They shrink the production set to the line marked AB. If this was all there was to it, the consumer would choose point A. The production
distortion shown in the figure is measured by the term in \( \sigma_e \) in (13). However, taxes and subsidies reduce the implicit price of household production below its (distorted) opportunity cost. This causes consumers to choose B instead of A. The resulting consumption distortion in the figure is measured by the term in \( \sigma_e \) in (13). The total distortion is the sum of these two effects.

Subsidies imply that \( \rho \) is negative in (13). If the percentage marginal subsidy is set equal to the marginal income tax rate, then all welfare distortions in household production are eliminated and the term in \( \sigma_e \) vanishes, exactly the second-best intuition. However, the subsidy necessarily increases the distortion in the relative allocation of time between material goods and household goods, and the terms multiplying \( \sigma_e \) in (13) become larger. These drawbacks of subsidies have to be weighed against their virtues.

The applicability of this model for assessing welfare distortions in Sweden has been questioned, on the grounds that subsidized child-care is not available in unlimited supply to an individual buyer. Rather, women may be off the margin and the State may implicitly ration it, e.g., by only making it available when women are at work. If so, then a quantity constraint is imposed on the choice problem modeled above. If the quantity constraint is not binding the analysis is unaffected. If it is binding, then the formula in (13) has be applied to a "virtual" subsidy rate. That is, it is necessary to find the unrestricted subsidy that would voluntarily induce women to freely choose the rationed quantity and apply (13) to those rates of subsidy. If Swedish women are not getting all the child-care they desire at the actual subsidized prices, it is clear that the appropriate "virtual" subsidies are smaller than .90 used in table 3 and the deadweight losses calculated there are too large. For example, if the State were sufficiently well informed, it could impose the quantity ration that minimized distortions (of course if it had that information it could achieve the same result by choosing the unrestricted subsidy properly).

This point has merit, but fully assessing it requires ascertaining the extent to which Swedish women are rationed in their use of subsidized public child-care. To be sure, subsidized care largely is tied to job-holding of women in Sweden, and it is said that women use all that is made available to them.
However, this is not decisive on rationing in my judgement because it does not tell us what they would use if unconstrained. After all, child care of the kind in question largely is associated with job-holding, even in countries where it is not provided through the public sector. It is not implausible that the demand price for systematic, day-in and day-out child care may fall off sharply during after-work hours and weekends. Furthermore, women typically work fewer hours than men, in Sweden and elsewhere, and many Swedish women choose not to work full-time, often for family related reasons. Certainly rations don’t bind in these cases. Finally there is the extensive margin to consider. Most of the response of women’s labor supply to market opportunities is known to arise at the participation margin. However, all or nothing work/not work decisions are also distorted by income taxes and subsidies. An analysis along the lines of Bergstrom and Blomquist, 1993 would imply supply and demand curves much like the analysis presented here, and formula (13) also applies to them. I conjecture that quantitative assessment of welfare distortions using participation analysis among heterogeneous agents must produce similar numbers as those in table 3, because that is where most of the empirical data on female labor supply parameters come from. This, however, is a subject for future research.

V. A Calculation for Sweden

Combining income taxes, payroll taxes, and value-added taxes, the average marginal income tax wedge in Sweden today is in the 50-65 percent range, down from 65-80 percent of a few years ago, but still one of the largest in the democratic world. Taxes of this magnitude cause families to overuse own inputs in household production. Large subsidies to purchased household inputs are necessary to correct these distortions in household production. Since Swedish local governments pay approximately 90 percent of the total costs of day care and home time (leave from work) of mothers with very small children, the average marginal subsidy also must be about 0.9.

It is important to notice that the empirical weight of the terms multiplying \( c_r \) in (13) for Sweden must be much smaller than the weight on \( c_e \), because the share weighted difference in the absolute values
of marginal tax and subsidy rates is much smaller than their sum. The share of own time ($\theta$) in household production involving small children is substantial, even for full-time labor force participants. Whatever it is, the maximum possible value of $\theta(1-\theta)$ in the first term of (13) is 0.25. Using the large tax and subsidy rates at the upper limits of the ranges in the paragraph above implies $[\tau + \rho]^2 = 0.04$, so $\theta(1-\theta)[\tau + \rho]^2$ multiplying the term in $\sigma_r$ is .01 at most. But $[\theta \tau - (1-\theta)\rho]^2$, the coefficient multiplying $\sigma_r$ in (13) is 0.065 with these same tax parameters assuming, conservatively, that $\theta = 1/2$. Furthermore, $(1-\phi)$, the share of material goods in full income, must be substantial, at least 0.75, considering that $z$ is confined to preschool children activities here. The net result is a coefficient on $\sigma_r$ in (13) of 0.25, at least 25 times larger than the coefficient on $\sigma_c$. Unless $\sigma_r$ is extremely large relative to $\sigma_c$, the welfare loss calculation for Sweden must be much more sensitive to $\sigma_c$ than to $\sigma_r$.

The division of the model economy into material and household goods sectors does not map into direct econometric estimates of $\sigma_c$ and $\sigma_r$. However, estimates can be backed out of the formulas in (3), since the elasticity of market labor supply is $\eta_{hw} = \delta \log(1-h)/\delta \log w = -(h/1-h)\eta_{hw}$. This and the Slutsky decompositions in imply

$$[-(h/1-h)]\eta_{hw} = (1-\theta)\sigma_p + \theta(1-\phi)\sigma_c - (1-\theta\phi)\eta_{st}$$

The wage elasticity of female labor supply $\eta_{hw}$ in Sweden is in the range $[0.1,0.9]$ (Blomquist and Hansson-Brusewitz, 1990 and Aronsson and Walker, 1994). Economic growth and increasing income everywhere are associated with relative expansion of the material goods sector and relative contraction of the household sector, implying $\eta_{st} < 1.0$. However, the declining share of the household sector has been affected by technical changes in both sectors, so the true income elasticity probably is greater than what is implied by trends alone. Certainly it is no larger than unity. We use $\eta_{st} = 1.0$ here. Working mothers with small children spend as much of their time in own household production of child services
as in the labor market. Splitting their time 50-50, so $1-h/h = 1.0$, and using the values for $\theta (= 1/2)$, and $\phi (= 1/4)$ above in (14) gives a linear equation restricting $\sigma_p$ and $\sigma_c$ for a given value of $\eta_w$. The possibilities are shown in table 3 in the columns labeled $\sigma_c$. Each of three possible female labor supply elasticities 1/3, 2/3, and 1.0 within the empirical range, combined with each of the four alternative values of $\sigma_p$ implies an estimate of $\sigma_c$. For instance, if $\sigma_p = 1.0$ and the labor supply elasticity is 1/3, then equation (14) requires $\sigma_c = 1.88$. It requires that $\sigma_c = 3.67$ if $\sigma_p = 1.0$ and the labor supply elasticity $\eta_w$ is unity.

The columns of table 3 headed DWL show the estimated deadweight loss in equation (13), expressed as a fraction of $qz$ for marginal tax and subsidy rates of .70 and .90 respectively. State subsidies for child care in Sweden are about SEK 50 billion (see table 2) so $qz = \text{SEK 5500}$ per child is a minimum bound on $qz$ per child because it does not include any imputed values for either parental time or material inputs into $z$ production. To illustrate, if the labor supply elasticity is 0.33 and $\sigma_p = 1.0$ the deadweight loss is $0.46(qz)$, on the order of SEK 25 billion or SEK 32,000 (roughly $4000) per child. The estimates are sensitive to the assumed decomposition of labor supply elasticity into its $\sigma_c$ and $\sigma_p$ components in (14), but almost all of them are positive. Note also that most of these numbers are large, on the order of half or more of government child care related expenditures. In assessing the plausibility of table 3, readers might compare them with Hansson’s (1984) larger estimates of deadweight losses for other tax distortions in Sweden.

Table 3 reveals two strong regular patterns in the calculated deadweight losses. First, the estimated deadweight loss falls if $\sigma_c$ is larger and $\sigma_p$ is smaller, for each labor supply elasticity. The reason is that the currently large taxes and subsidies eliminate a small production distortion when $\sigma_p$ is small and create a large consumption distortion when $\sigma_c$ is large. Second, the distortion is larger the larger the labor supply elasticity, because large labor supply elasticities imply greater substitution elasticities. Only if $\sigma_p$ is relatively large and female supply elasticities relatively small are the welfare
distortions in table 3 of no economic significance.

Of course, substantial portions of the DWL multipliers in table 3 are attributable to the high marginal income tax rates, not to child-care subsidies per se. Nonetheless, there is evidence that child-care subsidies are too high in Sweden today. Consider an experiment where the subsidy is reduced a little and the marginal tax rate also is reduced by the amount required to maintain budget balance. Expressing the taxes and subsidies as percentages, equation (7) and the budget constraints imply that

\begin{equation}
(15) \quad d\tau = \left[\frac{(1-\theta)/(1 - \theta \phi)}{d\rho}\right]d\rho
\end{equation}

is necessary for government finances.

Totally differentiate equation (13) and substitute (15). Evaluating the resulting equation at \( \theta = 1/2, \phi = 1/4, \tau = .7 \) and \( \rho = -.9 \), the parameters used in table 3, yields the gradient

\begin{equation}
(16) \quad \delta D = (\partial \log \text{DWL}/\partial \log \rho)_{\text{budget balance}} = -0.0571 \sigma_p - 0.241 \sigma_c
\end{equation}

Since the two substitution elasticities are positive, equation (16) must be negative. Therefore if the subsidy is reduced a little (e.g. from -.9 to -.8, so \( d\rho \) is positive), the deadweight losses in table 3 decrease and it can be concluded that the current subsidy is too large.

The percentage rate of decline in (16) is calculated for corresponding values of the substitution parameters in the column labeled \( \delta D \) in table 3. Remarkably, the estimates cluster around unity for most possible parameter values (except when DWL is itself quite small, where the efficiency gain from lowering the subsidy is estimated as much larger because the denominator DWL is itself small). To a first approximation, the estimates in table 3 strongly suggest that the dead weight loss is locally linearly declining in \( |\rho| \), so long as budget balance is maintained. For example, a ten percent reduction in the
subsidy from its current level of -0.90 to -0.81 would reduce the deadweight loss, whatever it is, by about 10 percent.

Remember that these derivatives apply in a neighborhood around current tax and subsidy rates. Were the experiment actually implemented, the gradient would change because budget shares and elasticities of substitution would change. It cannot be ascertained theoretically whether the second derivative of (16) is positive, negative, or zero, nor is there enough empirical evidence to make an educated guess. Hence caution must be exercised in extrapolating reductions in the subsidy beyond, say, 10 percent or so. Local linearity does not imply that total elimination of child-care subsidies would remove the dead weight loss of high income taxation in Sweden! Rather, the estimates in table 3 imply that welfare of the average family would improve if the subsidy were reduced because substitution in consumption is heavily distorted under current policy. It is entirely possible that, were we to start from a baseline of no subsidy, an increase in the subsidy would have improved welfare because the household production margin would be so heavily distorted.

A case can be made that the numbers in the upper right hand corner are likely the most relevant for Sweden today.

First, child-care tax and subsidy distortions largely work on the female labor supply margin and it is well known that the wage elasticity of female labor supply is much larger than that of males. The estimates in the labor economics literature vary depending on whether participation as well as hours of work are included, but a value of $\eta_w$ pushing toward 1.0 certainly is well within the range of estimates found over the years for women in many countries. The most sophisticated estimates for Sweden only examine a restricted range of variation in panel data and are sensitive to specification. For instance, sick leave policy and switching the tax basis from family to individual accounts are thought to have had a large effect on female labor supply.

Second and more speculatively, there is reason to think that the elasticity of substitution in
production, \( \sigma_p \), might be small at current time allocations in Sweden. The family leave policy implicitly recognizes that hired help is a poor substitute for full parental time of very young children. The argument can be extended to older, preschool children by imagining a hierarchy of uses of adult time devoted to children, with parents allocating their own time to "higher quality" uses and hiring the time of others for "lower quality" uses.\(^{10}\) Hired time in Sweden is so large that even more of it would be a very poor substitute for parental time. If this is true, the deadweight losses in Table 3 are half or more of total spending, or upward of SEK 30,000 ($4,000) per child per year.

V. Conclusion

The economic measurement of welfare losses quantifies tax distortions between private and social allocations, and assesses their aggregate social costs in monetary terms. These calculations suggest sizable efficiency losses caused by the marginal taxes and subsidies needed to implement the Welfare State. Large estimated losses are practically inevitable given standard economic methods and the enormous taxes found in Sweden. Evidently Swedish citizens regard the social value of the Welfare State and related egalitarian policies as worth their social costs. Nevertheless, it is worthwhile every now and then to assess how large those costs might be.

Available methods do not recognize the formal linkages between work and social claims that allow the Swedish economy to function on reasonably workable terms. From an economic point of view these links may be analytically equivalent to a multi-part tariff form of government finance. If so, in implicitly extrapolating the marginal tax and subsidy distortions all the way back to the hypothetical efficient allocations, the welfare triangle calculation in (13) may overstate the total amount of wasted resources.

\(^{10}\) Think of a continuum of child care activities, distinct in terms of the ratio at which parental time can be substituted by purchased time. Denote this ratio by \( r(s) \) for activity \( s \), and choose \( s \) so that \( r(s) \) is ordered from highest to lowest. If the relative price of purchased time compared to own time is \( \rho \), the household will purchase time for all activities for which \( r(s) \geq \rho \) and use own time for those satisfying \( r(s) < \rho \). The marginal rate of substitution for own and purchased time is \( \rho = r(s^*) \) and \( \sigma_p \) can be shown to depend on the curvature of \( r(s) \) in the neighborhood of \( s^* \). See Rosen (1978) for details.
in Sweden. Whatever that case may be, there is no getting around the staggering marginal taxes and subsidies inherent in the current system. These have to create substantial inefficiencies at the margin.

In many ways and at least in the aggregate, government-provided services replace what would have been purchased in other, more decentralized ways without the associated tax burdens. The fundamental manifestations of these costs are tendencies to over-consume subsidized government-provided goods, and to engage excessively in personal activities that are beyond the reach of the tax collectors. By reducing the linkages between personal contributions to production and claims on social output, the Welfare State encourages people to produce utility in ways that don’t have to be shared with others. The real household sector in Sweden is too large on both counts. The monetization of subsidized household services provided through the subsidized State bureaucracy increases the demand for publicly provided services and the size of the public sector, but reduces the value of social output and living standards in the overall economy. Total output is smaller than it would have been if household services had been paid privately and transacted through the market.
REFERENCES


1. Notation

h: time spent in household self-production.
m: time spent working in market household sector.
l: time spent working in material goods sector.
t = m + l: total time spent working in the market sector.
x: market good.
z: good produced in household.
M: purchased inputs in household production.
p: price of M.
w: wage rate.
q: average and marginal cost of z.
I: full income.
θ: cost share of own labor in household production.
1-θ: cost share of M in household production.
φ: budget share of z in consumption.
1-φ: budget share of x in consumption.
η_m: uncompensated own price elasticity of demand for z.
η_d: income elasticity of demand for z.
σ_p: elasticity of substitution between h and M in production.
σ_c: elasticity of substitution between x and z in consumption.
g: government revenue.
τ: unit income taxation rate.
ρ: unit tax or subsidy on M.
2. Elasticities

Equations (1) and (2) imply the following system of equations

\[ q = Q(w, p) \quad \text{marginal cost of } z \]
\[ h = H(w, p, z) \quad \text{derived demand for } h \]
\[ M = M(w, p, z) \quad \text{derived demand for } M \]
\[ z = Z(q, l) \quad \text{consumer demand for } z \]

When \( w \) or \( p \) changes, optimal factor proportions change. This is the production substitution effect. However, factor price changes affect \( q \) (and \( l \)) and also change optimal consumption. This is the consumption substitution effect.

For example

\[ \frac{\partial h}{\partial w} = H_w + H_z (Z_q Q_w + Z_l \frac{\partial l}{\partial w}) \]

From the Slutsky decomposition

\[ Z_q = Z_q^s - zZ_l \]

Constant returns implies \( H_z = Q_w = h/z \). Noting the \( l = w \) so that \( \partial h/\partial w = 1 \) and making all the substitutions

\[ \frac{\partial h}{\partial w} = H_w + (h/z)^2 Z_q^s + (h/z)Z_l (1-h) \]

Finally, noting that \( H_w = -(1-\theta)\sigma_z h/w \) and \( Z_q^s = -(1-\phi)\sigma_z z/q \) and \( \eta_{zl} = Z_l (l/z) \), we have

\[ \eta_{hw} = (w/h)(\partial h/\partial w) = -(1-\theta)\sigma_z - \theta(1-\phi)\sigma_z + (1-\theta\phi)\eta_{zl} \]

Note in (3) that \( \eta_{eq} = -(1-\phi)\sigma_z - \phi \eta_{zl} \). The other formulas in (3) are derived in the same way.

3. Cross-hauling

The consumer sees the budget
\[ w^*(1-h) = x + p*M \]

where \( w^* = w - \tau \) and \( p^* = p + \rho \). Differentiating, the demand functions must satisfy

\[
(1-h) \frac{\partial h}{\partial w^*} - \frac{\partial x}{\partial w^*} - p^* \frac{\partial M}{\partial w^*} = 0
\]

\[
M + p^* \frac{\partial M}{\partial p^*} + \frac{\partial x}{\partial p^*} + w^* \frac{\partial h}{\partial w^*} = 0
\]

However, the social budget is

\[ wh + x + pM = w - g \]

where \( w \) and \( p \) are fixed at their constant supply prices. Totally differentiating the social budget and noting that \( \rho M + \tau(1-h) = g \),

\[
\left( w^* \frac{\partial h}{\partial w^*} + \frac{\partial x}{\partial w^*} + p^* \frac{\partial M}{\partial w^*} \right) dw^* d\tau + \left( w^* \frac{\partial h}{\partial p^*} + \frac{\partial x}{\partial p^*} + p^* \frac{\partial M}{\partial p^*} \right) dp^* d\rho = 0
\]

so

\[
(1-h)d\tau + M d\rho = 0
\]

keeps the budget balanced. Equations (8) and (9) in the text follow from the expressions in equation (3) and the condition that \( d\tau = -[M/(1-h)]d\rho \)

Equation (2) implies

\[
du = dG = -\lambda d\tau - \lambda \left( -\frac{\partial q}{\partial w^*} d\tau + \frac{\partial q}{\partial p^*} dp^* \right) = -\lambda [(1-h)d\tau + M d\rho]
\]

so \( du = 0 \) in this experiment, as asserted in the text.

4. Separability

It is well known that much of the power of the household production model derives from its separability assumptions. For example, the income elasticities of demand for \( h \) and \( M \) are identical and so are the partial elasticities of substitution \( \sigma_{hx} = \sigma_{hx} \). These restrictions are relaxed by using the general
utility function $u = u(x, h, M)$ with budget $x + wh + pM = w$. Expanding the associated expenditure function, the deadweight loss formula becomes

$$
\phi(1-\theta)((1-\phi)[\sigma_{xx}x^2 + \phi\theta\sigma_{hx}x^2] + \phi\theta\sigma_{hh}(h+\tau)^2)qz/2
$$

Comparison with the expression in the text implies the following restrictions in the text model

$$
\sigma_{hx} = \sigma_{hx} = \sigma_x
$$
$$
\phi\sigma_{hh} = \sigma_y - (1-\phi)\sigma_x
$$
$$
\eta_{zl} = \eta_{hl} = \eta_{hl}
$$

Estimates of own and cross elasticities of labor supply and child-care demand provide enough information to calculate the more general formula above along the lines indicated in the text. The only such estimates known to us are Ribar (1993) for the United States. Using those numbers and Swedish tax and subsidy rates yields losses that are of the same order as those in Table 3.
TABLE 1

The size of the public sector, shares of total employment and GDP, 1990, percent.

<table>
<thead>
<tr>
<th></th>
<th>public employment</th>
<th>public consumption</th>
<th>public investment</th>
<th>public outlays</th>
<th>taxes</th>
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<tr>
<td>Canada</td>
<td>6.6</td>
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<td>na</td>
<td>na</td>
<td>36.1</td>
</tr>
<tr>
<td>USA (1989)</td>
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<td>17.9</td>
<td>1.7</td>
<td>36.3</td>
<td>29.6</td>
</tr>
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<td>Japan</td>
<td>6.0</td>
<td>9.1</td>
<td>5.2</td>
<td>32.0</td>
<td>31.1</td>
</tr>
<tr>
<td>France</td>
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<td>na</td>
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<td>42.6</td>
</tr>
<tr>
<td>West Germany</td>
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<td>2.3</td>
<td>na</td>
<td>40.3</td>
</tr>
<tr>
<td>United Kingdom</td>
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<td>2.4</td>
<td>41.6</td>
<td>35.5</td>
</tr>
<tr>
<td>Sweden</td>
<td>31.7</td>
<td>27.1</td>
<td>3.1</td>
<td>61.6</td>
<td>56.4</td>
</tr>
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</table>

Source: OECD, National Accounts

TABLE 2

Summary of Direct Expenditure on Child Care Programs, 1991/2

<table>
<thead>
<tr>
<th>Program</th>
<th>Expenditure SEK billion</th>
<th>Comments</th>
</tr>
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<tbody>
<tr>
<td>Day care subsidies</td>
<td>26</td>
<td>Central and local government</td>
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<tr>
<td>Parental insurance</td>
<td>13</td>
<td>Net of taxes</td>
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<tr>
<td>Housing allowances</td>
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<td>Excluding housing allowances for school children</td>
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<tr>
<td>Child allowances</td>
<td>7</td>
<td>Preschool children only</td>
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<tr>
<td>Total</td>
<td>48</td>
<td></td>
</tr>
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Note: the table does not give the full budget effects because impacts on tax revenues are ignored.
### TABLE 3

Deadweight Loss Multipliers for Alternative Substitution Elasticities

<table>
<thead>
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<th>DWL</th>
<th>$\delta D$</th>
<th>$\delta \gamma$</th>
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<td>3</td>
<td>N.A.$^5$</td>
<td>N.A.</td>
<td>N.A.</td>
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<td>0.11</td>
<td>-11.28</td>
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Notes:

1. Alternative values of substitution in production.
2. Implied by equation (14) for indicated values of $\sigma_\gamma$ and $\eta_{lw}$.
3. Proportionate deadweight loss from (13) for $\tau = 0.7$ and $\rho = 0.9$. These should be applied to a per child base for $qz$ of at least SEK 5500 (see text).
4. $\delta D = (\partial \log DWL/\partial \log \rho)_{\text{partial balance}}$. See text.
5. N.A. means that the substitution parameter is outside the economically feasible range.
Figure 1

- Total population
- Total employment
- Total labor force actually on the job
Fig 2: Private and Public Employment
Fig. 3 male employment by sector
Fig. 4 female employment by sector
Fig 5: Average hours worked per year
Fig 6: Average annual hours by gender
Fig 7: Relative hourly wages
Figure 8: Public Employment by Sector
Fig 9: Public Day Care, Children 0-7 years old
FIGURE 10