Identity capital and wealth accumulation

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Abstract

We develop a theory of identity capital. Identity is built up in connection with a “life project”, in which the individual invests time and/or money, and identity capital is a stock measure capturing these investments. In general, a life project can take many forms—building a firm, becoming a skilled professional, playing the guitar, or being a parent—and although it builds on investments it is not necessarily associated with wealth accumulation. In this paper, however, we focus on the kind of identity building that goes along with monetary payoffs and explore its relevance for wealth inequality. We hypothesize, in particular, that identity has a built-in utility asymmetry: although building it up is costly in terms of monetary resources, it does not yield an additional utility payoff, whereas decumulating it leads to a utility loss: the process of damaging, or losing, one’s identity is painful. Identity capital, more precisely, evolves like a habit—it is a weighted average of past activity—and affects utility payoffs asymmetrically.

1 Introduction

What drives individuals to accumulate wealth? In this project we examine a new factor behind wealth accumulation: identity management. We have in mind the notion that the build-up and, in particular, the persistence of individual wealth can be associated with a

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life project around which an individual builds her identity —think IKEA. We formalize a notion of identity capital, alongside asset wealth, and examine its management from the perspective of a utility-maximizing consumer. In addition to standard utility drivers, the consumer experiences costs/benefits associated with the identity capital. A key feature in our utility specification is an asymmetry that imposes high costs when the value of the life project around which one builds identity falls. This formulation opens up a mechanism for a reluctance to decumulate wealth and identity-related capital that is not present in standard frameworks. Thus, our theory focuses on a possible reason behind why the very richest choose to maintain their high wealth levels, at the same time as it also has implications for the process of wealth buildup.

The data on wealth inequality, and the challenges in understanding it based on standard models, is an important factor motivating our work. Wealth is persistently concentrated at the top of the distribution, and top wealth households together control an economically meaningful capital stock. In Sweden, for example, the wealth of the top 0.01% households was equivalent to four times the size of public yearly investments in 2000–2007 (Bach et al., 2018).

The portfolio composition is also strikingly different across the wealth distribution, as documented by recent empirical studies using high quality registry data. Two thirds of the top 0.01% Swedish portfolios consist of private equity, and in Norway, the corresponding figure was 85% between 2005 and 2015 (Bach et al., 2018; Fagereng et al., 2019). In both countries, the portfolios in the bottom half of the wealth distribution have negligible shares of private equity. The pattern is similar in the U.S. Using administrative tax data and the capitalization approach, Smith et al. (2019) estimate that the top 0.1% U.S. portfolios consist of over 40% private equity. Partly as a consequence of these systematic portfolio heterogeneities, returns also vary systematically with wealth. The expected return as well as the variance of the idiosyncratic return component increase sharply with net worth in both Sweden and Norway.

The savings behavior of the super-rich—characterized by high saving rates—and their portfolio behavior—characterized by poor diversification and, for many, private equity—are the main subjects of analysis in the present paper. An effort to understand these observations arises not just out of intellectual curiosity: from a macroeconomic perspective, insights into these issues are important for understanding the formation of the
aggregate capital stock and investments and, moreover, have implications for the effects of taxation policies. More generally, to the extent that one is worried that extreme wealth inequality leads to economic and social instability, the sources behind these developments are likely to matter. The primary aim of this paper is not to answer these broader and motivating questions, but to build toward a richer microeconomic structure that can be used in addressing them. Our efforts here consist of developing some new theory which will help us understand the available microeconomic data and, perhaps, give us a different perspective on some aspects of this data. Our endeavor also includes using survey methods to obtain further insights into our hypothesis as well as into other drivers of wealth accumulation among the very richest households.

Our main thesis is that the answer to the question of what drives top wealth accumulation may in part lie in the answer to the question of why top portfolios are dominated by private equity. We therefore develop a theory where the driver of the savings behavior of the super-rich also serves as a microeconomic foundation behind the pattern that the portfolio share of private equity rises in wealth: namely that people care about, and can invest in, projects that simultaneously strengthen their identity capital. We take identity to capture the idea that individuals may get utility from a “life project”, in addition to that arising from consumption and leisure, the traditional economic variables. A life project can be just about anything: becoming a good guitar player or a skilled professional, contributing to society or academia, building a company, or managing an inherited family firm—simply anything that an individual comes to identify with. By investing time and money into this life project, individuals accumulate what we call identity capital. We view this notion as potentially useful for understanding a range of phenomena but in this paper we focus on life projects that involve significant monetary engagements/success, such as those associated with building a company or managing an inherited family firm.

It is clear that people might benefit from building an identity—becoming excellent in some dimension—in ways that go beyond monetary values. Being a good classical guitarist, so good that one is able to play the full Concierto de Aranjuez without making mistakes, is one example, one that also hints at the possibly limited monetary upside. Thus, significant monetary investments in guitar lessons and equipment, as well as significant time investments, can be understood from this perspective. However, here our focus is less on the buildup of identity than on the perils—from the individual’s perspective—of
identity dents: large losses in the value of the project around which one builds identity, such as those arising to a entrepreneur whose market share is weakened, a politician whose changes to society are reversed, or a skilled professional who loses her job (for reasons other than purely financial ones).

We implement these notions by first distinguishing between two kinds of assets that can be accumulated: a riskless asset and firm capital, the latter being the source of identity buildup. We define identity capital as a (habit-like) weighted average of past values of business capital. We then theorize that individuals dislike that their level of firm capital is (far) below the level of identity capital. Thus, we think of identity capital as a reference point. The monetary returns from firm capital is subject to shocks and depreciation. As a result, there is an additional savings motive: negative returns to the investments make it harder to maintain the identity level. Thus, the costs of losing one’s identity capital also introduces a risk of building “too much” identity capital in the first place. One of the purposes of our theory is to examine how these forces play out under different assumptions on primitives.

We embed our identity capital analysis into a dynamic model of consumption and savings with uninsurable idiosyncratic risk, building on Huggett (1993) and Aiyagari (1994) but abstracting from general-equilibrium feedbacks; the rates of returns on assets and wages are therefore exogenous. To this model, we add entrepreneurs who, as indicated above, have the choice to invest in either a risk-free asset or in firm capital, whose return is subject to uninsurable idiosyncratic shocks that follow a two-state Markov chain. Next, to transform the entrepreneur to an identity capitalist, the firm becomes the entrepreneur’s life project, through which she builds her identity. The firm is formalized by firm capital in our model, and the stock of identity capital is then a geometric weighted average of the past levels of firm capital. To complete the transformation from an entrepreneur to an identity capitalist, we introduce a utility cost of having the current firm capital be below the current level of identity capital. In this sense, identity capital acts like a reference point for the capitalist. The utility cost is convex: on the margin, losses in identity hurt much more when the identity loss is large. Moreover, we assume that this cost becomes more important—relative to the utility derived from consumption—as wealth/consumption increases.

We consider two model versions. In one, the firm technology has decreasing returns
to scale, so that (under a scenario without shocks) there will be an optimal firm size. Identity capital in this version, once shocks are introduced, mean that if the optimal firm size is high—due to a (temporarily) high outcome for productivity—it is costly for the entrepreneur to decrease firm size in response to a negative productivity shock. By the same token, it becomes costly to fully expand firm size in response to a positive productivity shock, since downsizing is so costly from an identity perspective, making a forward-looking entrepreneur hold back investments to some extent ex ante. In an extension of the model, we weaken (or even reverse) the latter effect by also allowing identity to have an independent and positive effect on utility.

In the second model, the same mechanisms are at play, but the technology does not have decreasing returns to scale. I.e., this second version is similar to a standard household portfolio problem: there are two assets and two exogenous returns, one of which is riskless and the other one random (and related to the entrepreneur’s identity). Here, given the linear entrepreneurial technology, and absent any shocks, there is no natural long-run size. However, we also assume that the entrepreneur receives some other, exogenous and random income and the presence of this other income gives rise to a stationary distribution for entrepreneurial capital, much like in an Aiyagari-Huggett setting.

As of writing the present version of this introduction, no definitive results have been obtained but the numerical analysis works well and, overall, confirms the intuition laid out above. I.e., the identity capital elements make the entrepreneur engage in decisions that are “financially irrational”: she will hold back investment if the firm return is expected to be very good and, more to our point, she will resist downsizing when the firm’s financial prospects are weak. We also find that for a range of values of the entrepreneur’s wealth and identity capital levels, higher wealth increases the share of the risky asset that she chooses to hold. This is interesting since the risky share increases in wealth in the data (as we illustrate below using a study on Sweden) and yet standard, calibrated portfolio models cannot replicate this fact. It should be noted, however, that our results are only theoretical possibilities at this point; a notable challenge is the precisely the calibration. In particular, there is no data that directly bears on how we could formulate the new habit-like element in the utility function. We therefore proceed by examining to what extent different formulations of this new element can help account better for saving
behavior. That is, we follow the approach typically taken in the exploratory asset-pricing literature by simply searching for habit formulations that help explain the data.¹

Given our focus on the very wealthiest, data on their saving behavior would be especially helpful. Their appearance in surveys is rare if at all existent, so this project also sets out to add to the state of knowledge by conducting a special survey directed at this group. We will thus target wealthy Swedes; Sweden offers an opportunity to collect such data since it is a small country with high concentration of wealth at the top and with somewhat publicly known wealthy individuals and industrial families, both comprising young and old “dynasties”. The Swedish business magazine *Veckans Affärer* yearly publishes a list of “Swedish billionaires”, like *Forbes 400*, with 191 persons on the list in 2018. We intend to sample individuals using this list and by recommendations. We plan to ask about the interviewee’s wealth within broad intervals; hours worked per week and weeks of vacation per year; which generation of a potential family firm that they belong to; and the relative importance of a number of potential savings motives. We will also, in a qualitative and more open-ended part of the interview, give the subject the opportunity to provide more depth to her quantitative answers and to contrast her own savings motives to those that she believes govern the behavior of other, both equally and much less wealthy, individuals. The interview survey has been prepared and is awaiting an ethical review (which it needs to clear before it is launched).

We comment on the macroeconomic literature on inequality in detail in Section 2 below. The key points we make there is that a basic model along the lines of Aiyagari (1994) falls quite short of generating highly dispersed wealth, that the literature has proposed a number of mechanisms for generating more right-tail wealth, and that we offer a new avenue that potentially can generate higher wealth dispersion. In particular, our model makes wealthy individuals very reluctant to decumulate their wealth, in contrast to how they behave in the basic Aiyagari model.

Section 3 presents the main motivating empirical evidence for our theory of identity capital. In Section 4, we describe our model of identity capital. The implementation of the model is presented in Section 5. Preliminary results are presented in Section 6.

¹The same procedure is practiced in the applied business-cycle literature where not only habit specifications but also the formulation of investment adjustment costs are selected to fit the aggregate data rather than based on direct observations from microeconomic data.
Section 7 then looks at some extensions and Section 8 briefly describes our planned data collection. Section 9 presents our conclusions once they have emerged.

2 Related literature

Our work builds on a large macroeconomic literature on wealth inequality, starting with consumption-savings models with uninsurable idiosyncratic income risk and precautionary savings as in Huggett (1993) and Aiyagari (1994). For these models, however, generating a persistently extreme wealth concentration is a challenge: the equilibrium interest rate is below the rate of discount, so the well-insured wealthy decumulate systematically.

Several contributions make extensions to standard consumption-savings models with precautionary savings motives to generate more wealth inequality. Some of these extensions add stronger savings motives for the rich. For example, Krusell and Smith (1998) use shocks to the time discount rates to generate a mechanism by which the patient become rich and hence have higher saving propensities than the average consumer. Carroll (2000) develops a model in which wealth enters consumers’ utility functions directly as a luxury good relative to consumption. Carroll’s model accommodates numerous potential motives behind accumulating wealth in a reduced-form way. He discusses some of these, such as status-seeking; a metric for comparing oneself to others; philanthropic ambitions; power-lust; and a way of keeping score of one’s job performance. Similar in spirit to Carroll’s model, De Nardi (2004) studies a life cycle model economy with luxury-good bequests that give parents warm-glow utility from a utility function with low curvature. Our work relates to Carroll (2000) and De Nardi (2004) in adding a type of capital to the utility function, but instead of assuming that there is a utility gain from higher levels of capital, we assume that there is a utility cost from losing capital. This makes our results less “hard-wired”, since there is both an incentive to save once a high wealth level has been obtained and an incentive not to build wealth ex ante, since it may be painful to decumulate it when it is financially costly to maintain high wealth.

Another strand of the macroeconomic literature on wealth inequality generates a more dispersed distribution by considering an alternative earnings process to that normally adopted. In particular, Castañeda et al. (2003) assume that individuals can experience a rare but persistent state of extraordinarily high labor productivity—about a thousand
times larger than that of the lowest state—that at the same time carries non-negligible risk of a large drop in earnings. In this model, individuals who have become extraordinarily productive continue accumulating assets due to the risk of becoming much less productive. Kaymak and Poschke (2016) explore the same mechanism as Castañeda et al. (2003), but with the purpose of accounting for the changes in wealth inequality in the U.S. over the last decades. We relate to these studies by also introducing a mechanism that dissuades successful entrepreneurs from decumulating wealth, in particular when the prospects for the business weaken.

The contributions by Quadrini (2000) and Cagetti and De Nardi (2006) instead explore the role played by introducing an occupational choice for a household between supplying its labor services to the market (“labor”), or employing them in its own firm (“entrepreneurship”). The household chooses whichever occupation is the financially most beneficial, given its constraints and abilities. A key here is some extent of increasing returns: only if one is rich enough will it pay to become an entrepreneur, but then it can generate higher returns. We add to this literature by modeling entrepreneurs who engage in accumulating firm capital not solely due to financial reasons, but also because the firm is their life project that they come to identify with.

A recent strand of the literature emphasizes the role played by heterogeneity in portfolios and asset prices for understanding top wealth accumulation. Hubmer et al. (2019) perform a similar exercise to Kaymak and Poschke (2016) in accounting for drivers of wealth inequality over time, but in their model economy, the heterogeneity in household portfolios and the dispersion in household returns to assets are key variables in matching steady-state and medium-run wealth concentration at the top. Also in a similar accounting exercise but with a particular focus on social mobility, Benhabib et al. (forthcoming) find that both savings rates that rise in wealth and stochastic idiosyncratic returns to wealth contribute to generating a thick right tail of the wealth distribution, consistent with data, while stochastic earnings are important in matching social mobility but less so for wealth inequality.

Hubmer et al. (2019) and Benhabib et al. (forthcoming) make important contributions in noting that portfolio heterogeneity and asset prices are central in accounting for top wealth concentration. Model-wise, however, they make short-cuts in both these respects. First, they do not explicitly model portfolio choice; instead, the return process is given
(and, in the case of Hubmer et al. (2019), positively dependent on wealth by assumption). Second, none of these contributions attempt to solve for asset prices by market clearing for each asset class. We add to this literature by allowing portfolio choice. We do not allow for investments in the projects of others—one can only invest in one’s own project or in a risky asset—nor do we solve for asset prices from market clearing. Relaxing these assumptions may be relevant but would be challenging.

While no prior contribution in the wealth inequality literature has explored the concept of identity, it is not a new idea to economics. Dating back to (at least) Akerlof and Kranton (2000) (A&K), there is a literature on the economics of identity that our work also relates to. A&K argue that the prominence of identity in psychology suggests that economists should consider identity as an argument in utility functions. In their theory, identity describes both a person’s self-categorization as well as the social categories to which other people assign her. Individuals thus choose their identity to some extent. The utility derived from a person’s identity depends not only on these categories, but also on how her own and others’ behaviors correspond to the “ideal” behavior in the respective category, which, in turn, is affected by individual action. Relative to A&K’s theory, in our model, there is only one category of identity, within which a person chooses her “intensity”. Similarly to A&K’s theory, a person compares her identity to a reference point (the “ideal” behavior in A&K’s model), which in our model is affected by her own behavior. In contrast to A&K, however, we do not consider how other individuals’ beliefs and behaviors affect a person’s identity. Indeed, we do not set out to provide a complete model of a person’s identity but rather focus on how identity can interact with wealth accumulation.

Our project also relates to a strand of the sociology literature that attempts to get at more fundamental mechanisms among wealthy entrepreneurs. One example is Kantula and Kuusela (2019), who study how morals evolve among wealthy Finnish entrepreneurs (those in the top 0.1% of the income distribution). The focus is to understand how a growing socioeconomic distance from the rest of the population can be “legitimized”. More similar in spirit to our theory is Schervish (2016), who interviews 49 wealthy entrepreneurs in the U.S. with the purpose of “constructing a sociology” around entrepreneurs. His main thesis is that entrepreneurship is a joint process of making money and making a “self”, such that the entrepreneurial production of financial capital also creates what the author
calls “moral capital”. We add to this literature by focusing on the effects of identifying with a life project, instead of the process of building morals along with building a company. We also emphasize that this trait is shared by all individuals, and not unique to wealthy entrepreneurs.

Finally, while there are several potential theoretical mechanisms that drive top wealth accumulation, there is little data to evaluate them with, since survey data on the very richest is scant. For example, in the U.S. Survey of Consumer Finances (SCF), which especially targets and over-samples the most wealthy households, the survey questions aim primarily at documenting the respondents’ income, assets and debts on a detailed level, rather than getting at savings motives that can explain the behavior of the wealthiest households. To boot, the response rate among the most wealthy households in the SCF is quite low. Around 1,000 of the 1.22 million households in the top 1% of the wealth distribution are part of the survey. These households are, in turn, divided into four strata, whose response rates decrease in stratum household wealth (Bricker et al., 2016). In the stratum with the most wealthy households, the response rate is 12 percent, compared to 70 percent in the part of the survey that targets the population in general. The SCF is also prohibited from sampling individuals on Forbes’ list of the top 400 wealthiest U.S. individuals. Relative to the lack of data on savings motives among the super rich, we add to the state of knowledge by collecting survey data and interview data from wealthy Swedes, with a particular focus on their savings motives.

3 Top wealth, portfolios and returns

In recent years, high-quality administrative datasets have made it possible to document wealth, portfolio compositions and asset returns on a household level across the wealth distribution. Bach et al. (2018) use an administrative panel containing the full balance sheet of every Swedish resident between 2000 and 2007 to study portfolio composition and returns across the full wealth distribution. Fagereng et al. (2019) do the equivalent exercise for Norway between 2005 and 2015. Both contributions document a number of key facts about top wealth, portfolio compositions and returns to wealth. For a country

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The question on savings motives asked in the SCF is oriented towards consumption motives or precautionary savings motives, which arguably are of second-order importance in understanding the savings behavior of the super-rich.
like the U.S. where no such high-quality administrative data is available, estimates using the capitalization method and U.S. tax data show similar patterns (Smith et al., 2019).

The first fact, which is well documented elsewhere too and for a number of countries, is that wealth is extremely concentrated at the very top of the distribution. The top 0.1% of the net wealth distribution holds around 10 percent of total wealth in both Norway and Sweden, out of which, half is owned by the top 0.01% in Sweden\(^3\). In the U.S., the equivalent top groups hold even more wealth, but with similar relations between the top 0.1% and the top 0.01%, according to recent estimates (Smith et al., 2019).

The second fact is that the share of risky assets in general, and of private equity in particular, increases in net worth. In lower brackets, the share of private equity is negligible in both Sweden in Norway, but reaches almost a fifth of top 1%-0.5% portfolios, and nearly two-thirds for the top 0.01% in Sweden and 85% of the top 0.01% Norwegian portfolios. This systematic pattern is illustrated for Sweden in figure 1, which shows the average portfolio holdings of Swedish households across different brackets of the net wealth distribution, between 2000 and 2007. As figure 1 illustrates, private equity plays a crucial role for top wealth dynamics. In total, the top 0.1% in Sweden held almost half of all private equity, out of which, half was owned by the top 0.01%. A similar pattern appears in the U.S. In a new data series from the Federal Reserve called the *Distributional Financial Accounts*, the top 1% U.S. portfolios are dominated by business assets, both public and private, while in bottom portfolios, these assets rarely show up (Batty et al., 2019). Smith et al. (2019) use administrative tax data and the capitalization method to estimate top 0.1% portfolios in the U.S. and find that they consist of over 40% private equity.

Partly as a consequence of these systematic portfolio patterns, returns covary with the level of wealth. Both expected and average returns as well as the idiosyncratic risk component increase sharply in net worth. This is the third fact documented by Bach et al. (2018) and Fagereng et al. (2019), and is a phenomenon called *scale dependence* (Gabaix et al., 2016). Heterogeneity in returns is, according to Bach et al. (2018), the dominant mechanism in capturing the average evolution in top shares over the sample period. In particular, they find that private business owners disproportionately contribute to return heterogeneity at the top of the distribution.

\(^3\)Fagereng et al. (2019) do not report the wealth share held by the top 0.01% for Norway.
Figure 1: Portfolio holdings

Note: Figure from Bach et al. (2018). The figure shows average Swedish household portfolios and leverages (defined as total household debt to gross wealth) in different brackets of the net wealth distribution in Sweden between 2000 and 2007.

Bach et al. (2018) argue that these facts do not reflect that the wealthy have exceptional investment skill, but rather that risk exposure predict historical returns well, and that these historical returns explain the level and volatility in top wealth shares with high accuracy. Fagereng et al. (2019), on the other hand, argue that while return persistence partly arises from stable differences in risk exposure and assets scale, it also reflects heterogeneity in sophistication and financial information, as well as entrepreneurial talent. Common for both views, however, is that investment behavior seems first-order for top wealth concentration. In particular, since private equity dominates top portfolios, investment behavior among entrepreneurs or firm owners seems key for top wealth accumulation. Indeed, Fagereng et al. (2019) show in their online appendix that entrepreneurs have five times higher probability of moving from the bottom decile in 2004 to the top 1% in 2015, compared to non-entrepreneurs. Further adding to the picture, Smith et al. (2017) characterize top income earners in the U.S. using administrative data and find that active private business owner managers are key for top income inequality. They also find that the top business income comes from extremely undiversified business assets.
4 Model

The basic building block in our model is a partial-equilibrium version of the frameworks in Huggett (1993) and Aiyagari (1994). We then first add entrepreneurs, who can invest time and money into their business, which is subject to idiosyncratic productivity shocks. Next, we formalize the model implications from our theory of identity capital. Adding these to the model, the entrepreneurs become identity capitalists.

4.1 From consumers to identity capitalists

Time is discrete and the economy is populated by a continuum of infinitely lived, ex-ante identical households (dynasties). In the standard Bewley-Huggett-Aiyagari type model, households maximize the expected lifetime utility

\[ E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \]  

where \( \beta \) is the intertemporal discount rate, \( u(c_t) \) has constant relative risk aversion (CRRA), and \( E_0 \) is the expectation operator at time zero (von Neumann-Morgenstern preferences are assumed).

In the simplest consumption-saving version of this model, labor supply is exogenous, constant, and normalized to 1. Each consumer earns the competitive wage rate \( w_t \), which is subject to idiosyncratic shocks \( \epsilon_t \). This can be thought of as shocks to labor productivity. Asset markets are incomplete, so consumers cannot fully insure against idiosyncratic shocks. As in the standard model, consumers have access to a riskless asset. However, they can also invest in a risky asset that we think of as related to entrepreneurship.

Entrepreneurs The household is also an entrepreneur. Their labor supply can thus be allocated to two activities: they allocate \( l_t \) to running their business and \( 1 - l_t \) to market work at the competitive wage rate \( \epsilon_t w \), where \( w \) is the aggregate wage level. In particular, \( l_t \) may be greater than 1, in which case the entrepreneur has chosen to hire labor to work in the firm at the competitive wage rate. In our baseline setup, entrepreneurs do not get disutility from working, but such a feature will be added in one of our model extensions.

Entrepreneurs choose to save \( a_{t+1} \) at the risk-free rate and to invest financial assets \( e_{t+1} \) in their business in order to build firm capital \( h_{t+1} \). The investment \( e_{t+1} \) is subject
to either decreasing or constant returns given by $\gamma \leq 1$, and is scaled by a potentially random $\theta_t$, which can be thought of as entrepreneurial ability or investment skills. In the baseline model we will set $\gamma = \theta_t = 1$. Firm capital depreciates at rate $\delta$ each period. In period $t$, the entrepreneur uses the firm capital chosen in the previous period $h_t$, firm labor $l_t$, and a potentially non-linear Cobb-Douglas technology to produce output. This output is subject to idiosyncratic productivity shocks $\Phi_t$. The entrepreneur thus solves

$$\max_{\{c_t, a_{t+1}, e_{t+1}, h_{t+1}, l_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

subject to

$$c_t + a_{t+1} + e_{t+1} = (1 + r)a_t + \epsilon_t w(1 - l_t) + \Phi_t h_t^\nu l_t^{1-\nu},$$

$$h_{t+1} = (1 - \delta)h_t + \theta_t e_{t+1}^\gamma,$$

$$a_{t+1} \geq a, h_{t+1} \geq h.$$}

Let us now implement the baseline and set $\theta_t = \gamma = 1$, which allows us to solve for $e_{t+1}$ from the law of motion of firm capital $h_t$ in equation (3) and insert it into the budget equation (2). Now, instead of choosing investment $e_{t+1}$, the entrepreneur directly chooses the level of firm capital tomorrow $h_{t+1}$.

We now consider two model cases, each of independent interest: $\nu = 1$ (first) and then $\nu < 1$.

4.1.1 Standard portfolio choice (with a twist)

When we set $\nu = 1$, the entrepreneur optimally allocates all labor to wage work. Making these simplifications, the entrepreneur’s problem becomes

$$\max_{\{c_t, a_{t+1}, h_{t+1}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

subject to

$$c_t + a_{t+1} + h_{t+1} = (1 + r)a_t + \epsilon_t w(1 - l_t) + (\Phi_t + 1 - \delta)h_t,$$
accumulation behavior implies that wealth goes to zero or to infinity. With idiosyncratic
labor-income risk, matters are, in general, different. We make the assumption that this
risk ($\epsilon$) is independent of the entrepreneurial investment risk ($\Phi$). This implies that there
will be a stationary distribution of wealth.

In this baseline model, it is a standard result from the associated literature that the
risky share will be decreasing in wealth and, in the limit—as wealth goes to infinity—
approach the value in the model without independent labor income. The intuition behind
the risky share decreasing in wealth is that the independent labor income, to the extent its
variance is limited, is similar to a riskless endowment; hence, a wealth-poor agent already
has something akin to the riskless asset and thus does not need to invest in the such an
asset at low wealth levels. At high wealth levels, the endowment plays a subordinate role
and hence the outcome looks more like the case without this endowment.

**Identity capital** We denote identity capital by $\bar{h}_t$: a weighted average of past levels of
firm capital. It is thus a form of habit, and we assume for concreteness that

$$\bar{h}_{t+1} = \bar{h}_t(1 - \rho) + h_t \rho,$$

where $\rho \leq 1$. The stock of identity capital could, of course, be a direct addition to the
utility from consumption, but in our baseline formulation we assume that it is not: it
is only a reference point to which current firm capital is compared. In particular, the
individual compares her current firm capital $h_t$ to her identity capital $\bar{h}_t$ and, to the
extent the former is higher, there is no effect on utility, but if it is lower, there is a utility
loss. We use $f(h_t, \bar{h}_t)$ to capture this additional utility loss at time $t$. We assume a
particular functional form below but, aside from being differentiable and monotone, the
key features we wish to entertain are that the losses become larger the further $h_t$ is from
$\bar{h}_t$ and that, as an entrepreneur becomes wealthy, identity matters more. We will discuss
these issues in the context of a specific example in Section 5.

Note that in a dynasty model, the entrepreneur can also be thought of as an individual
that inherits and keeps running the family business. Such an interpretation is consistent
with the theory of identity capital, if this individual identifies herself with the family
business. An individual who inherits the family firm and has her identity associated with
it, is modelled as an agent with a high initial reference point $\bar{h}_0$. 

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Adding these features to the entrepreneur’s problem, the identity capitalist solves

\[
\max_{\{c_t, a_{t+1}, h_{t+1}, \bar{h}_{t+2}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [u(c_t) - f(h_t, \bar{h}_t)]
\]

subject to

\[
c_t + a_{t+1} + h_{t+1} = (1 + r)a_t + \epsilon_t + (\Phi_t + 1 - \delta)h_t;
\]

\[
a_{t+1} \geq a_t, h_{t+1} \geq h_t, \bar{h}_{t+1} = \bar{h}_t(1 - \rho) + h_t\rho.
\]

Notice that the formulation makes clear that \((a_0, h_0, \bar{h}_0, \bar{h}_1)\) is given. This is one of our key formulations and we investigate its properties numerically in Section 6 below.

**Analytical special cases**  Consider a very simple version of the loss function: \(f(h, \bar{h}) = f(\bar{h} - h)\), where \(f\) is increasing and convex, \(f(0) = 0\), and \(f'(0) = 0\) (assumptions we will also use below). Moreover, we set \(\epsilon_t = 0\), thus dispensing with independent endowments, and set \(1 - \delta + \Phi_t = 1 + r\), so that the return to firm capital is non-random and the same as for the riskless asset. Then we obtain an analytical solution whenever \(\beta(1+r) > 1\) and the borrowing constraint \(a\) is not too tight. Intuitively, then, assets will be accumulated without bound, and identity capital will as well, though lagging behind since it is a weighted average of past levels of firm capital. More precisely, total saving out of wealth (cash on hand) is a fraction \(s = \beta(1 + r)^{\frac{1-\sigma}{\sigma}}\), so that total wealth evolves at the rate \(s(1 + r) = (\beta(1+r))^{\frac{1}{\sigma}} > 1\). In this case, no utility loss from identity capital is ever experienced because initially savings can be allocated towards \(h\), if initial \(\bar{h}\) is high, so that the two will be equal. Then, the value for \(a\) will be correspondingly low, and the borrowing constraint needs to allow this. As time passes, \(h\) will rise, and \(\bar{h}\), which is a weighted value of past levels, will rise but lag behind and hence \(h > \bar{h}\) will always hold. At this point, the portfolio choice has indifference over a range: as long as \(h\) does not fall below \(\bar{h}\), the entrepreneur can Thus, identity does not matter for consumption allocations here, though it matters for portfolio choice initially.

Suppose instead \(\beta(1+r) < 1\). Then total saving will move downward and there will be utility losses whenever \(\bar{h}_t > h_t\). Given that \(\bar{h}\), lagging behind, would then tend to be higher than \(h\) if savings fall over time, can the entrepreneur avoid losses? The only chance is to set \(h_t = \bar{h}_t = \bar{h}_1\) at all points in time, i.e., to not let \(h\) fall. This is again feasible if the borrowing constraint on \(a\) is not too tight: the sum of the two assets will
be going to zero, as will consumption, but \( h = \bar{h} \) will stay positive and finite. In relation to consumption, \( h \) will be infinitely large, and the entrepreneur will be borrowing to maintain her identity level.

In sum, this example—where total wealth grows in one case and shrinks in the other—shows how the portfolio choice will be guided by identity, in the second case in rather an extreme way. In the example, no identity losses are ever experienced, and consumption is thus not affected. It is clear, of course, that minor alterations of the assumptions would also make identity affect consumption. If, for example, the borrowing constraint \( \bar{a} \) is less lax, then consumption would be affected. Another interesting case to consider is when the return on \( h \) is below the riskfree rate. Our main focus below, however, is on a quantitatively oriented model where there are shocks: a straight extension to the Aiyagari-Huggett settings.

### 4.1.2 A neoclassical entrepreneur (with a twist)

If \( \nu < 1 \), the entrepreneur has decreasing returns to her project. On the other hand, since she can also hire labor flexibly, this formulation again leads to a reduced-form production function that is linear in \( h \). Let us therefore set \( \epsilon_t = 0 \) in order to preserve decreasing returns.\(^4\) We also adopt the same identity capital formulation as in the previous structure, thus leading to

\[
\max_{\{c_t, a_{t+1}, h_{t+1}, \bar{h}_{t+2}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ u(c_t) - f(h_t, \bar{h}_t) \right] \\
\text{subject to } c_t + a_{t+1} + h_{t+1} = (1 + r)a_t + \Phi_t h_t^\nu + (1 - \delta)h_t; \\
a_{t+1} \geq 0, h_{t+1} \geq 0 \\
\bar{h}_{t+1} = \bar{h}_t(1 - \rho) + h_t \rho.
\]

Clearly, for this model, the entrepreneur’s accumulation of firm capital will be stationary whether or not identity plays a role.

**Analytical special cases** Let us, as in the case with \( \nu = 1 \) in Section 4.1.1, look at a special case where there is no risk—we assume that \( \Phi_t = \Phi \) for all \( t \)—and where the

\(^4\)Alternatively, the Cobb-Douglas production function could be raised to a power less than one, with the same implied reduced form.
borrowing constraint $g$ is lax enough. As for the earlier special case we make the same assumptions on $f$, i.e., it is a function of $\bar{h} - h$ only and has regularity properties. Let us now also assume that $\beta(1 + r) = 1$, for easier characterization (the other cases can be solved as well but the algebra is somewhat more involved).

Assume first that $\bar{h}_1$ is low enough. Then the entrepreneur will immediately set

$$h \equiv h_{ss} = \left(\frac{r + \delta}{\nu \Phi} \right)^{\frac{1}{\nu - 1}}.$$  

That is, there is immediate convergence to steady state, regardless of initial wealth. Thus, $\bar{h}_1$ being low enough simply means lower than $h_{ss}$. Here, identity is never a problem for the entrepreneur; rather, the identity is initially low and then immediately begins climbing toward $h_{ss}$—it converges toward this value as $t$ goes to infinity.

However, suppose that $\bar{h}_1 > h_{ss}$. If a falling identity were not associated with utility losses ($f(\bar{h} - h) = 0$ globally), we would again see immediate convergence to steady state. The presence of utility losses, however, makes convergence slow. The economy will, eventually, converge to $h_{ss}$, so the long-run outcome is independent of initial identity, but the path there is not.\footnote{The exact nature of the convergence depends on the specific properties of $f$.} This case illustrates the asymmetry implied by our asymmetric loss function.

5 Implementation

5.1 Functional forms

We start with functional-form choices for the utility functions for consumption and for identity capital. In order to be consistent balanced saving rates under long-run growth, the utility of consumption is in CRRA form, with a relative risk aversion parameter $\sigma$. When $\sigma = 1$, individuals have log utility of consumption.

The identity capitalist gets utility based on a comparison between her levels of firm capital $h'$ and identity capital $\bar{h}'$. This speaks in favor of having a habit-type of utility function associated with the identity capital. As described in Section 4, the identity capitalist also dislikes when her firm capital is below her identity capital, and in particular if it is far below. Dropping the prime superscripts for notational simplicity, one candidate
functional form is then

\[ f(h, \bar{h}) \equiv \begin{cases} 
A\bar{h}^{\alpha_1} \left(\frac{\bar{h} - h}{\bar{h}}\right)^{\alpha_2} & \text{if } h < \bar{h}, \\
0 & \text{else} 
\end{cases} \quad (6) \]

with \( \alpha_1, \alpha_2 \geq 0 \).

Except for the corner case with \( \alpha_2 = 0 \), the functional form in (6) implies that the utility of identity capitalists are not directly affected by level of their identity capital. Instead, their utility is only affected by deviations of their firm capital from their identity capital. With \( \alpha_2 \in (0, 1) \), the loss function is concave in the loss region. The identity capitalist is then more sensitive to losing her first unit of identity capital than latter units. And, with \( \alpha_2 > 1 \), the loss function is convex in the loss region. She is then less sensitive to losing the first unit of identity capital than she is to losing latter units. We believe that the latter case better captures the mechanism we have in mind and thus use \( \alpha_2 > 1 \) in our analysis.

With \( \alpha_1 = 0 \), the utility loss from depleting identity capital is a purely a function of the percentage deviation of firm capital from identity capital. It implies that the identity capitalist would be equally distraught by halving her firm capital irrespective of the level of identity capital. With \( \alpha_1 \) sufficiently high, a given percentage gap between \( \bar{h} \) and \( h \) plays a more important role for utility than does consumption, which obeys a power function with exponent \( 1 - \sigma \). Mostly to save on parameters, we will impose \( \alpha_1 = \alpha_2 \equiv \alpha \), so that the utility loss form depleting identity capital is purely a function of the level deviation of firm capital from identity capital, and we will impose that \( \alpha > 1 \).

Finally, given our choices, the function is conditional on whether firm capital is below or above identity capital. The identity capitalist thus does not derive utility from building identity capital, that is investing in firm capital above and beyond her level of identity capital, but only suffers from depleting it. We use this setup in our benchmark model to avoid hard-wiring the results with a utility benefit from building identity capital. As one of our model extensions, we will consider the case when the identity capitalist also obtains utility from having \( h > \bar{h} \).
5.2 Parameter values

For most parameter, we pick values that are standard in the literature. For those that are special to our setup, as there is a general lack of data on the saving motives among the super-rich and as our own data is not yet collected, we have had to make parameter choices mostly to highlight different model mechanisms. The parameters are presented in table 1.

Notice that we assume that the two states have different expected returns to firm investments. We assume that productivity follows a Markov chain. In the baseline model, the transition probabilities between the two states are

\[
\Pi = \begin{bmatrix}
\pi_{g|g} & \pi_{b|g} \\
\pi_{g|b} & \pi_{b|b}
\end{bmatrix} = \begin{bmatrix}
0.5 & 0.5 \\
0.3 & 0.7
\end{bmatrix}.
\]

Thus, the productivity states are positively serially correlated and in the good state the expected return is higher.

5.3 Computation

To solve the model with risk and incomplete insurance, it is necessary to use computational methods. Because of the non-linearities involved, moreover, we use dynamic
programming. To state our problem, define cash-on-hand to be

$$\omega_t \equiv (1 + r)a_t + \epsilon_t + (\Phi_t + 1 - \delta)h_t.$$ 

Let $\Phi_t$ be governed by a two-state Markov process coming from the firm productivity shocks $\Phi_t$, and assume that labor productivity $\epsilon_t$ is a two-state iid shock. Since $h_0, \bar{h}_0,$ and $\bar{h}_1$ are predetermined, so that also $f(h_0, \bar{h}_0)$ is given, redefine the current utility flow in period 0 to be determined by $f(h_1, \bar{h}_1)$. Then the state variables today are cash-on-hand $\omega$ and identity capital $\bar{h}$ and the decision problem of the identity capitalist can be stated as follows:

$$V_{\Phi}(\omega, \bar{h}') \equiv \max_{a' \geq 0, h' \geq 0} u(\omega - a' - h') - \beta f(h', \bar{h}')$$

$$+ \beta E \left[ V_{\Phi'} (a'(1 + r) + \epsilon' + (\Phi' + 1 - \delta)h', (1 - \rho)\bar{h}' + \rho h') \right].$$

We solve our dynamic program using the endogenous grid method; the iteration is performed from a final period with $V = 0$. 

---

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>Risk aversion</td>
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</tr>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
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<tr>
<td>$\delta$</td>
<td>Depreciation of $h$</td>
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<tr>
<td>$1 - \rho$</td>
<td>Persistence of $h$</td>
<td>$1 - \delta$</td>
</tr>
<tr>
<td>$(\alpha_1, \alpha_2)$</td>
<td>Curvature of identity loss function</td>
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</tr>
<tr>
<td>$A$</td>
<td>Strength of identity loss function</td>
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</tr>
<tr>
<td>$(\Phi_g, \Phi_b)$</td>
<td>Productivity states</td>
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</tr>
<tr>
<td>$(\epsilon_h, \epsilon_l)$</td>
<td>Exogenous income</td>
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</tr>
<tr>
<td>$(\pi_h, \pi_l)$</td>
<td>iid income shocks</td>
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</tr>
<tr>
<td>$r$</td>
<td>Interest rate</td>
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</tr>
<tr>
<td>$(\mathbb{E}(r_h</td>
<td>g), \mathbb{E}(r_h</td>
<td>b))$</td>
</tr>
</tbody>
</table>
6 Preliminary results

6.1 The case with $\nu = 1$

With $\nu = 1$, the return to the risky asset $h'$ displays constant returns to scale. If we first consider the case without utility-affecting identity in Figure 3, we see the standard result that we elaborated on in Section 4. That is, the risky portfolio share is decreasing in wealth. This is the case both when individuals are in the low and in the high productivity state; the difference is simply that the risky portfolio share starts to decrease at different wealth levels.

![Figure 3: Policy function for the risky portfolio share without identity](image)

This behavior is mirrored in Figure 4 which shows the level of the firm (risky) asset.

![Figure 4: Policy function for firm capital without identity](image)

Here, we see that while low-productivity entrepreneurs choose a risky portfolio share
strictly between zero and one, this share does not appear as they are borrowing con-
strained and do not invest in either asset.

For identity capitalists, the portfolio behavior is less clearcut. We will begin with
Figure 5, which shows the level of the firm asset purchased.

![Figure 5: Policy function for firm capital with identity](image)

We begin with the left-hand side of this figure. In that case, i.e., the bad state,
firm capital has the same expected return as the riskless asset—and thus is a relatively
unattractive asset financially since it also bears risk. Notice first that behavior now de-
pends on \( \bar{h} \), though not in the upper left corner. In fact, the picture has a “diagonal”
situated slightly below the 45-degree line. Above the diagonal, the behavior is qualita-
tively similar to the case without identity, Figure 4: for a given level of wealth, a higher or
lower identity does not matter. This looks like it may contradict the relevance of identity,
but the reason for this is simply that above the diagonal, it is not possible to increase \( h \)
more as \( \bar{h} \) goes up (again, for a given \( \omega \), since above the 45-degree line, spending more on
\( h \) would require negative consumption). Still, compared to the case without identity we
see that the \( \bar{h} \) level is maintained, and not lowered as in the case of when no identity losses
are experienced: the entrepreneur invests more in her firm here than without identity,
despite the own firm giving an unfavorable (risk-adjusted) return.

Turning to the area below the diagonal in the left panel (bad state) of Figure 5, we
see that increased cash on hand does not make for more a higher investment in the own
firm. The reason is that identity is only a minus—it will generate losses if it is raised and
then needs to be reduced—and given the poor returns in the bad state this is why higher
wealth always goes into the other asset in this case.
The right-hand side of Figure 5 describes the good state, where the firm gives a favorable expected return. It is, in part, more nontrivial. Above the diagonal, as in the bad state, there really is no choice but to simply invest so as to roughly maintain the same identity—given that consumption cannot be negative. Below the diagonal, unlike under the bad state, there are counteracting forces. It is a minus to invest in the own firm identity because it can lead to future identity losses, but the own firm now also gives a very high return. For high levels of identity, we see the same behavior as in the bad state: the fear of future identity losses is the dominant force: because we assume $\alpha_1$ is large, so that at high levels of wealth or identity, the latter matters more for utility than does consumption.

When the level of identity is low in the good state, however, the investments can be more aggressive for the richer entrepreneurs: more wealth allows higher investment into the own firm. This will not cause a radical shift up in next period’s identity, because identity is a slow-moving average of past identities. Hence, more wealth will mean higher investments in the own firm, causing future identity to slowly move up. Perhaps curiously, there is a “second diagonal” in the good state, with a positive but much lower slope, defining two qualitatively different areas: above it, behavior is dominated by “managing identity”, thus not accumulating it, and below it, financial concerns dominate. Along this second diagonal, $h$ is chosen so as to roughly keep identity constant between now and next period. The level at which this occurs is increasing (slowly) in wealth.

These observations also imply that, to the left of the second diagonal, if $\bar{h}$ is held constant and $\omega$ increases, firm investment is first falling in wealth and then increasing, reaching a minimum in the middle. That minimum is around where $h = \bar{h}$. Intuitively, when wealth and identity are similar in size, with the former being slightly larger, more money will mean that the entrepreneur will invest slightly less in the firm, since the lower the level of one’s future identity, the less painful it will be to lose it. Beyond a certain wealth level, however, the financial concerns dominate, since there is now enough wealth to also counter possible future low firm returns.

Finally, the risky portfolio share implied by the above analysis is shown in Figure 6.
We note again, as in standard portfolio-choice models, that more wealth means a lower risky share, except in the area to the right of the second diagonal in the good state, where higher wealth actually raises the risky share, for reasons discussed above.

In sum, our portfolio-choice model delivers intuitive results given the importance of identity capital and the asymmetric loss function we assume. In particular, there is a trade-off between managing one’s identity and purely financial concerns, and the relative importance of these two forces varies with the expected return and the entrepreneur’s current state. We also obtained, as a result of these trade-offs, some surprising predictions for the portfolio share’s dependence on wealth: it is increasing in parts of the state space.

6.2 The case with $\nu < 1$

With $\nu < 1$, the return to the risky asset $h'$ is decreasing in scale, such that there will always be a point above which the marginal return of $h'$ is below the risk-free interest rate $r$. There will thus be an upper bound to the $h'$'s chosen by identity capitalists, beyond which, the portfolio share of $h'$ will fall with cash-on-hand. [MORE TO FOLLOW]
7 Extensions

7.1 Utility gains from identity capital

7.2 Disutility from labor

7.3 Two risky assets

8 Interviewing wealthy Swedes

It is possible to construct a utility function to justify any conceivable behavior: just assume that the behavior in question yields more utility than its alternatives. Any postulated utility function should therefore be defensible on grounds other than its ability to match the facts it was created to match. Also, while there are several theories of why the rich save so much, there is little empirical evidence to evaluate them with, since the most wealthy individuals rarely show up in surveys. As another contribution to the literature, we therefore collect data on the savings behavior among super wealthy Swedes.

Sweden offers a unique opportunity to conduct precisely the type of interviews that are required to collect this data, being a small country with concentrated wealth at the top of the distribution and somewhat publicly known wealthy individuals and industry families. In our study, we want to evaluate potential savings motives with survey questions, in combination with open-ended questions where the wealthy individuals can give more depth to their quantitative answers. In particular, we aim to study not only the motives behind high wealth concentration at the top, but also behind the dominance of private equity in top portfolios. The interviewees will also be given the chance to contrast their own savings motives to those they believe govern the behavior of much less wealthy individuals.

We are currently awaiting the ethical approval to conduct this data collection. This part of the project is thus very much work in progress.

9 Conclusion

TBD.
References


