Comparing Cross-Section and Time-Series Factor Models

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Discussion by

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Overview

▶ Much previous discussion of which characteristics to use
  - Momentum, profitability, investment, . . .
  - Operating profitability, cash profitability, or ROEQ? Annually or monthly updated book-to-market?

▶ Now discussing how to define factors from given characteristics, after sorting to form portfolios
  - High-minus-low for big and small stocks separately and returns minus $R_f$ rather than high-minus-low (Fama-French, 2018)
  - Tangency portfolio return, computing tangency using all other months (Grinblatt-Saxena, 2018)
  - This paper: Slopes from FM regressions on portfolio characteristics rather than high-minus-low

▶ 2nd innovation: Characteristics as factor loadings
Models in this paper:

Model TS: \[ R_i - R_f = a_i + b_{im} (Mkt - R_f) + \sum_{\text{TS factors}} \beta \times \text{factor} + e_i \]

Model CS: \[ R_i - R_f = a_i + b_{im} (Mkt - R_f) + \sum_{\text{CS factors}} \beta \times \text{factor} + e_i \]

Model (2): \[ R_i - R_z = \sum_{\text{CS factors}} \text{char} \times \text{factor} + e_i \]

TS factors are SMB, HML, RMW, and CMA (and maybe UMD). CS factors are FM slope coefficients. Cross-section consists of portfolios from 2 × 3 sorts on size and BM, OP, and INV (and maybe MOM). \( R_z \) is the intercept in the cross-sectional regression. The right-hand side variables in the regressions are characteristics (MC, BM, OP, INV and maybe MOM).

Main Results:

- CS and TS models work equally well for pricing test assets
- Pricing errors (mean absolute mean \( e_i \)) are smaller for Model (2) than are the alphas from the TS and CS models.
Differences between Model CS and Model (2):

1. Model (2) does not include the market beta
2. Model (2) has time-varying loadings
3. Model (2) explains returns in terms of characteristics, not covariances

Lewellen (CFR, 2015) shows that Model (2) has predictive power for returns. He uses past FM coefficients to predict future returns for the assets in the FM regressions.

- This paper uses FM coefficients to explain returns of different assets.

Outline of discussion:

- Removing the market beta may be more important than using characteristics as loadings.
- Some results for the CS model with different cross-sectional factors
Removing the Market Beta from TS and CS

Model TS: \( R_i - R_f = a_i + b_{im}(Mkt - R_f) + \sum_{\text{TS factors}} \text{beta} \times \text{factor} + e_i \)

Model CS: \( R_i - R_f = a_i + b_{im}(Mkt - R_f) + \sum_{\text{CS factors}} \text{beta} \times \text{factor} + e_i \)

Model (2): \( R_i - R_Z = \sum_{\text{CS factors}} \text{char} \times \text{factor} + e_i \)

Model TS*: \( R_i - \hat{R}_Z = a_i + \sum_{\text{TS factors}} \text{beta} \times \text{factor} + e_i \)

Model CS*: \( R_i - \hat{R}_Z = a_i + \sum_{\text{CS factors}} \text{beta} \times \text{factor} + e_i \)
The intercept in a Fama-MacBeth regression is

\[ R_z = R_{ew} - \sum \bar{x}_k R_k \]

where \( R_{ew} \) is the equally weighted return of the assets in the cross-section, the \( \bar{x}_k \) are the cross-sectional means of the RHS variables, and the \( R_k \) are the slope coefficients (Fama, Fundamentals of Finance, Chapter 9).

So, Model (2) is

\[ R_i - R_z = \sum x_{ik} R_k + e_i \iff R_i - R_{ew} = \sum (x_{ik} - \bar{x}_k) R_k + e_i \]

Model CS*: \[ R_i - R_{ew} = a_i + \sum b_{ik} R_k + e_i \]

Model TS* same as CS* but use time-series factors as \( R_k \)
Results for Models CS* and TS*

Mean absolute alphas or pricing errors of test assets (25 portfolios sorted on size and another characteristic). TS factors are Mkt-RF, SMB, HML, RMW, and CMA. CS factors are Mkt-RF and FM slope coefficients on Size, BM, OP, and INV characteristics. Cross-section for the FM regressions consists of the 18 portfolios from the $2 \times 3$ sorts on Size and either BM, OP, or INV.

<table>
<thead>
<tr>
<th>Assets</th>
<th>Model</th>
<th>TS</th>
<th>CS</th>
<th>Mod (2)</th>
<th>CS*</th>
<th>TS*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paper</td>
<td>MC × BM</td>
<td>0.090</td>
<td>0.084</td>
<td>0.059</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>MC × OP</td>
<td>0.064</td>
<td>0.073</td>
<td>0.043</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>MC × INV</td>
<td>0.082</td>
<td>0.106</td>
<td>0.045</td>
<td></td>
<td></td>
</tr>
<tr>
<td>My repl.</td>
<td>MC × BM</td>
<td>0.089</td>
<td>0.090</td>
<td>0.064</td>
<td>0.068</td>
<td>0.076</td>
</tr>
<tr>
<td></td>
<td>MC × OP</td>
<td>0.061</td>
<td>0.080</td>
<td>0.051</td>
<td>0.058</td>
<td>0.073</td>
</tr>
<tr>
<td></td>
<td>MC × INV</td>
<td>0.081</td>
<td>0.112</td>
<td>0.050</td>
<td>0.081</td>
<td>0.078</td>
</tr>
</tbody>
</table>
Variations of Cross-Sectional Factors

- If we are going to define factors from FM regressions, there are many possible variations.
  - High-minus-low returns from 2x3 sorts have advantage of simplicity. But, if we’re going to run regressions, why not enlarge the cross-section?
  - Montone transformations of characteristics matter. This paper uses log ME, but not other logs (I think).
  - Should we scale the FM portfolios and slopes each month so the portfolios are 100% long and 100% short?

- To compare alternative factor definitions, we can use result of Barillas-Shanken (2017, 2018).
  - The factors that minimize alphas of test assets (including alternative factors) in the sense of minimizing $\alpha' \Sigma^{-1}_{\text{resid}} \alpha$ are the factors with the maximum Sharpe ratio.
  - See also FF (2018).
### Sharpe Ratios of Some Cross-Sectional Factors

<table>
<thead>
<tr>
<th></th>
<th>Size</th>
<th>BM</th>
<th>OP</th>
<th>Inv</th>
<th>Agg</th>
</tr>
</thead>
<tbody>
<tr>
<td>TS (usual FF factors)</td>
<td>0.082</td>
<td>0.123</td>
<td>0.118</td>
<td>0.143</td>
<td>0.318</td>
</tr>
<tr>
<td>CS (2x3)</td>
<td>-0.102</td>
<td>0.064</td>
<td>0.175</td>
<td>-0.143</td>
<td>0.344</td>
</tr>
<tr>
<td>logINV</td>
<td>-0.095</td>
<td>0.074</td>
<td>0.197</td>
<td>-0.153</td>
<td>0.369</td>
</tr>
<tr>
<td>5x5</td>
<td>-0.088</td>
<td>0.066</td>
<td>0.159</td>
<td>-0.142</td>
<td>0.323</td>
</tr>
<tr>
<td>5x5 industries</td>
<td>-0.073</td>
<td>0.118</td>
<td>0.185</td>
<td>-0.192</td>
<td>0.344</td>
</tr>
<tr>
<td>5x5 industries logINV</td>
<td>-0.067</td>
<td>0.127</td>
<td>0.209</td>
<td>-0.211</td>
<td>0.369</td>
</tr>
</tbody>
</table>

Agg includes the market return. Other than TS, all factors are FM slopes. logINV replaces the INV characteristic with log(1+INV). 5x5 uses the portfolios from the 5x5 sorts as the cross-section. Industries includes 12 industry weights as characteristics (but not as factors) so all factors have zero net weight in each industry.

Controlling for industries improves factors (next paper!). Value matters more when we control for industries (Cohen-Polk, 1998; Chou et al., 2012)
The portfolios that produce the CS factors are are x% long and x% short, where $x > 100$. For example, the Size factor in 1963-07 is produced by a portfolio that is 122% long and 122% short.

They are levered because they are scaled to have the same standard deviations as the standard TS factors. W/o scaling the CS factors are low risk – maximally diversified subject to constraints.

The leverage in the portfolios decreases substantially over time.
## Cross-Sectional Factors from Individual Stocks

<table>
<thead>
<tr>
<th></th>
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<th>Agg</th>
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</tr>
<tr>
<td>CS (2x3)</td>
<td>-0.102</td>
<td>0.064</td>
<td>0.175</td>
<td>-0.143</td>
<td>0.344</td>
</tr>
<tr>
<td>All</td>
<td>-0.145</td>
<td>0.196</td>
<td>0.106</td>
<td>-0.296</td>
<td>0.498</td>
</tr>
<tr>
<td>All, industries</td>
<td>-0.141</td>
<td>0.280</td>
<td>0.150</td>
<td>-0.361</td>
<td>0.597</td>
</tr>
<tr>
<td>All, weighted</td>
<td>-0.088</td>
<td>0.105</td>
<td>0.177</td>
<td>-0.181</td>
<td>0.343</td>
</tr>
<tr>
<td>All, wtd, industries</td>
<td>-0.086</td>
<td>0.165</td>
<td>0.199</td>
<td>-0.225</td>
<td>0.401</td>
</tr>
</tbody>
</table>

- Factors from FM regressions on all stocks. Agg includes Mkt-RF.
- Industries includes dummy variables for 12 industries so all factors are industry-neutral (industry slopes not included in Agg).
- Weighted is FM regressions weighted by market cap to the power 0.7. The reliance on micro-caps and small stocks is the same or less than for standard FF factors.
Conclusion (or, rather, Questions)

- Are we really happy plugging in characteristics for betas and just assuming (I guess) that betas are linear in characteristics?
  - Isn’t this just giving up on pricing by covariances (pricing by an SDF)?

- How much do we care about whether factors are tradeable? We just want to find an SDF, but
  - Test that $\alpha = 0$ is only right if factors are tradeable.
  - Barillas-Shanken idea that best factors are those with maximum Sharpe ratio is only right if factors are tradeable.

- How much are we going to optimize over this dataset in defining factors?