Equity Term Structures
without Dividend Strips Data

Discussion by
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Main results

- The equity term structure is slightly upward sloping.
- This rejects the Bansal and Yaron (2004) model in simulations.
- Differences in equity yields were near zero prior to early 1990s, and then widened.
- 2-year yields reflect expected dividend growth mainly; 5-year yields are split between expected dividend growth and discount rates.
- The term structure inverts during recessions: short yields rise above long yields.
A simplified model

- An $k \times 1$ vector of standard normal shocks $\epsilon_t$.
- An $k \times 1$ vector of risk prices $\lambda_t$.
- An essentially affine SDF (Duffee, 2002)

\[ m_{t+1} = -r_f - \frac{1}{2} \lambda_t^\top \lambda_t - \lambda_t^\top \epsilon_{t+1} \]

- Then for gross return $R_{t+1}$ lognormally distributed, with $1 \times k$ volatility vector $\sigma_R$,

\[ \log E_t R_{t+1} - r_f = \sigma_R \lambda_t \]
Dividend growth and the price of risk

- Log dividend growth \( d_t = \log D_t \)

\[
\begin{align*}
\Delta d_{t+1} & = g + z_t - \frac{1}{2}\sigma_d\sigma_d^\top + \sigma_d\epsilon_{t+1} \\
z_{t+1} & = \phi_z z_t + \sigma_z\epsilon_{t+1}
\end{align*}
\]

- Price of risk \( \lambda_t = \lambda_0 + \lambda_1 x_t \) for scalar \( x_t \) such that

\[
x_{t+1} = \phi_x x_t + \sigma_x\epsilon_{t+1}
\]

- Assume \( \sigma_d\sigma_z^\top = \sigma_d\sigma_x^\top = \sigma_z\sigma_x^\top = 0. \)
Where do strip yields come from

- **Equity strip price:**
  \[ P_{nt} = E_t^Q [D_{t+n}] = \exp\{-na_n + b_{zn}z_t + b_{xn}x_t\} D_t \]
  for \( b_{zn} > 0 \) and increasing, and \( b_{xn} < 0 \), and usually decreasing.

- **Yield:**
  \[ y_{nt} = \frac{1}{n} (d_t - p_{nt}) = a_n - \frac{1}{n} (b_{zn}z_t + b_{xn}x_t) \]
  where
  \[ a_n \approx r_f + \left( \sigma_d + \frac{1}{n} \sum_{m=1}^n b_{xm}\sigma_x + \frac{1}{n} \sum_{m=1}^n b_{zm}\sigma_z \right) \lambda_0 - g \]

- By comparing \( E[y_{nt}] \) for different \( n \), we learn something about \( \sigma_x\lambda_0 \) and \( \sigma_z\lambda_0 \).
▶ No time-variation in the price of risk: $\sigma_{x} = 0$.
▶ Let $R$ be the market return, with loadings on shocks $\sigma_{R}$.
▶ BY assume parameters such that: $\sigma_{R}\sigma_{z}^{\top} > 0$ (increases in expected CF growth increase prices)
▶ Moreover,

$$\lambda_{0}^{\top} = RRA \sigma_{d} + (RRA - 1/EIS) b\sigma_{z}$$

for $b > 0$. Shocks to long-run CF growth have a positive price.
▶ This implies an upward slope.
Recursive utility more generally

- Prices of risk

\[ \lambda_0^\top \approx \sigma_d + b_z\sigma_z + b_x\sigma_x \]

with \( b_z > 0 \) and \( b_x < 0 \) [assets that go up in price when volatility or disaster risk rises have a lower risk premium than otherwise].

- Returns

\[ \sigma_R \approx \sigma_d + b_{Rz}\sigma_z + b_{Rx}\sigma_x \]

To generate risk premia, parameters chosen so that \( b_{Rz} > 0 \), implies \( b_{Rx} < 0 \).

- Any such model implies an upward slope
Consider an adverse change to the consumption distribution.

Relative to the Consumption CAPM, the agent requires a premium to hold securities that go down in price when this adverse change occurs.

Longer-lived assets will be more affected by the adverse change (duration!).

This causes them to carry risk premia, and thus will be still more affected (implying yet a steeper slope).
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Conditional Consumption CAPM-type models

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These models have $\sigma_z = 0$.

They have $\lambda_0 \propto -\sigma_x \propto \sigma_d$

Marginal utility rises with risk premia, which rise in response to negative normal-times shocks.

Any such model implies an upward slope.
In general

- General expression for a risk premium:

\[
\log E[R] - r_f = b_d \sigma_{Rd} \sigma_d^\top + b_x \sigma_{Rx} \sigma_x^\top + b_z \sigma_{Rz} \sigma_z^\top
\]

- Economic logic implies \( \sigma_{Rx} < 0 \) and \( \sigma_{Rz} > 0 \).
- Must have prices of risk \( b_x < 0 \) and \( b_z > 0 \) to explain the equity premium.
- But then, the equity term structure will be upward sloping.
- \( \Rightarrow \) if the term structure is not upward sloping, the equity premium arises from \( b_d \sigma_{Rd} \sigma_x^\top \) (or rare-event equivalent).
- This would appear to rule out any explanation of the equity premium by means of a dynamic model.
So is the term structure upward sloping?

Growth stocks have lower returns than value stocks ⇒ No

But perhaps other things besides duration drive value and growth.

Equity yields on long-term dividend strips are below those on short-term strips ⇒ No

But perhaps the sample is too short to tell if this is a population mean.

This paper: the equity term structure extracted from factors ⇒ No

Where is this result coming from?
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- Where is this result coming from?
This is hard.

You need an estimate of dividend growth (to establish duration).

For this, you need to estimate the persistence of dividend growth changes away from their mean.

At the same time, this persistence also determines the shape of the term structure.
Assumption: only shocks to returns are priced.

This seems sensible: because if there are risk premia, the shocks must be priced.

But consider this within the lens of the previous model:

$$\lambda_0^T \propto \sigma_R = \sigma_d + b_{xR}\sigma_x + b_{zR}\sigma_z.$$  

we know that the last two terms should be large in magnitude: only so much of returns can come from realized dividend growth.

Thus the model implies an upward sloping term structure by construction.

Why is the slope shallow? Because most of the variation explained by $z$, and $\phi_z << 1$
Some advantages of this explanation

- It explains why short-term yields load more on expected dividend growth.
- It explains why short-term yields are much more volatile, and spike up in a recession
  - Dividends are expected to be low and then recover.
- It explains why the gap in yields opened in the late 1990s (when the dividend growth component became more variable).
Conclusions

▶ This paper recovers equity strips from long-lived assets
▶ Conjecture: the equity yield curve is upward sloping in this paper by construction
▶ Even bigger conjecture: the equity yield curve “wants” to be downward sloping, which is why authors find a slight upward slope
▶ Findings suggest that thinking carefully about dynamic models can only hurt you when it comes to explaining the equity premium.