Origins of International Factor Structures*

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Abstract

We develop and test a model of the global trade network. This model connects international co-movements to a simple measure of network closeness, constructed from observed trade weights. We report three findings: (1) Countries that are closer in the network tend to have more correlated consumption growth rates, more correlated stock returns, and more correlated exchange rate movements. (2) International comovements can be decomposed into a component driven by primitive productivity shocks and a component due to network transmissions. Asset price correlations tend to be explained by the network structure, while consumption correlations by the correlations of primitive shocks. (3) The trade network generates factor structures in equity returns and exchange rate movements. It helps to explain the existence of the dollar and the carry factors, and gives rise to regional factors. These findings offer a network-based account of the origins of factor structures in international economic quantities and asset prices.

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The world is connected through a global trade network that transmits shocks across countries. In this paper, we develop a model of this global trade network and test its implications. We show how the network structure gives rise to related co-movements in international asset prices and economic quantities.

In our model, a measure of network closeness describes how productivity shocks propagate through the international trade network. Importantly, this measure is only constructed using trade quantities — the shares of goods each country imports from other countries for its production and consumption. We find that this closeness measure explains a great deal of international co-movements in economic quantities and asset prices. Figure (1) illustrates on of our key findings. Countries with higher closeness have more correlated consumption growth rates, more correlated equity returns, more correlated currency strength\(^1\), and less volatile bilateral exchange rates.

These empirical findings are direct implications of our model. In our model, each country produces its tradable goods using inputs from other countries. This interdependence gives rise to a global production network, which transmits productivity shocks across countries that trade with each other. Countries are exposed through higher-order connections, too. For example, if China’s production relies on Brazil’s output and Germany’s production relies on China’s output, then Germany is also exposed to Brazil’s productivity shocks.

On top of the production layer, the representative households in each country consume both home and foreign goods with different marginal rates of substitution. As a result, the equilibrium consumption and asset prices are determined by the interaction between the global production network, the consumption weights, and the primitive productivity shocks. This structure allows us to decompose international co-movements into two components: network closeness and exposure closeness. Network closeness accounts for the structure of input-output linkages in the production and consumption networks, whereas exposure closeness is the correlation structure of the primitive productivity shocks that are necessary to match observed consumption growth correlations in the presence of the trade networks.

In the data, we find that consumption growth correlations are mainly explained by ex-

\(^1\)We define each country’s base currency factor as the equal-weighted average of its log exchange rate against all other currencies, which measures the relative strength of its currency.
posure closeness, while stock return correlations and exchange rate correlations tend to be explained by network closeness. This helps to reconcile our results with the literature that finds little relation amongst international asset prices and quantities. Our decomposition demonstrates that the cross-country variation in asset prices is related to the global production and consumption networks, which are separate from the correlation structure of the primitive fundamental shocks. In this way, we provide a framework to understand the sources of covariation in economic quantities and asset prices across countries.

We also demonstrate how the network structure gives rise to international factor structures in asset prices and economic quantities. In particular, we show that the covariation that arises from the structure of the global trade network is related to currency factors such as dollar, carry, and peripheral-minus-central (Lustig, Roussanov, and Verdelhan 2014, 2011; Richmond 2019). To show this, we extract principal components from the covariance matrix implied from our measure of network closeness. We find that each country’s loadings on these principal components explain much of the variation in the country’s loadings on standard currency risk factors. Currency portfolios constructed from these network-based principal components are also highly correlated with standard currency risk factors. These findings suggest a close link between economic fundamentals, as measured by the structure of the global trade network, and the factor structures in international asset prices.

Finally, we show how the structure of the global trade network can give rise to common regional variation. In the model, a block structure in the international production network will manifest itself as a factor structure in consumption growth and asset prices. In particular, if each country trades only with countries in the same region, then a weighted average of idiosyncratic productivity shocks within each region becomes a systematic factor.

To test this prediction, we use a clustering algorithm to assign countries into blocks based on their network closeness, and construct asset pricing factors as weighted averages across countries in each block. We find that this approximate block structure from the consumption and production network is apparent in the factor structure in global equity market returns and exchange rates. Countries within each block are more exposed to the factor within the block than factors in other blocks. Asset prices and economic quantities also tend to be more correlated within the same block. This demonstrates how idiosyncratic country-level shocks
can give rise to common variation in asset prices across countries and how exposure to these common factors arise from the structure of the global trade network.

Related Literature  Our work is most related to the literature on the common factors in international asset prices. Lustig, Roussanov, and Verdelhan (2011); Verdelhan (2018) show that exchange rate movements are driven by a few common factors. Lustig and Richmond (2017) find that this factor structure is related to bilateral measures of physical, cultural, and institutional distance. Aloosh and Bekaert (2019) use a clustering algorithm on currency base factors and show that cluster-based factors explain variation currency base factors quite well. They also present evidence that the factor structure is apparent in retail sales growth data across countries. Forbes and Rigobon (2002); Bekaert, Hodrick, and Zhang (2009) also find co-movements in international equity returns. Our paper studies a global trade network that provides a structural way to understand the origins of this factor structure.

This literature also relates this factor structure to other real quantities. For example, Richmond (2019) shows how currency risk premia are related to trade network position and how this position affects consumption growth correlations with aggregate consumption. Ready, Roussanov, and Ward (2017) show how each country’s trade composition between final and commodity goods impacts its exchange rate behavior. Corte, Riddiough, and Sarno (2016) discover a priced risk factor based on external imbalances. Jiang (2018) finds that the factor structure in exchange rates is related to the factor structure in government surpluses.

Our work is also related to the literature on the co-movements in international business cycles. Gregory and Head (1999); Kose, Otrok, and Whiteman (2003); Imbs (2004); Calderon, Chong, and Stein (2007); Burstein, Kurz, and Tesar (2008); Rose and Spiegel (2009) find trade, specialization, and financial integration are relevant for determining the international co-movements in business-cycle fluctuations. Bayoumi and Eichengreen (1998); Devereux and Lane (2003) show that trade links and common economic shocks also affect bilateral exchange rate volatility. Our work structurally relates these co-movements to the input-output linkages in the international production network.

In terms of theory, our model is a simplified version of international business cycle model (Backus, Kehoe, and Kydland (1992)). Colacito et al. (2018) study a generalization of this
framework with recursive preferences. Our model is also related to network and granularity models such as Long Jr and Plosser (1983); Gabaix (2011); Foerster, Sarte, and Watson (2011); Acemoglu et al. (2012); Chaney (2014), as well as to the literature that connects asset pricing to input-output linkages across firms such as Herskovic et al. (2016); Herskovic (2018); Gofman, Segal, and Wu (2018). We adapt the network model in an international context, and confirm that international co-movements align with the predictions of a simple network model. Contemporaneous to our work, Huo, Levchenko, and Pandalai-Nayar (2018) build a production model with trade network, and show how the distribution of international GDP comovements would change if technology and non-technology shocks are turned off. In comparison, our work proposes a trade-based closeness measure and shows how it explains business cycle and asset price comovements between each pair of countries.

Moreover, the literature on optimal currency areas since Friedman (1953) and Mundell (1961) suggests that countries with correlated business cycles should form currency unions. This is because, on net, countries with more correlated business cycles benefit more from lower transactions costs in international trade. Frankel and Rose (1998) show that business cycles are endogenous: Countries that trade more tend to have more correlated business cycles. Our paper provides a network-based explanation for the relationship between trade and business cycle correlations and connects these findings to asset prices and factor structures.

I. Model

A. Set-Up

Time is discrete and infinite, indexed by \( t = \{1, 2, \ldots\} \). There is no storage technology that allows agents to transform consumption goods across periods.

There are \( N \) countries. Each country produces a distinct tradable goods. The representative household in each country has the following production function:

\[
X_{it} = A_i L_{it}^{\theta_i} \left( \prod_{j=1}^{N} X_{ijt}^{w_{ij}} \right)
\]

where \( A_i \) measures the productivity, \( L_{it} \) is the labor input, and \( X_{ijt} \) is the tradable goods.
from country $j$ that are used as production inputs in country $i$. The parameter $\theta_i$ measures the contribution of country $i$'s labor, and the parameter $w_{ij}$ measures the contribution of country $j$'s input. These parameters satisfy

$$\theta_i + \sum_{j=1}^{N} w_{ij} = 1,$$

$$\theta_i, w_{ij} > 0.$$

The aggregate consumption $\bar{C}_{it}$ in country $i$ at time $t$ is assembled from each country’s tradable goods:

$$\bar{C}_{it} = \prod_{j=1}^{N} C_{ijt}^{v_{ij}}.$$  

The parameters satisfy

$$\sum_{j=1}^{N} v_{ij} = 1,$$

$$v_{ij} > 0.$$

The market clearing condition for country $i$'s tradable goods is

$$\bar{X}_{it} = \sum_{j=1}^{N} (C_{jit} + X_{jit}).$$

It is worth noting that country $i$ imports country $j$’s tradable goods not only for consumption, but also for country $i$’s production of its tradable goods.

Labor supply is fixed. The market clearing condition for country $i$’s labor is

$$L_{it} = L_i.$$  

Define lower case letters to be the logs. A variable with its country subscript omitted is a vector. For example, $\bar{c}_i$ is the vector where each element is log $\bar{C}_{it}$. A capitalized parameter with two country indices omitted is a matrix. For example, $W$ is the matrix with each
element being \( w_{ij} \).

All households have log preferences and discount future utility at rate \( \beta \). Markets are complete. In each period, households in all countries share risk before any shock is realized. Therefore, the competitive equilibrium can be characterized by the solution to a social planner’s problem. The social planner assigns Pareto weights \( \lambda_i \) to country \( i \), and maximizes:

\[
\sum_{i=1}^{N} \lambda_i \sum_{t=1}^{\infty} \exp(-\beta t) \log C_{it}.
\]

The log productivity follows a random walk:

\[
a_{it+1} = a_t + \varepsilon_{it+1}.
\]

These shocks \( \varepsilon_{t+1} \) are normally distributed with mean zero. They are mutually independent across time, but they can be correlated across countries. Their covariance matrix is defined as

\[
\mathbb{E}_t[\varepsilon_{t+1}\varepsilon'_{t+1}] = \Omega.
\]

B. Real Quantities and Trade Network

**Lemma 1 (Real Quantities).** For some constants \( \kappa^x \), the log production growth rate is

\[
\pi_{t+1} = (I - W)^{-1}(\kappa^x + a_{t+1}),
\]

\[
\Delta \pi_{t+1} = (I - W)^{-1}\varepsilon_{t+1}.
\]

The log consumption growth rate is

\[
\Delta \pi_{t+1} = V(I - W)^{-1}\varepsilon_{t+1}.
\]

The equilibrium consumption is determined by each country’s productivity shock \( a \) and the trade network \( V \) and \( W \).
We can also use the quantity data to map out the input-output network. Let \( q_{jt} \) denote the price of the tradable goods in country \( j \). Then, \( q_{jt}X_{ijt} \) is the total value of inputs of intermediate goods from country \( j \) to country \( i \), and \( q_{jt}C_{ijt} \) is the total value of inputs of consumption goods from country \( j \) to country \( i \).

**Lemma 2 (Input-Output Network).** (a) Input shares of intermediate goods reflect matrix \( W \):

\[
\frac{q_{jt}X_{ijt}}{q_{kt}X_{ikt}} = \frac{w_{ij}}{w_{ik}}.
\]

(b) Input shares of consumption goods reflect matrix \( V \):

\[
\frac{q_{jt}C_{ijt}}{q_{kt}C_{ikt}} = \frac{v_{ij}}{v_{ik}}.
\]

(c) The share of value added in total output reflects the labor share \( \theta_i \):

\[
\frac{q_{it}X_{it} - \sum_{j=1}^{N} q_{jt}X_{ijt}}{q_{it}X_{it}} = \theta_i.
\]

We note that the labor factor in the production function allows us to separate a country’s value add from the cost of its inputs. Moreover, as in Cole and Obstfeld (1991), the Cobb-Douglas aggregation of consumption goods implies that the price of good \( q_{it} \) is inversely proportional to the aggregate quantity \( \overline{X}_{it} \). Under certain parametric restrictions, the gains from having complete markets are minimal. The real exchange rates and the stock returns are nevertheless still well-defined, as shown below.

**C. Real Exchange Rates and Stock Returns**

We define the bilateral log real exchange rate \( e_{ijt} \) between countries \( i \) and \( j \) as the log exchange rate between their consumption bundles. For country \( i \), we define its base currency factor \( \overline{e}_{it} \) as its equal-weighted average real exchange rate against all countries, including
itself:

\[ \bar{e}_{it} = \frac{1}{N} \sum_{j=1}^{N} e_{ijt}. \]

This base currency factor can be regarded as the exchange rate index of country \( i \).

We define each country’s stock market as the claim to the future consumption stream. Since the household uses its production to purchase its inputs \( X_{ijt} \) and consumption \( C_{ijt} \):

\[ q_{it}X_{it} = \sum_{j=1}^{N} q_{jt}(C_{ijt} + X_{ijt}), \]

the value of the country’s future consumption stream also equals the value of its future production minus production costs.

Let \( P_{it}^{eq} \) denote the ex-dividend stock price in the unit of local consumption bundle. Let \( r_{it+1}^{eq} \) denote the log cum-dividend return:

\[ r_{it+1}^{eq} = \log \frac{C_{it+1} + P_{it+1}^{eq}}{P_{it}^{eq}}. \]

**Lemma 3 (Asset Prices).** (a) The base currency factor movement is

\[ \Delta \bar{e}_{it+1} = \frac{1}{N} \sum_{j=1}^{N} \Delta \bar{e}_{jt+1} - \Delta \bar{e}_{it+1}. \]

(b) The log cum-dividend return of the stock market is

\[ r_{it+1}^{eq} = \beta + \Delta \bar{e}_{it+1}. \]

**D. Characterizing the Covariance Structure**

Define network profile \( H \) as the vector

\[ H \equiv V(I - W)^{-1}, \]
and define the closeness between two countries $i$ and $j$ as

$$C(i, j) \equiv \{H\Omega H'\}_{ij}.$$ 

Define the average closeness of country $i$ as the average closeness between country $i$ and all countries:

$$\overline{C}(i) = \frac{1}{N} \sum_{j=1}^{N} C(i, j).$$

The next two propositions characterize the relationship between the closeness, consumption growth, stock returns, and real exchange rate movements.

**Proposition 1 (Consumption Growth and Equity Returns).** Closer countries have more correlated consumption growth rates and more correlated stock returns:

$$cov(\Delta c_{it}, \Delta c_{jt}) = C(i, j),$$

$$cov(r^e_{it}, r^e_{jt}) = C(i, j).$$

**Proposition 2 (Exchange Rate Movements).** (a) Controlling for country-level fixed effects, closer countries also have more correlated base currency factor movements and less volatile bilateral real exchange rate movements:

$$cov(\Delta e_{it}, \Delta e_{jt}) = C(i, j) - \overline{C}(i) - \overline{C}(j) + \kappa^e,$$

$$var(\Delta e_{ijt}) = -2C(i, j) + C(i, i) + C(j, j),$$

where $\kappa^e$ is a constant that applies to all countries:

$$\kappa^e = \frac{1}{N^2} \sum_{k=1}^{N} \sum_{\ell=1}^{N} C(k, \ell).$$

(b) The variance of country $i$'s base currency factor movement is

$$var(\Delta e_{it}) = -2\overline{C}(i) + C(i, i) + \kappa^e.$$
Fixing its closeness to itself $C(i,i)$, the country’s base currency factor movement is less volatile if it has a higher average closeness.

(c) The loading of each bilateral exchange rate on the base currency factor (Lustig and Richmond 2017) is given by:

\[
\frac{cov(\Delta \bar{e}_{it}, \Delta e_{ijt})}{var(\Delta \bar{e}_{it})} = \frac{\bar{C}(j) - C(i) - C(i,j) + C(i,i)}{-2\bar{C}(i) + C(i,i) + \kappa e}.
\]

E. Some Examples

We present three examples that help understand this network structure. In these examples, we assume the home bias in consumption tends to 1 in the limit:

\[
v_{ij} \to \begin{cases} 
1, & \text{if } i = j, \\
0, & \text{if } i \neq j,
\end{cases}
\]

which allows us to abstract away the consumption trade network $V$ and focus on the production trade network $W$. In the data, households have high degrees of home bias in consumption.

Example 1: Three Countries

Consider two similar economies, both with three countries, $a$, $b$, and $c$. In economy 1, countries $a$ and $b$ do not import from other countries for production input, whereas country $c$ needs to. In economy 2, countries $a$ and $b$ need to import from country $c$ for production input, whereas country $c$ does not.

\[
W_1 = (1 - \theta) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1/3 & 1/3 & 1/3 \end{bmatrix}, \quad W_2 = (1 - \theta) \begin{bmatrix} 2/3 & 0 & 1/3 \\ 0 & 2/3 & 1/3 \\ 0 & 0 & 1 \end{bmatrix}.
\]

where the labor share $\theta_i = \theta$ is the same for each country.

In both economies, the only international links are between countries $a$ and $c$, and between
countries \( b \) and \( c \), both with weight \( 1/3 \). The average closeness in economy 1 and 2 are

\[
(I - (1 - \theta)W_1)^{-1}((I - (1 - \theta)W_1)^{-1})' \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix} = \begin{bmatrix} \frac{1}{\theta^2(2+\theta)} & \frac{1}{\theta^2(2+\theta)} & \frac{2 - 2\theta + 3\theta^2}{\theta^2(2+\theta)^2} \end{bmatrix},
\]

\[
(I - (1 - \theta)W_2)^{-1}((I - (1 - \theta)W_2)^{-1})' \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix} = \begin{bmatrix} \frac{1 - 1\theta + 3\theta^2}{\theta^2(1+2\theta)^2} & \frac{1 - 1\theta + 3\theta^2}{\theta^2(1+2\theta)^2} & \frac{1}{\theta^2 + 2\theta^3} \end{bmatrix}.
\]

Country \( c \) has the smallest average closeness in economy 1 and the largest average closeness in economy 2:

\[
\frac{1}{\theta^2(2+\theta)} - \frac{2 - 2\theta + 3\theta^2}{\theta^2(2+\theta)^2} = \frac{3\theta - 3\theta^2}{\theta^2(2+\theta)^2} > 0,
\]

\[
\frac{1 - 1\theta + 3\theta^2}{\theta^2(1+2\theta)^2} - \frac{1}{\theta^2 + 2\theta^3} = \frac{3(\theta - 1)}{\theta(1+2\theta)^2} < 0.
\]

Country \( c \) also has the smallest consumption variance \( C(i, i) \) in economy 1 and the largest consumption variance \( C(i, i) \) in economy 2.

This example shows that the network is directed in this framework. Productivity shocks propagate downstream. All else equal, a country that exports to other countries is more central than a country that imports from other countries.

**Example 2: Linkage with the Giant**

Consider a different economy with three countries, \( a \), \( b \), and \( c \). Country \( c \) is the only country that exports to other countries. Country \( a \) imports less from country \( c \) than country \( b \) does:

\[
W = (1 - \theta) \begin{bmatrix} 0.9 & 0 & 0.1 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \end{bmatrix},
\]

where the labor share \( \theta_i = \theta \) is the same for each country.
Country \( c \) always has the greatest average closeness \( C(i) \). Between countries \( a \) and \( b \), if the labor share \( \theta \) is small, i.e.

\[
\theta < \frac{3 + 4\sqrt{6}}{29} \approx 0.44,
\]

then country \( b \) has a greater average closeness \( C(i) \). Otherwise country \( a \) has a greater average closeness.

As \( \theta \) decreases, production relies more on the inputs from domestic and foreign goods, thereby amplifying the shocks from foreign countries. Since a greater average closeness means a greater exposure to the major shocks in the world, a country can also become more exposed by importing more from the country that produces major shocks. Indeed, in our sample, Ireland has the 5th greatest average closeness. It barely exports to any foreign countries, but imports a large fraction of production inputs from other central countries such as United States.

**Example 3: Integrated Regions**

We can further flesh out the origins of international co-movements by specifying the following structure:

**Assumption 1.** (a) The productivity shocks load on systematic shocks \( u_t \) and idiosyncratic shocks \( \eta_t \):

\[
\varepsilon_t = \Psi u_t + \Xi \eta_t,
\]

which \( u_t \) is a \( K \)-by-1 vector of standard normal random variables and \( \eta_t \) is a \( N \)-by-1 vector of standard normal random variables. \( u_t \) and \( \eta_t \) are mutually independent. \( \Xi \) is a diagonal matrix with elements \( \xi_{ii} \), so other countries’ idiosyncratic shocks \( \eta_{jt} \) do not affect \( \varepsilon_{it} \).

(b) The production trade network has a block structure. There are \( M \) regions, and each country only imports from other countries in the same region for production inputs. Let
$R(i)$ denote the set of countries in the same region as country $i$. We assume

$$w_{ij} = \begin{cases} 
\omega_0, & \text{if } i = j, \\
\omega_1, & \text{if } i \neq j \text{ and } j \in R(i), \\
0, & \text{if } j \not\in R(i),
\end{cases}$$

where the parameters satisfy $\omega_0 + (|R(i)| - 1)\omega_1 = 1 - \theta_i$. The parameters $\omega_0$ and $\omega_1$ can vary across regions, but we drop the index of the region for notational simplicity.

Assumption 1(a) allows the productivity shocks to have $K$ systematic components. The covariance matrix $\Omega$ for productivity shocks can be expressed as

$$\Omega = \Psi \Psi' + \Xi \Xi'.$$

Since $\eta$ is a country-specific shock, $\Xi \Xi'$ is a diagonal matrix. So, only $\Psi \Psi'$ generates co-movements between different countries’ productivity shocks. However, later we will show that the trade network propagates the idiosyncratic shocks within the region.

Assumption 1(b) highlights the fact that trade partners are related through geographical or industrial proximity. For example, United States, Canada and Mexico mainly trade with each other because they are close. Similarly, countries linked by supply chains also import each other’s goods.

Under these simplifying assumptions, the following proposition characterizes how the aggregate factor structures in quantities and asset returns comes from productivity shocks and the trade network.

**Proposition 3 (Regions and the Factor Structure).** (a) The consumption growth in each country is driven by the systematic shocks $v_k$, a regional shock that is a weighted average of the idiosyncratic shocks from countries in the same region, and the country-specific shock $\eta_i$:

$$\Delta \bar{c}_{it} = \sum_{k=1}^{K} \kappa^h \left( \theta_i \psi_{ik} + \sum_{j \in R(i)} \omega_1 \psi_{jk} \right) u_{kt+1} + \kappa^h \omega_1 \left( \sum_{j \in R(i)} \xi_{ij} \eta_{jt+1} \right) + \kappa^h \theta_i \xi_{ii} \eta_{it+1},$$

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where $\kappa^h > 0$ is a constant that depends on $\omega_0$ and $\omega_1$.

(b) The closeness between country $i$ and country $j$ is

$$C(i, j) = \begin{cases} 
(\kappa^h)^2 \left( \theta_i \psi_i + \sum_{k \in R(i)} \omega_1 \psi_k \right) \left( \theta_j \psi_j' + \sum_{k \in R(i)} \omega_1 \psi_k' \right)' + (\kappa^h \omega_1)^2 \sum_{k \in R(i)} \xi_{kk}, & \text{if } j \in R(i), \\
(\kappa^h \theta_i)^2 \psi_i \psi_j', & \text{if } j \notin R(i).
\end{cases}$$

(c) If all countries have the same idiosyncratic variance $\xi_{ii}^2$ and the same loadings on the systematic shock $\psi_i$, then the first $K + M$ principal components in consumption growth are the systematic shocks $u_{kt+1}$ and the regional shocks $\sum_{j \in R(i)} \xi_{jj} \eta_{jt+1}$. Countries are closer to countries in the same region than to countries in other regions.

This proposition shows that a single regional factor arises from the production network within each region. Within each region, a country with a higher productivity volatility $\xi_{jj}$ has a higher weight in the regional factor. Each country’s consumption growth $\Delta \tilde{c}_{it+1}$ loads on the systematic productivity shocks $u_{kt+1}$, this regional factor, and its own idiosyncratic productivity shock $\eta_{it+1}$. Both the systematic shocks and the regional factors affect aggregate fluctuations in real quantities and asset returns: Two countries are close if they have similar loadings on the systematic shocks or if they are in the same region.

II. Data

A. Data Sources

We consider two samples. Our primary sample contains developed countries as classified by MSCI: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom, and United States. The second sample of developed and emerging countries adds Brazil, China, Czech Republic, Greece, Hungary, India, Indonesia, Mexico, Poland, Russia, South Korea, and Turkey. The sample of countries is primarily limited by the coverage of the World Input-Output Database.

Our World Input Output Tables (WIOT) are from World Input-Output Database (Timmer et al. (2015)). The data is annual from 2000 to 2014.
Our spot exchange rate data are monthly from Global Financial Data from April 1973 to December 2014. We end the sample in 2014 to align with the last available year of the World Input-Output Database. We refine the base currency factor of country $i$ as in Lustig and Richmond (2017) and Verdelhan (2018) as the average log real exchange rate with respect to all foreign countries:

$$ e_{it} = \frac{1}{N-1} \sum_{j \neq i} e_{ijt}; $$

(1)

a positive value means a stronger currency in country $i$ relative to other currencies.

The consumption data are from the World Bank’s World Development Indicators from 1973 until 2014. Gravity variables are from CEPII.

B. Estimating the Trade Network

We use the WIOT to measure the bilateral consumption and production weights that our model takes as parameters. Following Lemma 2, we recover the production matrix $W$ and the labor share $\theta_i$ from the input shares of intermediate goods and final consumption shares.

For our main results we take the average of $\theta$, $W$ and $V$ across all years in the WIOT. This is equivalent to assuming that the structure of global production remained stable over our sample period. Then, we use these average parameters from WIOT to explain asset prices and quantities in our sample period that start before the WIOT data are available. While the time-series evolution of the global structure of production and consumption is interesting, we leave the study of this for future work.

Figure (2) and Figure (3) visualize the intermediate trade network $W$ and the consumption trade network $V$. Countries that are closer in these figures have stronger connections in the matrices $W$ and $V$. We notice that the intermediate trade network and the consumption trade network do not necessarily coincide.
C. Estimating the Covariances of Productivity Shocks

We estimate the covariance matrix \( \Omega \) of productivity shocks from the covariance between consumption growth rates. According to Proposition 1,

\[
\text{cov}(\Delta \tau_{it}, \Delta \tau_{jt}) = C(i, j) \equiv \{H \Omega H'\}_{ij}
\]

(2)

where \( H = V(I - W)^{-1} \).

Given our estimate of the trade networks \( W \) and \( V \), we solve for \( \Omega \) using the covariance matrix of consumption growths. Figure (4) depicts the covariance matrix \( \Omega \). Countries that are closer in this figure have larger covariances in \( \Omega \).

D. Measures of Closeness

As the closeness is defined as \( C(i, j) = \{H \Omega H'\}_{ij} \), two countries can be close because of either their locations in the trade network \( H \) or their systematic risk exposures \( \Omega \). To separate these components, we examine five different measures of closeness.

First, we examine the total closeness as driven by both the network and the implied covariance of shocks:

\[
C^{Consumption}(i, j) = \{H \Omega H'\}_{ij},
\]

which, by construction, is the covariance in consumption growth rates \( \text{cov}(\Delta \tau_{it}, \Delta \tau_{jt}) \).

Second, we examine the closeness as driven by systematic risk exposures:

\[
C^{Exposure}(i, j) = \{\Omega\}_{ij},
\]

which amounts to assuming autarky in consumption and production, \( H = I \).

Third, we examine the closeness as driven by the trade network:

\[
C^{Network}(i, j) = \{H H'\}_{ij},
\]

which amounts to assuming the productivity shock in each country is i.i.d., \( \Omega = I \).
Fourth, we further reduce the trade network to production network by assuming perfect home bias in consumption, i.e. $V = I$. Since $H = V(I - W)^{-1}$, this assumption gives rise to a closeness measure that is only based on the production trade network:

$$C^{Production}(i, j) = \{(I - W)^{-1}((I - W)^{-1})'\}_{ij}.$$ 

Lastly, we further apply a first-order approximation to $C^{Production}(i, j)$. Noticing

$$(I - W)^{-1} = I + W + W^2 + \ldots,$$

we define

$$C^{ProductionFO}(i, j) = \{I + W + W'\}_{ij} = w_{ij} + w_{ji} \text{ for } i \neq j.$$ 

This expression is very similar to the trade intensity in the optimal currency area literature, which is usually defined as

$$\mathcal{I}(i, j) = \frac{q_j(X_{ij} + C_{ij}) + q_i(X_{ji} + C_{ji})}{q_iX_i + q_jX_j} = w_{ij} \frac{q_iX_i}{q_iX_i + q_jX_j} + w_{ji} \frac{q_jX_j}{q_iX_i + q_jX_j} + \frac{q_jC_{ij} + q_iC_{ji}}{q_iX_i + q_jX_j},$$

which is a value-weighted average of $w_{ij}$ and $w_{ji}$ plus the shares of consumption imports. In the data, we find that $C^{ProductionFO}(i, j)$ and $\mathcal{I}(i, j)$ have a correlation of 68% across country pairs.

All these measures of closeness are covariances. To compare across countries with different volatilities, we normalize them by the two countries’ corresponding standard deviations. We define correlation based closeness for each type of closeness (network, exposure, etc.) as

$$\tilde{C}^{Type}(i, j) = \frac{C^{Type}(i, j)}{\sqrt{C^{Type}(i, i)} \sqrt{C^{Type}(j, j)}}.$$ 

To understand what drives variation in our closeness measures, we regress bilateral closeness on gravity variables. Table 1 reports the results. Countries that are physically closer,
share a common language, and have common borders tend to have higher network-based
closeness. Interestingly, these distance measures are not correlated with the consumption
correlation and the exposure-based closeness.

III. Empirical Tests

A. Closeness and International co-movements

Proposition 1 predicts that countries that are closer have more correlated production growth
rates, more correlated consumption growth rates, more correlated real exchange rate move-
ments, more correlated stock returns, and less volatile bilateral real exchange rate move-
ments. We use the five measures of closeness as defined in Section II.D.

Following Proposition 1(a), we run regressions of the correlations in consumption, $corr(\Delta c_{it}, \Delta c_{jt})$, and stock returns in local currency, $corr(r_{it}^{eq}, r_{jt}^{eq})$, on the correlation based bilateral close-
ness, $\tilde{C}(i, j)$:

$$corr(\Delta c_{it}, \Delta c_{jt}) = \alpha + \beta \cdot \tilde{C}(i, j) + \varepsilon_{ij}, \quad (3)$$

$$corr(r_{it}^{eq}, r_{jt}^{eq}) = \alpha + \beta \cdot \tilde{C}(i, j) + \varepsilon_{ij}. \quad (4)$$

Table (2) reports the result in the sample of developed countries and Table (3) for the
sample of developed and emerging countries. For all bilateral regressions we only select the
set of unique country pairs. That is, we keep $(i, j)$, but do not include $(j, i)$, $(i, i)$, and $(j, j)$.

The first 4 columns explain consumption correlations. The majority of variation in con-
sumption growth correlations appears to be explained by variation in exposure based close-
ness, with an R-squared of 43% versus 8% for network closeness in the developed sample.
The third and fourth columns explain consumption correlations with production based close-
ness measures. The R-squared in both of these columns is about 8%, similar to that of the
network based closeness measure and substantially lower than exposure closeness.

The last 5 columns of Table (2) and Table (3) explain bilateral equity market correlations.
Network closeness has the highest explanatory power for equity market correlations with an
R-squared of 32% and 23% for the developed and the full samples respectively. This is in
contrast to consumption closeness of which only explains about 11% in both samples. This suggests that correlation in equity markets is mostly related to the structure of the global production and consumption network rather than the primitive shocks measures by exposure closeness.

The last two columns of Table (2) and Table (3) present results using production closeness and its first order approximation. These have similar explanatory power of about 29% and 18% in each of the samples. This suggests that first order connections in the production network capture much of the explanatory power for equity market correlations. That said, the R-squared using production based closeness measures is lower than when using network closeness. This implies that it is important to take into account the structure of the global consumption network as well as the production network when explaining equity market correlations.

Following Proposition 2(a), we run similar regressions for the correlations of base currency factors and the volatility of the bilateral exchange rate movement, with country fixed effects:

\[
\text{corr} (\Delta \tau_{it}, \Delta \tau_{jt}) = \delta_i + \delta_j + \beta \cdot \tilde{C}(i, j) + \epsilon_{ij},
\]

\[
\text{std}(\Delta \epsilon_{i,jt}) = \delta_i + \delta_j + \beta \cdot \tilde{C}(i, j) + \epsilon_{ij}.
\]

The results are presented in Table (4) for the developed sample and Table (5) for the developed and emerging sample. We find similar results for exchange rates to those that we found for equity market correlations. Network closeness has the highest explanatory power for currency base factor correlations with an R-squared of 22% and 17% in the developed and full samples respectively. In comparison, the exposure closeness and the consumption correlation capture much smaller variations in these exchange rate moments. Production based closeness measures explain more than exposure closeness, but again not as much as network closeness. We conclude that by taking into account the structure of global consumption and production, our network closeness measure explains variation across countries that drives co-movement in their asset prices.

Finally, we present evidence for Proposition 2(b) in Figure (5). We plot the variance of each country’s currency base factor versus the implied one from the model using network
based values. As the figure illustrates, there is a strong positive relation between actual currency base factor variances and those that are implied by the global production and consumption network.

Overall, these findings confirm Proposition 1 and 2 in our model, showing that the network closeness explains a large amount of variation in bilateral co-movements in consumption growth, equity returns, and exchange rate movements.

IV. International Factor Structures

A. Principal Component Analysis

We next study whether the co-variance structure in our measures of closeness are related to the factor structures in international asset prices. To do so, we apply a principal component analysis to the covariance matrices from network closeness $C^{\text{Network}}$ and exposure closeness $C^{\text{Exposure}}$. Specifically, for any covariance matrix $\mathcal{C}$, we can apply the eigenvalue decomposition:

$$\mathcal{C} = U \Lambda U',$$

where $U$ is an orthogonal matrix, and $\Lambda = \text{diag}(\lambda_1, \ldots, \lambda_N)$ is diagonal. The scalars $\lambda_j$ are the eigenvalues of the matrix $\mathcal{C}$.

If $\lambda_k$ is the largest eigenvalue among $\{\lambda_1, \ldots, \lambda_N\}$, the $k$-th column of $U$ is the first principal component $PC_1(\mathcal{C})$ implied by the covariance matrix $\mathcal{C}$. This eigenvector will explain a fraction of the total variance given by

$$\frac{\lambda_k}{\sum_{j=1}^{N} \lambda_i}.$$ 

Similarly, the second principal component $PC_2(\mathcal{C})$ corresponds to the second largest eigenvalue among $\{\lambda_1, \ldots, \lambda_N\}$, and so on.

Table 6 reports the fractions of variance explained by the first 3 PCs for four covariance matrices: network closeness ($C^{\text{Network}}$), exposure closeness ($C^{\text{Exposure}}$), the covariance matrix of consumption growth ($C^{\text{Consumption}}$), and the covariance matrix of currency base factor
movements \((C^{FX})\). The first 3 PCs explain the majority of variation in \(C^{Exposure}\), \(C^{Consumption}\), and \(C^{FX}\).

In comparison, the first 3 PCs explain very little variation in \(C^{Network}\). This low explanatory power of the principal components for network closeness is a result of the large degree of home bias in consumption. Since each country’s consumption loads heavily on domestic goods, it is largely affected by its own productivity shocks. In this case, the covariance matrix based on network closeness is largely diagonal, with each principal component explaining a small amount of the variation. That said, the common variation that does arise from network closeness explains variation in asset prices, as we show next.

This eigenvalue decomposition allows us to study the relation between the principal components in exposure and network closeness and common currency factors studied in the literature. To do so, we first note that currency base factors are simply linear combinations of log exchange rate movements, and the change in the value of a currency portfolio can be expressed as a linear combination of its constituent currencies’ base factors\(^2\). For example, the dollar and the carry factors (Lustig, Roussanov, and Verdelhan 2011, 2014) can be expressed as some linear combinations of currency base factors.

A particularly interesting set of portfolios in our context are those derived from principal components of our closeness measures. Given a covariance matrix \(\mathcal{C}\), each principal component \(PC_n(\mathcal{C})\) can be interpreted as a currency portfolio. The \(i\)-th element \(PC_{n,i}(\mathcal{C})\) indicates the position on currency base factor \(i\), which itself is a linear combination of all exchange rates versus currency \(i\). The currency portfolio based on the first principal component \(PC_{1,i}(\mathcal{C})\) then represents the combination of currencies that account for the largest fraction of variance in implied exchange rate movements.

To see how these portfolios relate to typical currency factors, we first regress each currency’s factor loading on the portfolio loadings based on the first three principal components. These principal components are obtained from either \(C^{Network}\) or \(C^{Exposure}\):

\[
\beta_i = \alpha + \beta_1 PC_{1,i}(\mathcal{C}) + \beta_2 PC_{2,i}(\mathcal{C}) + \beta_3 PC_{3,i}(\mathcal{C}) + \epsilon_i. \tag{7}
\]

\(^2\)This follows directly from application of triangular arbitrage on log exchange rate changes. For useful discussions of the spanning properties of currency base factors see Appendix A of Lustig and Richmond (2017) and Aloosh and Bekaert (2019).
As an example, $\beta_i$ is the loading of currency $i$’s base factor movement $\Delta \bar{e}_t$ with respect to the dollar factor. We regress this dollar factor loading $\beta^i$ on that country’s loadings on the first 3 principal components of network closeness: $PC_{1,i}(C^{Network})$, $PC_{2,i}(C^{Network})$, and $PC_{3,i}(C^{Network})$. The R-squared in these regressions answers the following question: how much of the cross-sectional variation in currency factor loadings can be explained by variation in common exposures that arises in our measures of closeness?

Table 7 reports the results for 4 currency factors: dollar, carry (HML), unconditional carry (UHML), and peripheral-minus-central (PMC as in Richmond (2019)). The currency loadings based on the first 3 PCs implied from $C^{Network}$ explain 64% of the variation in each currency’s dollar beta. In other words, the major currency comovements implied from $C^{Network}$ align with the dollar beta. These currency loadings from $C^{Network}$ also explain 9% of the variation in each currency’s conditional carry beta, 21% of the variation in each currency’s unconditional carry beta, and 38% of the variation in each currency’s peripheral-minus-central beta. In comparison, the currency loadings based on the first 3 PCs implied from $C^{Exposure}$ explain much smaller variations in currency betas.

Next, we study how variation in the actual portfolios correlates with standard currency factors. Given that the vector of currency base factors is given by $\Delta \bar{e}_t$, the exchange rate movement on the currency portfolio based on $PC_n(C)$ can be expressed as

$$PC_n(C)'\Delta \bar{e}_t.$$  

We regress each currency risk factor $f_t$ (dollar, conditional carry, unconditional carry, and peripheral-minus-central) directly on the exchange rate movements of the currency portfolios based on the first 3 PCs implied from $C^{Network}$:

$$f_t = \alpha + \beta_1 PC_1(C)'\Delta \bar{e}_t + \beta_2 PC_2(C)'\Delta \bar{e}_t + \beta_3 PC_3(C)'\Delta \bar{e}_t + \varepsilon_t. \quad (8)$$

Table 8 reports the results. The currency portfolios based on the first 3 PCs implied from $C^{Network}$ explain 88% of the variation in the dollar factor, 22% of the variation in the conditional carry factor, 42% of the variation in the unconditional carry factor, and 44% of the variation in the peripheral-minus-central factor. In comparison, the currency loadings
based on the first 3 PCs implied from $C^{\text{Exposure}}$ explain much smaller variations in these currency factors.

In summary, we show that each currency’s loadings on the first 3 principal components of network closeness explain its exposures with respect to currency risk factors, and that currency portfolios based on these 3 principal components are highly correlated with standard currency risk factors. We conclude that the international risk factors are mostly related to the structure of the trade network, rather than the covariance structure of primitive shocks.

B. Geographic Clusters

In addition to the standard risk factors, there may exist other factor structures in the co-variance matrix based on the trade network. From our model, Proposition 3 shows that if there is a block structure in the world trade network, then this block structure will also manifest itself in consumption correlations, equity market correlations, and in FX correlations. Additionally, this block structure would lead to a factor structure in these asset prices and quantities. In this section, we link the regional comovements to factor structure in the trade network.

We begin by extracting the block structure in the intermediate and consumption trade network for developed countries, as suggested by Proposition 3. To do this, we start with our matrix of network based closeness measures. We then apply a hierarchical clustering algorithm (Johnson (1967)) to these closeness measures using the inverse of closeness as a measure of distance. As an illustration, we then partition countries into 3 clusters that are furthest apart.

We note that Aloosh and Bekaert (2019) also apply a clustering algorithm. In their case, they start with currency base factors (currency baskets in their terminology) and apply a clustering algorithm. This is in contrast to our clustering which is done on the world input output network. Interestingly, there are numerous similarities to the clusters which we observe, which is consistent with the gravity in the exchange rate factor structure findings of Lustig and Richmond (2017).

To see how the block structure in the world trade network relates to that in consumption, FX markets, and equity markets, we plot each country’s average correlations with countries
in the same cluster and with countries in other clusters. We refer to them as the within correlation and the outside correlation.

**Figure (6)** presents the clustering of the countries. The first cluster has Australia, Canada, Japan, and the US. The second cluster has mostly main land European countries. The third cluster has Scandinavian countries. Each bar represents the average correlation or volatility of a countries economic quantity or asset price with all countries within and outside of each cluster.

The top panel presents consumption growth correlations. Notably, only countries in the second block have higher within correlations than outside correlations. This is not particularly surprising given our finding that the majority of variation in consumption growth correlations is explained by exposure closeness rather than network closeness. We could conduct a similar exercise constructing clusters using exposure closeness, but we focus on network closeness to emphasize the implications of our theoretical model.

The third and fourth panels present results for equity market correlations and currency base factor correlations. For almost all countries, correlations are higher within the cluster than outside of the cluster. Interestingly, for currency base factor correlations, most of the outside cluster correlations are positive. The most noticeable exception to this is the US, which has a negative base factor correlation with respect to base factors of countries outside of cluster 1. This is likely consistent with Lustig, Roussanov, and Verdelhan (2014); Verdelhan (2018); Jiang, Krishnamurthy, and Lustig (2018) that show that the US dollar has unique properties relative to most other currencies. A similar result holds for FX volatility in second panel, where the within cluster volatility being lower than the outside of cluster volatility.

It is important to note that the block structure implied by clustering network closeness is only approximate. Proposition 3 has an exact block structure and thus would imply that the correlation outside of each countries block would be zero. As seen in Figure (6), the correlation outside of each countries block is non-zero, which implies that the block structure is only approximate or that there is cross-sectional correlation due to the primitive shocks. Nevertheless, the structure of the global trade network still gives rise to phenomena that is consistent Proposition 3, which we continue to explore in this section.
We next turn to understand how this commonality within clusters can generate a factor structure. For each cluster and for each quantity or asset price, we construct a factor that is the average change in that quantity or asset price within the cluster. We then regress each country’s quantity or asset price on each of these factors to obtain factor loadings. When we regress country $i$’s quantity and asset price, we omit this country from the cluster factor so as to not have any mechanical link between the country and the cluster factor. For example, when we regress the Australian dollar base factor on cluster 1’s base currency factor, we construct cluster 1’s base currency factor as the average change in all countries in cluster 1 except Australia. We also regress the Australian dollar base factor on factors constructed from cluster 2 and cluster 3.

Figure (7) presents the factor loadings for each country across clusters. For consumption, there is not much variation in the loadings. This is again consistent with our finding that the majority of variation in consumption is explained by exposure closeness and not by network closeness. For equity return loadings we find that countries’ within cluster loadings are almost always higher than the loadings on cluster factors other than their own. The same is true for the FX base factors. These results show that when currencies within a cluster systematically appreciate, they tend to do so in tandem. Due to the construction of currency base factors, this also implies that currencies outside of the cluster tend to systematically depreciate on a relative basis.

The findings in this section suggest that the structure of the world trade network is an important determinant of the factor structure that we observe in global equity returns and exchange rate movements. Specifically, factors implied from the world’s input-output linkages are able to explain how exchange rates and equity markets co-move.

V. Conclusion

In this paper, we develop a model of the global production and consumption network and show that the network closeness explains numerous co-movements in economic quantities and asset prices. We empirically measure closeness and show that countries that are closer in this network tend to have more correlated consumption growth, stock returns, and exchange rate movements. The network also generates factor structures in equity returns and exchange
rates as found in the data. These results offer a network-based account of the origins of factor structures in international asset prices and economic quantities.
References


Gofman, Michael, Gill Segal, and Youchang Wu. 2018. “Production networks and stock returns: The role of vertical creative destruction.”


Equity market correlations, exchange rate volatility, and currency base factor correlations versus global production network closeness. Consumption growth correlations are constructed from yearly consumption growth. Equity market correlations are correlations equity market returns in local currency. Exchange rates are nominal, and currency base factors are the average appreciation of currencies against all other currencies in the sample. Network closeness is constructed as the implied correlation of consumption from the world input output network assuming that shocks across countries are iid. Exchange rate regressions include home country and foreign country fixed effects.
Plot of the bilateral world production network as implied by the World Input Output Table.
Plot of the bilateral world consumption network as implied by the World Input Output Table.
Figure 4
Covariances between Productivity Shocks

Plot of the implied exposure network from consumption growth and the world input output network. Exposure shocks are the implied shock structure that is necessary to explain bilateral consumption growth correlations using the world input output network.
Base factor variances versus those implied by the network closeness. Network closeness is constructed as the implied correlation of consumption from the world input output network assuming that shocks across countries are iid.
Average correlations of consumption growth correlations, equity market correlations, exchange rate volatility, and base factor correlations within and across clusters. Three clusters are constructed based upon network closeness. Equity market correlations are correlations equity market returns in local currency. Exchange rates are nominal, and currency base factors are the average appreciation of currencies against all other currencies in the sample.
Loadings on cluster factor in terms of consumption growth, equity market returns, and currency base factors. Three clusters are constructed based upon network closeness. Within each cluster and quantity, a factor is constructed as the average across all countries in that cluster. Factor loadings are from a regression of each country’s quantity on each cluster factor, omitting the country itself in the construction of the factor. Equity market returns are in local currency. Exchange rates are nominal, and currency base factors are the average appreciation of currencies against all other currencies in the sample.
Regressions of bilateral consumption growth correlations, network closeness, and exposure closeness on gravity variables. Consumption growth correlations are constructed from yearly consumption growth. Network closeness is constructed as the implied correlation of consumption from the world input output network assuming that shocks across countries are iid. Exposure closeness is the implied shocks structure that is necessary to explain bilateral consumption growth correlations using the world input output network.

Regressions of bilateral consumption growth correlations and equity market correlations on network closeness, exposure closeness, and bilateral consumption growth correlations. Equity market correlation are correlations equity market returns in local currency.
Table 3
Explaining Consumption and Equity Correlations (Developed and Emerging)

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***p < 0.01, **p < 0.05, *p < 0.1

Regressions of bilateral consumption growth correlations and equity market correlations on network closeness, exposure closeness, and bilateral consumption growth correlations. Equity market correlations are correlations equity market returns in local currency.

Table 4
Explaining FX Correlations and Volatilities (Developed)

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***p < 0.01, **p < 0.05, *p < 0.1

Regressions of exchange rate volatility and currency base factor correlations on network closeness, exposure closeness, and bilateral consumption growth correlations. Exchange rates are nominal exchange rates. Currency base factors are the average appreciation of currencies against all other currencies in the sample. Exchange rate regressions include home country and foreign country fixed effects.
Table 5
Explaining FX Correlations and Volatilities (Developed and Emerging)

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<tr>
<td>Exposure</td>
<td>0.11*</td>
<td></td>
<td>0.02**</td>
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<tr>
<td></td>
<td>(1.74)</td>
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<tr>
<td>Production</td>
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<td>1.53***</td>
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<tr>
<td>Production FO</td>
<td>27.28***</td>
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<td>2.56***</td>
<td></td>
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<td>(2.83)</td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>Within R²</td>
<td>0.04</td>
<td>0.02</td>
<td>0.17</td>
<td>0.11</td>
<td>0.11</td>
<td>0.06</td>
<td>0.02</td>
<td>0.04</td>
<td>0.04</td>
</tr>
</tbody>
</table>

Regressions of exchange rate volatility and currency base factor correlations on network closeness, exposure closeness, and bilateral consumption growth correlations. Exchange rates are nominal exchange rates. Currency base factors are the average appreciation of currencies against all other currencies in the sample. Exchange rate regressions include home country and foreign country fixed effects.

Table 6
Percent of variance explained by principal components

<table>
<thead>
<tr>
<th>PC</th>
<th>Network</th>
<th>Exposure</th>
<th>Consumption</th>
<th>Base Factors</th>
</tr>
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<tr>
<td>1</td>
<td>4.5</td>
<td>58.6</td>
<td>31.5</td>
<td>24.8</td>
</tr>
<tr>
<td>2</td>
<td>3.5</td>
<td>19.6</td>
<td>17.9</td>
<td>19.7</td>
</tr>
<tr>
<td>3</td>
<td>3.1</td>
<td>9.8</td>
<td>11.3</td>
<td>9.2</td>
</tr>
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</table>

Percent of variance explained by principal components of network closeness, exposure closeness, consumption growth, and FX base factors.
Table 7
Cross-Sectional Regressions of Factor Loadings on Closeness Principal Component Loadings

<table>
<thead>
<tr>
<th></th>
<th>Dollar</th>
<th>HML</th>
<th>UHML</th>
<th>PMC</th>
<th>Dollar</th>
<th>HML</th>
<th>UHML</th>
<th>PMC</th>
</tr>
</thead>
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<tr>
<td>Network 1</td>
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<td>4.10*</td>
<td>4.92**</td>
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<td></td>
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<td>(1.14)</td>
<td>(1.84)</td>
<td>(2.21)</td>
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<tr>
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<td>−0.37</td>
<td>0.28</td>
<td>0.54</td>
<td>0.37</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(−0.56)</td>
<td>(0.37)</td>
<td>(0.58)</td>
<td>(0.39)</td>
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<td></td>
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<td></td>
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<tr>
<td>Network 3</td>
<td>−1.69***</td>
<td>−0.28</td>
<td>−0.59</td>
<td>−1.33***</td>
<td></td>
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<tr>
<td></td>
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<td>(−0.69)</td>
<td>(−1.17)</td>
<td>(−2.59)</td>
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</tr>
<tr>
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<tr>
<td></td>
<td>(1.13)</td>
<td>(−0.32)</td>
<td>(−0.33)</td>
<td>(0.49)</td>
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<tr>
<td></td>
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<td>(−0.29)</td>
<td>(−0.07)</td>
<td>(0.10)</td>
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<td>Exposure 3</td>
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<td></td>
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<tr>
<td></td>
<td>(0.47)</td>
<td>(0.01)</td>
<td>(−0.06)</td>
<td>(0.16)</td>
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</tr>
<tr>
<td>Within R²</td>
<td>0.64</td>
<td>0.09</td>
<td>0.21</td>
<td>0.38</td>
<td>0.05</td>
<td>0.01</td>
<td>0.01</td>
<td>0.02</td>
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<td>31</td>
<td>31</td>
<td>31</td>
<td>30</td>
<td>31</td>
</tr>
</tbody>
</table>

***p < 0.01, **p < 0.05, *p < 0.1

Regressions of loadings of base factors on dollar, carry, unconditional carry, and centrality exchange rate factors on the loadings of countries on the first 3 principal components from network and exposure closeness.

Table 8
Explaining FX Factors with Principal Components of Closeness

<table>
<thead>
<tr>
<th></th>
<th>Dollar</th>
<th>HML</th>
<th>UHML</th>
<th>PMC</th>
<th>Dollar</th>
<th>HML</th>
<th>UHML</th>
<th>PMC</th>
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<td>1.19***</td>
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<td></td>
<td>(6.28)</td>
<td>(4.44)</td>
<td>(5.58)</td>
<td>(6.06)</td>
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<tr>
<td>Network 2</td>
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<td>(2.13)</td>
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</tr>
<tr>
<td>Network 3</td>
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<td>0.03</td>
<td>−0.13***</td>
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<tr>
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<td>(0.46)</td>
<td>(−2.05)</td>
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<tr>
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<td>−0.03</td>
<td>0.18**</td>
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<td>(−0.35)</td>
<td>(2.54)</td>
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<td>−0.29</td>
<td>−0.33</td>
<td>−0.32**</td>
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<td>Exposure 3</td>
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<td>0.06</td>
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<tr>
<td>Within R²</td>
<td>0.88</td>
<td>0.22</td>
<td>0.42</td>
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<td>0.02</td>
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</table>

***p < 0.01, **p < 0.05, *p < 0.1

Regressions of the dollar, carry, unconditional carry, and centrality exchange rate factors on the first 3 principal components from network and exposure closeness.
A. Proof Appendix

For proofs of the following two lemmas we omit time subscripts because there is no storage technology and therefore the model can be solved period by period.

Lemma 1

The Lagrangian is

\[
\sum_{i=1}^{N} \lambda_i \left( \sum_{j=1}^{N} v_{ij} \log C_{ij} \right) + \varphi_i \left( A_i L_i^\theta \left( \prod_{j=1}^{N} X_{ij}^{w_{ij}} \right) - \sum_{j=1}^{N} (C_{ji} + X_{ji}) \right) + \chi_i (\bar{L}_i - L_i) \tag{9}
\]

FOCs are

w.r.t. \( C_{ji} \):

\[
\lambda_j v_{ji} C_{ji}^{-1} = \varphi_i\tag{10}
\]

w.r.t. \( X_{ji} \):

\[
\varphi_j \overline{X}_j w_{ji} X_{ji}^{-1} = \varphi_i\tag{11}
\]

w.r.t. \( L_i \):

\[
\varphi_i \overline{X}_i \theta L_i^{-1} = \chi_i\tag{12}
\]

Substitute into the market clearing condition:

\[
\varphi_i \overline{X}_i = \sum_{j=1}^{N} (\lambda_j v_{ji} + \varphi_j \overline{X}_j w_{ji}).\tag{13}
\]

Define \( \Gamma_i = \varphi_i \overline{X}_i \). Then

\[
\Gamma = (I - W')^{-1} V' \lambda\tag{14}
\]

is determined by the primitive parameters. In particular, it is not determined by the productivity shock \( A_i \).

Then, the log production is

\[
\log \overline{X}_i = a_i + \theta \ell_i + \sum_{j=1}^{N} w_{ij} \log \left( \frac{\Gamma_i \overline{X}_j}{\Gamma_j w_{ij}} \right),\tag{15}
\]
which implies
\[ \tilde{x} = \kappa x + a + W \tilde{x} \]  
\[ = (I - W)^{-1}(\kappa x + a). \]  \hspace{1cm} (16)\hspace{1cm} (17)

The log consumption is
\[ \bar{c}_i = \sum_{j=1}^{N} v_{ij} \log \left( \frac{\lambda_i v_{ij} X_j}{\Gamma_j} \right) \]  \hspace{1cm} (18)
\[ \bar{c} = \kappa c + V \tilde{x} \]  \hspace{1cm} (19)
\[ \bar{c} = \kappa c + V (I - W)^{-1}(\kappa x + a) \]  \hspace{1cm} (20)

**Lemma 2**

Let \( q_i \) denote the price of country \( i \)'s tradable goods in the numeraire. Back to the optimization of country \( i \)'s representative household:
\[ \sum_{j=1}^{N} v_{ij} \log C_{ij} \]  \hspace{1cm} (21)

with budget constraint
\[ q_i X_i = \sum_{j=1}^{N} q_j (C_{ij} + X_{ij}) \]  \hspace{1cm} (22)

The Lagrangian is
\[ \sum_{j=1}^{N} v_{ij} \log C_{ij} + M_i \left( q_i A_i L_i^\theta \left( \prod_{j=1}^{N} X_{ij}^{w_{ij}} \right) - \sum_{j=1}^{N} q_j (C_{ij} + X_{ij}) \right) + N_i (L_i - L_i) \]  \hspace{1cm} (23)

FOCs are
\[ \text{w.r.t. } C_{ij} : v_{ij} C_{ij}^{-1} = M_i q_j \]  \hspace{1cm} (24)
\[ \text{w.r.t. } X_{ij} : q_i X_i w_{ij} X_{ij}^{-1} = q_j \]  \hspace{1cm} (25)
\[ \text{w.r.t. } L_i : M_i q_i X_i \theta L_i^{-1} = N_i \]  \hspace{1cm} (26)
which implies \( q_i \) equals \( \varphi_i \) up to a scalar. Then Lemma 2 also directly follows from the FOCs.

Lastly, from the social planner’s FOCs:

\[
\begin{align*}
\varphi_i X_i w_{ij} &= \varphi_j X_{ij} \quad (27) \\
\varphi_i X_i \theta &= \chi_i L_i \quad (28)
\end{align*}
\]

Given \( \theta + \sum_j w_{ij} = 1 \), the labor value added is \( \theta \) fraction of the total output.

**Lemma 3**

Let \( p_i \) denote the price of country \( i \)’s consumption basket. The household maximizes

\[
p_i \left( \prod_{j=1}^{N} C_{ij}^{v_{ij}} \right) - \sum_{j=1}^{N} q_j C_{ij} \quad (29)
\]

The zero-profit condition implies

\[
p_i = \frac{\sum_{j=1}^{N} q_j C_{ij}}{C_i} \quad (30)
\]

Then, the log real exchange rate between countries \( i \) and \( j \) is

\[
e_{ij} = \log \frac{p_i}{p_j} \quad (31)
= \log \sum_{k=1}^{N} q_k C_{ik} + \bar{v}_j - \bar{v}_i \quad (32)
= \log \sum_{k=1}^{N} \lambda_{j} v_{ik} + \bar{v}_j - \bar{v}_i \quad (33)
\]

Then the change in the base currency factor is

\[
\Delta \bar{v}_{it+1} = \frac{1}{N} \sum_{j=1}^{N} \Delta \bar{v}_{jt+1} - \Delta \bar{v}_{it+1}. \quad (34)
\]
The price of the claim to period $t + k$ consumption is

$$
E_t \left[ e^{-k \beta - \tau_{it+k} + \tau_{it} C_{it+k}} \right] = e^{-k \beta C_{it}}.
$$

(35)

So the price of the ex-dividend claim to the consumption stream is

$$
P_{it}^{eq} = \sum_{k=1}^{\infty} e^{-k \beta C_{it}} = \frac{e^{-\beta}}{1 - e^{-\beta} C_{it}}.
$$

(36)

The log cum-dividend return is

$$
r_{it+1}^{eq} = \log \frac{C_{it+1} + \frac{e^{-\beta}}{1 - e^{-\beta} C_{it}} C_{it+1}}{1 - e^{-\beta} C_{it}} = \beta + \Delta \tau_{it+1}.
$$

(37)

**Size**

Define size as the total value of production, $S_i = \varphi_i X_i$, then

$$
S_i = \sum_{j=1}^{N} (\lambda_j v_{ji} + S_j w_{ji}).
$$

(38)

Define $Q_i = \sum_{j=1}^{N} \lambda_j v_{ji} = V' \lambda$. Then

$$
S = (I - W')^{-1} Q.
$$

(39)

Define production size as $S_i^P = \varphi_i \sum_{j=1}^{N} X_{ji}$, then

$$
S_i^P = \sum_{j=1}^{N} \varphi_j X_j w_{ji}
$$

(40)

$$
S^P = W' S
$$

(41)

**Proposition 1 and 2**

The covariance between the changes in base currency factors is

$$
cov(\Delta \tau_{it+1}, \Delta \tau_{jt+1}) = cov \left( \frac{1}{N} \sum_{k=1}^{N} \Delta \tau_{kt+1} - \Delta \tau_{it+1}, \frac{1}{N} \sum_{k=1}^{N} \Delta \tau_{kt+1} - \Delta \tau_{jt+1} \right).
$$

(42)
Other claims are straight-forward to prove.

**Proposition 3**

If two matrices $A$ and $B$ have the same block structure,

$$
(AB)_{ij} = \sum_k A_{ik}B_{kj}
$$

which is non-zero only if $k \in R(i)$ and $k \in R(j)$. So, not only does $AB$ have the same block structure, the results in each block of $AB$ are the product between the corresponding blocks of $A$ and $B$.

Because

$$
(I - W)^{-1} = I + W + W^2 + \ldots,
$$

then the inverse $(I - W)^{-1}$ in each block is also the inverse of the corresponding block of $(I - W)$.

Next, we consider a $\ell$-by-$\ell$ matrix $U$ such that $U_{ij}$ is $\omega_0$ if $i = j$ and $\omega_1$ otherwise. We conjecture

$$
\{(I - U)^{-1}\}_{ij} = \begin{cases} 
\kappa^h(1 - \omega_0 - (\ell - 2)\omega_1), & \text{if } i = j, \\
\kappa^h\omega_1, & \text{if } i \neq j.
\end{cases}
$$

Then

$$
\{(I - U)^{-1}(I - U)\}_{ij} = \sum_{k=1}^{\ell} \{(I - U)^{-1}\}_{ik}\{(I - U)\}_{kj}
$$

$$
= \begin{cases} 
\kappa^h(1 - \omega_0 - (\ell - 2)\omega_1) \cdot (1 - \omega_0) + (\ell - 1)\kappa^h\omega_1 \cdot (-\omega_1), & \text{if } i = j, \\
\kappa^h(1 - \omega_0 - (\ell - 2)\omega_1) \cdot (-\omega_1) + \kappa^h\omega_1 \cdot (1 - \omega_0) + (\ell - 2)\kappa^h\omega_1 \cdot (-\omega_1), & \text{if } i \neq j \text{ and } j \in R(i).
\end{cases}
$$

$$
= \begin{cases} 
\kappa^h(1 - \omega_0 - (\ell - 2)\omega_1) \cdot (1 - \omega_0) + (\ell - 1)\kappa^h\omega_1 \cdot (-\omega_1), & \text{if } i = j, \\
0, & \text{if } i \neq j \text{ and } j \in R(i).
\end{cases}
$$

47
Thus, we confirm our conjecture, and $\kappa^h$ can be solved from $\kappa^h(1 - \omega_0 - (\ell - 2)\omega_1) \cdot (1 - \omega_0) + (\ell - 1)\kappa^h\omega_1 \cdot (-\omega_1) = 1$. It then follows that $\kappa > 0$ and

\[
\{(I - U)^{-1} H\}_{ij} = \begin{cases} 
\kappa^h\omega_1\xi_{jj} + \kappa^h\theta\xi_{jj}, & \text{if } i = j, \\
\kappa^h\omega_1\xi_{jj}, & \text{if } i \neq j.
\end{cases}
\]  

(47)

Similarly,

\[
\{(I - U)^{-1} \Psi\}_{ik} = \kappa^h\theta\psi_{ik} + \sum_{j=1}^{\ell} \kappa^h\omega_1\psi_{jk}
\]  

(48)