Investment Sophistication and Wealth Inequality

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Abstract

I study how differences in investment sophistication, the ability to identify profitable investment opportunities, contribute to wealth inequality and its dynamics. I analyze a financial market with a continuum of investors heterogeneously informed about the distribution of future returns. The stationary distribution of wealth shares features a thick right tail populated by the best-informed investors. Wealth inequality increases with the emergence of ever-costlier information production technologies, but the expected excess return achieved through such technologies is bounded in the long run. Top wealth shares have an inverted U-shaped relationship with the wealth share delegated to the best-informed investors to manage. Empirically, wealthier households indeed tend to have more precise beliefs, and the gap in their beliefs can rationalize inequality under reasonable parameters. My findings suggest subsidizing education and leveling the playing field regarding information can help reduce inequality.

1 Introduction

Wealth inequality has been rising for decades (Piketty (2014); Saez and Zucman (2016)). A key contributing factor is that households earn substantially different returns on their savings (Fagereng, Guiso, Malacrino, and Pistaferri (2016)). This return heterogeneity

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partly stems from the differences in households’ information about the available investment opportunities—the type of information that shapes households’ saving decisions and, consequently, their wealth accumulation. In this paper, founding on this intuitive interplay between such information asymmetries about asset returns and wealth inequality, I build a model that puts forward several empirical predictions about the portfolio dynamics, wealth distribution, distribution of expected returns, and composition of the wealthiest.

More specifically, I develop a dynamic model of a financial market with investors asymmetrically informed about the future returns. Therefore, endowed with different information, the investors trade assets, hold different portfolios, and consequently reap different returns on their savings, which translates into a change in the wealth distribution. In this baseline setup, the stationary (long-run) distribution of wealth shares features a thick right tail populated by the most informed investors. Extending the baseline model, I also study the role of endogenous information acquisition and investment delegations, resembling the $75 trillion dollar wealth management industry, in wealth inequality. The next paragraphs highlight the key insights the model generates.

Asymmetric information affects top wealth inequality, a measure of inequality among the wealthiest, only through the return of the most informed investors minus the economic growth rate. The intuition being that the top inequality is directly related to the rate at which the wealth share of the most informed investors grows, which is equal to their excess return with respect to that of the representative investor, who holds the average portfolio. As information advantage of the most informed shrinks, so does their expected excess return, which reduces the inequality. This result yields a heuristic rule for understanding the distributional impact of a change in the households’ information: if the change makes the wealthiest relatively more informed about the available investment opportunities compared to an average investor, then it also increases top wealth inequality. For instance, the de-
velopment of personalized financial advisors, such as robo-advisors, aiming to liberate the access to financial planning and information, is effective in reducing the inequality since they are directly targeting the less-wealthy segment of the population, reducing the extent of asymmetric information among investors.

In addition to the distribution, my model also generates predictions about the composition of super-rich. In equilibrium, as we focus on a smaller and smaller pool of the wealthiest, only the most informed investors remain and the others gradually wash out from the pool. In short, sophistication fully crowds out luck in the long-run: almost no lucky investor with a low sophistication makes it to the pool of super-rich. This result explains the substantial representation of fund managers among billionaires, known by their investment sophistication and advantage in access to information.

The presence of asymmetric information among investors might be best manifested in hedge funds’ access to highly expensive and exclusive information sources, such as satellite images\(^1\) and private polls\(^2\). To obtain an edge in financial markets, they search for novel, but potentially expensive, ways to produce information, resembling an arms race for generating \(\alpha\). Informing the consequences, the model implies wealth inequality generally increases with the emergence of costlier information production technologies. The intuition being that such costlier technologies are affordable for a smaller fraction of investors. As such, the adopters of such technologies enjoy a higher information advantage, increasing the inequality.\(^3\)

\(^1\)“Stock Picks From Space” The Atlantic, May 2019
\(^2\)“The Brexit Short: How Hedge Funds Used Private Polls to Make Millions” Bloomberg, June 2018
\(^3\)For instance, in the ’80s, the newspapers were providing high-quality information about the markets. At the time, the experts’ information advantage could be in the form of being the first to learn the information or having business insights on their implications. Nowadays, the only substantial change for retail investors is that they get the same high-quality information almost free, probably cheaper than the ’80s. However, the experts now generate information in too many new ways, process them faster and more accurately and turn information to trade orders at a much higher frequency. The point is that now we have expensive and advanced ways for information production that are affordable, directly or indirectly (through delegation of
The adopters of new information production technologies, as the new informed investors, populate the tail in the new equilibrium. Nonetheless, their superior performance diminishes over time. More specifically, their expected return converges to a bounded level, independent of the technology’s informativeness. The intuition being that as the adopters become wealthier, they push the equilibrium prices more strongly and make them more informative, blunting their information edge, and consequently, reducing their expected returns. Therefore, such technologies only boost the adopters’ short-term expected return, consistent with the recent mediocre performance of the hedge fund industry.\(^4\)

Fund managers managing others’ wealth has important implications for wealth inequality. As an extension of the baseline model, I let a fraction of investors delegate their investment decisions to the best-informed investors and collect a fraction of the extra return they obtain through the delegation. The collected fraction captures the competitiveness of the wealth management industry. I find the best-informed investors’ wealth share has an inverse U-shaped relationship with the wealth share under their management, for all levels of competitiveness. The reason is that although the best-informed investors gain from managing a larger wealth share, their average rent diminishes with the wealth share they manage. More specifically, managing a larger wealth share makes the prices more informative, narrowing the expected extra return their information advantage delivers. This result indicates that a potential reason for the rise of super-rich fund managers is the surge in investment delegations and the size of wealth management industry over the past decades. Nevertheless, a further increase in the investment delegations could actually reduce the inequality, calling for policies that advocate and facilitate investment delegations.

\(^4\)“Hedge Fund Trends For 2019: Industry Has Finally Hit Its Saturation Point” Yahoo Finance, January 2019
Regarding the comparative statics, I study the distributional impact of an increase in the precision of available information, say, obtained by the recent IT developments. First, the more precise the available information, the more informative the equilibrium prices, reducing both equity premium and asymmetric information among the investors. However, more strikingly, it might also increase wealth inequality. The intuition is that the investors, facing less uncertainty, react more aggressively to any signal not perfectly incorporated in the prices. Such aggressive behaviors magnify portfolio heterogeneity, and consequently, the return heterogeneity, increasing the inequality. Therefore, depending on whether this channel, the stronger reaction to beliefs, dominates the opposite channel, the belief convergence, the availability of more precise information might increase the inequality.

In addition to the precision, the allocation of information, namely who has access to and comprehends which source of information, impacts the wealth distribution. The concentration of information among a few increases inequality and does so even more when combined with the emergence of new exclusive sources of information. This result indicates that the policies tackling information inequality, such as subsidizing higher education or improving the transparency of financial markets, are both effective and crucial in reducing wealth inequality.

Along with the information environment, the financial markets have evolved tremendously in their size, liquidity, connectedness, and complexity over the past decades. The inequality increases with the liquidity of the markets and the magnitude of non-fundamental noises. More generally, the model implies the informativeness of equilibrium prices plays a crucial role in reducing the inequality; hence, any other change in the market structure that hampers the information dissemination through the prices also increases top wealth inequality.
Dynamic models of financial markets with asymmetrically informed investors are notoriously intractable. Nonetheless, my model delivers highly tractable dynamics for wealth distribution. I tackle the common issues and obtain tractability by using logarithmic preferences instead of CARA preferences. Assuming logarithmic preferences, I prove the wealth shares move with the investors’ probability assignments to the realized states, rendering all the intermediary steps, such as solving for the optimal portfolios or the equilibrium prices and asset allocations, unnecessary, once the dynamics are derived.\(^5\)

Moreover, according to the no-trade theorem of Milgrom and Stokey (1982), the presence of some “noise trades” is necessary to facilitate the trades among a group of traders with homogeneous preferences, but heterogeneous information about the asset values. Hence, to fully leverage the feature mentioned above, I introduce some noise traders for whom I exogenously specify their beliefs, instead of their asset demands. Therefore, the noise traders in my model are rational, in the sense that they are Bayesian and dynamically maximize their expected utility. However, they have misspecified beliefs about the underlying data generating process. Particularly, they are assumed to base their predictions about the asset returns on a wrong signal. That said, the wrong signal, operating through the noise traders’ decisions, works as a non-fundamental noise in the prices, obscuring the information of more informed investors from the less informed ones.\(^6\)

After building this model, several questions naturally follow. Do the wealthier actually have more precise beliefs? If so, is the gap in their precision large enough to significantly contribute to wealth inequality? In Section 2, I empirically find a significant difference in

\(^5\)This property of wealth dynamics under logarithmic preferences is employed in Blume and Easley (1992) and Mailath and Sandroni (2003). Yet they study asymptotic survival, while I study the equilibrium wealth distribution for asymmetrically informed investors.

\(^6\)In addition, the other studies mostly use CARA preferences, which disconnects the optimal portfolio from the wealth level, causing the price informativeness to be independent of the wealth distribution, which is an undesirable feature for the question at hand. Here, by using logarithmic preferences, the prices become more informative as the wealth becomes more concentrated in the hand of most informed investors.
the macroeconomic beliefs, confirming the earlier empirical finding of Das, Kuhnen, and Nagel (2017). Then, leveraging the wealth dynamics derived here, I estimate the impact of the belief heterogeneity on wealth inequality in Section 7.4. The estimates indicate a relatively large level of wealth inequality, suggesting the belief gap is large enough to contribute significantly to US wealth inequality. Moreover, in Section 7.3, I empirically show the model predictions about the top households’ portfolio dynamics are consistent with data. Finally, I discuss the role of bequest motive and security space in wealth inequality, in Appendix B.

The rest of the paper is organized as follows: In Section 2, I present two motivating facts. Section 3 lays out the model and Section 4 defines and characterizes the equilibrium. I find and analyze the wealth dynamics in Section 5 and discuss the stationary distribution. Section 6 studies the case of endogenous information acquisition. Section 7 contains the empirical analysis and some theoretical extensions of the model. Section 8 concludes.

**Literature**

With the recent surge in wealth inequality, its causes and consequences are currently at the center of political and academic debates. The earlier studies focused more on the mechanisms relating wealth inequality to income inequality (Aiyagari (1994), Castaneda, Diaz-Gimenez, and Rios-Rull (2003)). Nonetheless, they achieved limited success, especially in explaining the top inequality (Fagereng, Guiso, Malacrino, and Pistaferri (2016)). Alternatively, a new strand of theoretical and empirical studies have examined the role of return heterogeneities in wealth inequality. Supporting this channel, Bach, Calvet, and

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Sodini (2016) and Fagereng, Guiso, Malacrino, and Pistaferri (2016) document a vast heterogeneity in returns in Sweden and Norway respectively. Benhabib, Bisin, and Zhu (2011) and Benhabib, Bisin, and Luo (2015, 2019) demonstrate, theoretically and quantitatively, the importance of return heterogeneity to match the empirical wealth distribution. However, they do not discuss the root causes of return heterogeneity.

Several recent empirical studies suggest the answer lies in the portfolio heterogeneity across households. Garbinti, Goupille-Lebret, and Piketty (2017), Kuhn, Schularick, and Steins (2017) and Hubmer, Krusell, and Smith Jr (2018) show that the portfolio heterogeneity and asset price movements are essential to explain the dynamics of wealth distribution. Fagereng, Guiso, Malacrino, and Pistaferri (2016) are the first to demonstrate the presence of substantial return heterogeneity even within asset classes, highlighting the role of investment sophistication, which I primarily study here.8

A large body of empirical studies also lends support to the importance of investment sophistication in generating a higher risk-adjusted return. Grinblatt, Keloharju, and Linnainmaa (2011, 2012), Korniotis and Kumar (2013), Barber, Lee, Liu, and Odean (2014), Gargano and Rossi (2018), Clark, Lusardi, and Mitchell (2017) provide evidence that high-IQ, skilled and attentive investors hold portfolios with a higher Sharpe ratio.9 Frazzini, Kabiller, and Pedersen (2018) demonstrate Warren Buffet has generated a significantly positive alpha over Fama and French three factors and most of his wealth accrued by his stock-picking ability. Bartscher, Kuhn, and Schularick (2018) and Girshina (2019) emphasize the role of education in portfolio choice and generating higher returns on risky assets. Massa and

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8In fact, they show that the scale effects, the wealthier having access to more lucrative investment opportunities, and heterogeneity in risk preferences do not account for the whole cross-section of returns across households. Additionally, they find the individual fixed effects (after controlling for the observable characteristics and risk-taking behavior), interpreted as “investment sophistication”, substantially increases the explanatory power.

9Further on the role of experience, Kempf, Manconi, and Spalt (2017) show fund managers’ performance in a sector increases with his or her investment experience in that sector.
Simonov (2006) show investors invest in stocks closely related to their non-financial income and provide evidence that they rationally prioritize familiarity over income hedging in their portfolio decisions. My model sheds light on the distributional implication of the heterogeneities identified in the studies above.

Apart from investment sophistication, the returns are heterogeneous due to the differences in investment opportunity sets, risk preferences (Veronesi (2018)) and entrepreneurial skills (Cagetti and De Nardi (2006)). Pástor and Veronesi (2016) study a model of occupational choice with agents heterogeneous in their entrepreneurial skills and risk-aversion. Guvenen (2009) examines the role of market segmentation. However, none of these studies justify the over-representation of highly sophisticated and talented investors among the wealthiest, documented by (Kaplan and Rauh (2013)). Moreover, Fagereng, Guiso, Malacrino, and Pistaferri (2016) rigorously analyze the role of sophistication in the cross-section of returns and show it is essential to explain the vast heterogeneity in returns.

My paper also contributes to the literature of belief heterogeneity by characterizing the wealth dynamics induced by any arbitrary belief distribution. The belief heterogeneity, especially regarding the business cycle variables, is extensively documented (e.g., Mankiw, Reis, and Wolfers (2003), Vissing-Jorgensen (2003) Souleles (2004), Puri and Robinson (2007)). D’Acunto, Hoang, Paloviita, and Weber (2019) and Das, Kühnen, and Nagel (2017) empirically show high IQ investors and the ones with a higher socioeconomic status, respectively, have more precise macroeconomic beliefs. Giglio, Maggiori, Stroebel, and Utkus (2019) also find a tight connection between investors’ beliefs and their portfolio choice. These findings suggest economic beliefs directly impact the cross-section of households’ average returns, and subsequently, the wealth distribution. The setup presented here can be employed to study the distributional implication of such belief heterogeneities.
This paper is also related to nascent literature studying the role of sophistication in wealth inequality. In addition to Fagereng, Guiso, Malacrino, and Pistaferri (2016), An, Bian, Lou, and Shi (2019) and Campbell, Ramadorai, and Ranish (2018) provide evidence on how financial markets redistribute from the less to more sophisticated investors. Among the theoretical studies, Lei (2019) studies endogenous learning of the profitability of privately owned assets. Peress (2003) and Kacperczyk, Nosal, and Stevens (2018) find that wealth and capital income inequality increase with the emergence of new costly signals (costly to obtain or process), as they favor wealthier and more sophisticated investors. Alas, having finitely many periods, their models only inform the short-term consequences of such technological changes. Particularly, their models do not account for the fact that the price informativeness indeed increases with the informed investors gaining more wealth share, leading to a smaller information gap, and hence, return heterogeneity in the long-run. Accounting for this negative feedback effect, I find a sufficiently large increase in the information cost (or the emergence of a sufficiently expensive signal) increases wealth inequality. Nonetheless, the relationship might not be monotone. Overall, to the best of my knowledge, my paper is the first to discuss the long-run distributional impact of such technological changes. Furthermore, it puts forward empirical predictions regarding the composition of the wealthiest and distribution of expected returns, absent in the earlier studies.

2 Motivating Facts

The top wealth shares have increased substantially over the past decades (Saez and Zucman (2016); Smith, Zidar, and Zwick (2019)). However, along with the distribution, the composition of the wealthiest has drastically changed. For instance, Kaplan and Rauh
(2013) find that the number of fund managers in the Forbes 400 list has increased close to five times between 1982 and 2011. More strikingly, the number of hedge fund managers have increased from 2 in 1982 to 30 in 2011, indicating a fifteen-time increase (Figure 1). Notably, a salient distinguishing feature of the hedge fund managers is their direct access to the state-of-the-art technologies for information production and processing. Linking asymmetric information to wealth distribution, my model explains the presence of highly informed investors among the wealthiest. It further implies that the presence increases with the emergence of new and expensive information production technologies, justifying the empirical pattern in Figure 1.

![Figure 1: Top 0.1% wealth shares in the US, 1980-2016.](image)

The second observation is that the wealthier have more accurate beliefs about business cycles, which in turn improve their saving decisions. In order to demonstrate this, I use a sample of 218559 individuals asked about their macroeconomic expectation in Michigan Survey Data, between 1978q1-2018q1. Since the data does not provide the households' expectation

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10 The survey includes several questions, but I am particularly interested in the respondents’ expectation

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net worth, I use “investment amount in stock market” and “income” as proxy variables. Therefore, to see the relationship between wealth and beliefs, I divide the respondents into 20 groups, separately based on each proxy, and look at the fraction of respondents who expressed a high chance for having a good business condition in the year ahead. I compare this fraction with the realized growth over the next four quarters. The growth rate is considered to be high if it exceeds the historical mean, and it is considered to be low otherwise. Figures 2 and 3 show the average forecast errors for every income and investment level group respectively.\textsuperscript{11}

Figures 2 and 3 exhibit a negative relationships between the forecast errors in economic growth and the two proxies of wealth. Both figures show there is a substantial belief heterogeneity across the respondents and the wealthier individuals tend to have more accurate beliefs. Therefore, they are more likely to obtain a higher expected risk-adjusted return on their savings.

11 About 1-year-ahead economic growth, captured by variable “BUS12”. More specifically, it asks “Now turning to business conditions in the country as a whole, do you think that during the next 12 months we’ll have good times financially, or bad times, or what?” The data also includes information about the household income and their investment in stock market. I only consider the respondents without any qualification. For more information about the survey, see Das, Kuhnen, and Nagel (2017).

\textsuperscript{11}To construct the forecast errors, I compare the respondents’ answer with the realized grow in the following year (Including the survey’s quarter). I only use responds that have a clear good or bad expectation, where they constitute more than 75% of the total responds. Moreover, I consider an annual growth rate above its historical mean as “high”, and “low” otherwise. Then, I calculate and report the average value of \((\text{respond}_{i,t} - \mathbb{I}_{g_t+g_{t+1}+g_{t+2}+g_{t+3} \geq 2.4})^2\) for each of twenty half-deciles. \text{respond}_{i,t} is a binary variable indicating whether the respondent has a positive economic view. The result is qualitatively robust to all choices of the threshold between 1.5% to 2.5%.
Figure 2: Prediction error of the growth rate for different levels of total investment in the publicly listed stocks. The predictions are binary and the realized growth rates are divided into “high” or “low” growth rate, with a cutoff equal to the annual rate of 2.4%. The figure shows $E[(\text{respond}_{i,t} - \mathbb{I}_{g_{t} + g_{t+1} + g_{t+2} + g_{t+3} \geq 2.4})^2]$, where $i$ is the respondent indicator and $t$ is the quarter indicator and $g_t$ is the quarter-to-quarter growth at $t$.

Figure 3: Prediction error of the growth rate for different levels of income. The predictions are binary and the realized growth rates are divided into “high” or “low” growth rate, with a cutoff equal to the annual rate of 2%. The figure shows $E[(\text{respond}_{i,t} - \mathbb{I}_{g_{t} + g_{t+1} + g_{t+2} + g_{t+3} \geq 2.4})^2]$, where $i$ is the respondent indicator and $t$ is the quarter indicator and $g_t$ is the quarter-to-quarter growth at $t$. 

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Consider an economy with discrete periods, i.e. \( t = 1, 2, \ldots \). It is an endowment economy: There is a Lucas tree that pays out \( d_t \) units of consumption good at the beginning of each period. The dividend in the first period, \( d_1 \), is normalized to one. The dividend growth \( g_t \equiv \log d_t - \log d_{t-1} \) (\( t \geq 2 \)) follows a two-state Markov Process, as follows:

\[
\begin{align*}
    \tilde{z}_t^I & \sim U[-\bar{z}, \bar{z}] \\
    \tilde{z}_t^N & \sim U[-\bar{z}, \bar{z}] \\
    \tilde{z}_t^N, \tilde{z}_t^I & < \min\{1 - q^h, q^l\}
\end{align*}
\]

\[
P(\tilde{g}_{t+1} = g^h | \tilde{g}_t = g^j, \tilde{z}_t^I, \tilde{z}_t^N) = q(g^j) + \tilde{z}_t^I, \quad j \in \{h, l\} \quad \forall t
\]

In (1), \( \tilde{z}_t^N \) and \( \tilde{z}_t^I \) are the exogenous signals based on which the agents make their prediction about the next period’s growth rate \( g_{t+1} \). Following the literature, the variables with \( \sim \) are random variables. Note that, in this specification, the only signal informative about the future growth rate is \( \tilde{z}_t^I \). However, as elaborated later, some “Naive” investors have misspecified beliefs and use signal \( \tilde{z}_t^N \) for their predictions. \( q^j, j \in \{g, h\} \) is the conditional probability of a high growth rate realizing in period \( t + 1 \) (\( \tilde{g}_{t+1} = g^h \)) when \( g_t = g^j \), without knowing the realizations \( \tilde{z}_t^I \) and \( \tilde{z}_t^N \). For tractibility reasons, I also assume the unconditional probability of having a high or low growth rate is the same.\(^{12}\) It implies \( q^h = 1 - q^l \).

**Agents.** The economy is populated with a unit measure of investors. The investors have time-separable and logarithmic preferences over their consumption stream. Furthermore, they are Bayesian, that is they form and update their beliefs based on Bayes’ rule.

\(^{12}\)It is only relaxed for the quantitative analysis in Section 7.3.
There are two types of investors in the economy. First, there is a set of atomistic investors with beliefs consistent with the underlying data generating process. For the rest of the paper, I refer to this set of investors as “sophisticated investors”. They are assumed to constitute a total measure of $1 - b$, for some $b \in (0, 1)$. The sophisticated investors are further divided into finer types $i \in [0, 1]$. In addition to the sophisticated investors, there are also a set of Naive investors, with a misspecified beliefs about the underlying data generating process. I specify their beliefs later, after specifying the information environment among the sophisticated investors.

The information environment among the sophisticated investors is as follows. At the beginning of period $t$, an i.i.d random variable $\tilde{\lambda}_t \in (0, 1)$ is drawn with cumulative distribution $F(\cdot)$. Once $\lambda_t$ is realized, all sophisticated investors with type $i \geq \lambda_t$ perfectly observe $z^I_t$. The rest of the sophisticated investors do not receive any signal. Therefore, they need to make a Bayesian inference about $z^I_t$ based on the equilibrium prices. Furthermore, I denote the posterior of the informed and uninformed investors (after potentially observing the signal realization and equilibrium prices) by $q^I_t = P^I_t(\tilde{g}_t = g^h)$ and $q^U_t = P^U_t(\tilde{g}_t = g^h)$ respectively, where $P^J_t$, $J \in \{U, I\}$, represents the probability measure that an $J$-type investor assigns to the future events at time $t$.

To summarize, the sophisticated investors are only heterogeneous in their information about the future growth rates, more specifically, the unconditional probability that they observe the realization of informative signal $\tilde{z}^I_t$. However, the form of asymmetric information does not matter for the results, as shown in Section 7.2, where I consider an extension in which the investors are asymmetrically informed about multiple states. Despite having infinite types, after the realization of $\lambda_t$, all informed investors observe the same signal and there is no asymmetric information among the informed investors. Therefore, ex-post, the so-
phisticated investors are either informed or uninformed. Here, the information structure among the sophisticated investors is governed by cumulative distribution $F(\cdot)$.

The primary goal here is to study the evolution of wealth distribution among the sophisticated investors. However, to motivate trade among the sophisticated investors, we need to have an exogenous and stochastic supply of liquidity (Milgrom and Stokey (1982)). Here, the provision of exogenous liquidity is done by a set of “naive” investors, who constitute a measure of $b < 1$ in this economy. The key assumption about the naive investors is that they falsely believe that the growth rate follows the following Markov process, instead of the actual one described in (1):

$$P^N(\tilde{g}_{t+1} = g^h | \tilde{g}_t = g^j, z^I_t, z^N_t) = q^j + z^N_t, \quad j \in \{h, l\}$$  \hspace{1cm} (2)

Note that the only difference between the naive and sophisticated investors is that the naive investors falsely believe that signal $\tilde{z}^N_t$, instead of $\tilde{z}^I_t$, is informative about the next period’s growth rate. All naive investors perfectly observe $z^N_t$ at the beginning of each period. As a result, they do not update their beliefs based on the equilibrium prices and they just act based on their own signal $\tilde{z}^N_t$. That said, $\tilde{z}^N_t$ works as a non-fundamental uncertainty in the economy that distorts the prices, and hence, prevents $z^I_t$ from being perfectly revealed to the uninformed investors. Let $q^N_t \equiv P^N_t(\tilde{g}_{t+1} = g^h)$ be the posterior of the naive investors.

**Interpretation of the naive investors:** In the literature of financial markets and market microstructure, it is commonly assumed that there is a set of “noise traders” who have an exogenous and stochastic net demand across assets traded in the market (for example,  

\footnote{The assumption is made to simplify the exposition and avoid unnecessary complications in dealing with different information sets.}
see Grossman and Stiglitz (1980) and Kyle (1985)). The main role of this assumption is to obscure the signal of the more informed investors from the less informed ones, and consequently, open the possibility of trade among a set of heterogeneously informed investors with homogeneous preferences and a common prior. In my model, the naive investors serve a similar purpose by providing an exogenous, stochastic and stationary source of liquidity across markets. The only key difference is that, in my model, the naive investors’ belief, instead of their demand schedule, is exogenously specified. This modeling choice yields analytical tractability. Because, when investors have logarithmic preferences, the wealth dynamics can be specified just in terms of the investors’ beliefs, as shown in Lemma 5 and previously pointed out by Blume and Easley (1992).

Similar to Yaari (1965) and Blanchard (1985), I assume the economy is populated with “dynasties” of investors, who are subject to random death at rate $1 - \delta \in (0, 1)$. Particularly, at the beginning of period $t \geq 2$, each investor dies with probability $1 - \delta \in (0, 1)$, independent of the other shocks. Each deceased investor is replaced with his “child” with the same type.\footnote{This assumption implies that the sophistication level is perfectly persistent all generations of a family. I relax this assumption in Appendix B4 and show it modestly impacts the derivations.} The investors in such a sequence form a dynasty. There is a continuum of dynasties for each type of sophisticated investors ($i \in [0, 1]$). Therefore, each dynasty in the economy is indexed by a pair $(f, k)$, $f \in [0, 1]$ and $k \in [0, 1] \cup \{N\}$.$^{15}$

To ensure the existence of a stationary solution, there is an Estate tax on inherited wealth in this economy. Upon an investor’s death at period $t$, his wealth is taxed at rate $\tau \in (0, 1)$; as such, the child receives fraction $1 - \tau$ of the parent’s assets and the taxed assets are distributed evenly among the newborns at time $t$. Equation 5 specifies the inheritance process, once the assets available in the economy are introduced.

\footnote{This assumption is necessary to make sure the realization of the death shocks are independent from the wealth dynamics of each type. This property is essential to make sure the total wealth share owned by each type of investors follows a Markov Process independent of the death shocks.}
As mentioned earlier, I assume that the investors have logarithmic and time-separable preferences over consumption, that is the terminal utility of an investor with type \( k \in [0, 1] \cup \{N\} \) born in family \((f, k)\) at \( s_1 \) and dies at \( s_2 > s_1 \) from consumption path \( \{c_{f,t}^k\}_{t=s_1}^{s_2-1} \) is as specified in (3). Note that no bequeath motive for wealth accumulation embedded in the preferences. The bequest motive for saving is examined in Appendix B3.

\[
U(\{c_{f,t}^k\}_{t=s_1}^{s_2-1}) = \sum_{t=s_2}^{s_1} \beta^{t-s_1} \log c_{f,t}^k \quad k \in [0, 1] \cup \{N\} \tag{3}
\]

**Assets.** There are two assets in this economy. One is the shares of the tree, which is in unit supply and traded at equilibrium price \( p_t \), in the unit of consumption goods at period \( t \). In addition, there is a risk-free asset with a net zero supply. In particular, at period \( t \), the investors trade consumption claims for period \( t+1 \), at an equilibrium substitution rate \( R_t^F \). That is at time \( t \), they can trade \( R_t^F \) units of consumption goods at time \( t+1 \) for one unit of consumption goods at time \( t \).\(^{16}\) I denote the time-\( t \) share of the tree in the hand of investor of family \((f, k)\) by \( x_{f,t}^k \). Similarly, \( s_{f,t}^k \) is defined as the consumption claims of period \( t \) (traded at \( t-1 \)) that the investor owns, provided he is alive at period \( t \). Therefore, his wealth at time \( t \), in the unit of consumption goods at time \( t \), is:

\[
W_{f,t}^k = (p_t + d_t)x_{f,t}^k + s_{f,t}^k
\]

To avoid Ponzi-like consumption schemes, I assume an investor with a negative wealth cannot borrow. Therefore, all investors optimally maintain a positive wealth in all states to avoid zero consumption, leading to extremely low expected utility, with probability one.

\(^{16}\)Note that the market is dynamically incomplete. For instance, no long-term risk-free bond is available in this economy. However, in Appendix B2, I show this restriction is not binding. In other words, I prove even if the set of available securities fully span the space of future dividend realizations, the allocations and wealth dynamics would be exactly the same as the case I analyze here.
That said, their optimization problem at time $t$ is:

$$\max \{c_{f,t+j}^k x_{f,t+j+1}^k + s_{f,t+j+1}^k \}_{j=0}^\infty \mathbb{E}_t^k \left[ \sum_{j=0}^\infty (\beta \delta)^j \log c_{f,t+j}^k \right]$$

s.t. $c_{f,t+j}^k + p_t x_{f,t+j+1}^k + \frac{1}{P_{t+j}} s_{f,t+j+1}^k \leq (p_t + d_t) x_{f,t+j}^k + s_{f,t+j}^k \quad \forall j \geq 0$ \quad (4)

$$W_{f,t+j}^k > 0 \quad \forall j \geq 0$$

where the expectation operator $\mathbb{E}_t^k[\cdot]$ takes expectation with respect to the information set and belief of type-$k$ agents at time $t$. In (4), it is easy to see that the feasibility set scales with wealth and additionally, the preferences over different consumption paths are scale-invariant. Therefore, the optimal portfolio and consumption of all type-$k$ investors only differ by a scalar.

To simplify the expositions, it is useful to introduce the integrated variables $c_t^k, x_t^k, s_t^k$, where $c_t^k = \int_0^1 c_{f,t}^k df, x_t^k = \int_0^1 x_{f,t}^k df, s_t^k = \int_0^1 s_{f,t}^k df$. Similarly, the total wealth in the hand of type-$k$ investors is $W_t^k = \int_0^1 W_{f,t}^k df$.

**Inheritance.** As mentioned earlier, each investor dies with probability $1 - \delta$ and is replaced with his child, who has the same type. The inheritance takes place as follows: When the investor of family $(f,k)$ with wealth $W_{f,t}^k$ and asset holding $(x_{f,t}^k, s_{f,t}^k)$ dies at time $t$, his child replaces him and receives $1 - \tau$ fraction of the assets. The remaining fraction of the deceased’s wealth is distributed evenly among all the newborns at period $t$. Therefore, the initial asset holding of the replacing child in family $(f,k)$ at time $t$ is:

$$x_{f,t}^k = (1 - \tau) x_{f,t}^k + \tau x_t^N + \int_0^1 x_{i}^k di = (1 - \tau) x_{f,t}^k + \tau$$

$$s_{f,t}^k = (1 - \tau) s_{f,t}^k + \tau s_t^N + \int_0^1 s_{i}^k di = (1 - \tau) s_{f,t}^k$$

(5)
Therefore, the initial wealth of the replacing child is $W_{f,t}^k = (1 - \tau)\hat{W}_{f,t}^k + \tau(p_t + d_t)$.

4 Equilibrium

In this section, I solve for the equilibrium sequence of consumptions, asset allocations and prices, and the posterior beliefs, given the growth and signal realizations. The goal of this section is to derive the wealth dynamics based on the equilibrium characterization solved here. The notion of equilibrium is Markov Perfect Equilibrium. I formally define the equilibrium in Section 4.1.

4.1 Definition of Equilibrium

Definition 1 (Dynamic Competitive Equilibrium).

Define:

$$\mathcal{F}_t^P = \{(x_t^k)_{k \in S \cup \{N\}}, \{s_t^k\}_{k \in S \cup \{N\}}, \lambda_t, g_t\}$$

The sequence of demand functions $\{c_t^k(p_t, R_t^F, q_t^k; F_t^P), x_{t+1}^k(p_t, R_t^F, q_t^k; F_t^P), s_{t+1}^k(p_t, R_t^F, q_t^k; F_t^P)\}_{t=1}^\infty$ for $k \in [0,1] \cup \{N\}$ and the price functions $p(z_t^I, z_t^N, F_t^P), R(z_t^I, z_t^N, F_t^P)$, where $p_t = p(z_t^I, z_t^N, F_t^P)$ and $R_t^F = R(z_t^I, z_t^N, F_t^P)$, constitute an equilibrium if

1. [Optimality] The demand functions constitute a solution to the optimization problem (4) given beliefs $\{q_t^k\}_{k \in [0,1] \cup \{N\}, t \geq 1}$.

2. [Bayesian Inference] All investors make Bayesian inference based on their signals and equilibrium prices $p_t$ and $R_t^F$ at time $t$. In particular, if $g_t = g_j$, $j \in \{h, l\}$:
\[ q^k_t = \begin{cases} 
 q^j + z^I_t & k \in [0, \lambda_t] \\
 q^j + z^U(z^I_t, z^N_t; \mathcal{F}^P_t) & k \in [0, \lambda_t) \\
 q^j + z^N_t & k = N
 \end{cases} \]

, where \( z^U(\cdot, \cdot; \cdot) \) is obtained from Bayes’ rule.\(^\text{17}\)

3. [Market Clearing] For every \( t \), the prices \( p_t \) and \( R^P_t \) are such that the market for consumption goods, risky assets and riskless assets clear:

\[
\begin{align*}
\int_0^1 c^\prime_i \, di + c^N_i &= d_t \\
\int_0^1 x^\prime_{t+1} \, di + x^N_{t+1} &= 1 \\
\int_0^1 s^\prime_{t+1} \, di + s^N_{t+1} &= 0
\end{align*}
\]

4.2 Optimal Intertemporal Decisions

In this section, I derive the optimal consumption and portfolio of a type-\( k \) investor with wealth \( W^k_t = (d_t + p_t)x^k_t + s^k_t \) and posterior \( q^k_t, k \in [0, 1] \cup \{N\} \). First, I prove that the value function is logarithmic in wealth. As spelled out in Lemma 1, it has three important implications: 1) The price-dividend ratio of the tree is constant. 2) All investors consume a fixed fraction of their wealth in every period. 3) In their portfolio choice decisions, the investors act myopically and just maximize the expected log-wealth in the next period.

\(^\text{17}\)More specifically,

\[
z^U(z^I_t, z^N_t; \mathcal{F}^P_t) = \frac{\int_{\mathcal{F}^N} \int_{\mathcal{F}^I} z^I_t \mathbb{1}(p_t = p(z^I_t, z^N_t, \mathcal{F}^P_t), R^P_t = R(z^I_t, z^N_t, \mathcal{F}^P_t)) \, dz^I_t \, dz^N_t}{\int_{\mathcal{F}^N} \int_{\mathcal{F}^I} \mathbb{1}(p_t = p(z^I_t, z^N_t, \mathcal{F}^P_t), R^P_t = R(z^I_t, z^N_t, \mathcal{F}^P_t)) \, dz^I_t \, dz^N_t}
\]
In order to solve for the investors’ optimal consumption and portfolio decision, I first characterize their Bellman equation. Note that the state variable here is \( F_t \equiv (F^P_t, z^I_t, z^N_t) \), which includes the state of asset allocations at \( t \), the growth rate \( g_t \), \( \lambda_t \), and exogenous signal realizations \( z^I_t \) and \( z^N_t \). Therefore, the value function can be specified as follows:

\[
V^k(W_t; F_t) = \max_{c_t, x_{t+1}, s_{t+1}} \log c_t + \beta \delta \mathbb{E}^k[V^k(W_{t+1}; F_{t+1})]
\]

s.t. \( c_t + p_t x_{t+1} + \frac{1}{R^F_t} s_{t+1} \leq W_t \)

\[
W_{t+1} = (d_{t+1} + p_{t+1}) x_{t+1} + s_{t+1}
\]

\[
\text{Prob}(W_{t+1} > 0) = 1
\]

\[
p_t = p(z^I_t, z^N_t, F^P_t) \quad R^F_t = R(z^I_t, z^N_t, F^P_t)
\]

Lemma 1.

a) For every type \( k \in [0, 1] \cup \{N\} \), there exists function \( a^k(\cdot) \) such that

\[
V^k(W_t; F_t) = \frac{1}{1 - \beta \delta} \log W_t + a^k(F_t)
\]

b) All investors consume a fixed fraction of their wealth. In particular, \( c_t = (1 - \beta \delta) W_t \)

c) For every \( t \geq 1 \), \( p_t = \frac{\beta \delta}{1 - \beta \delta} d_t \)

Lemma 1 describes the optimal consumption and saving behavior of the investors. Part (a) shows that the value functions are logarithmic in wealth. Moreover, part (b) indicates that the investors always save a fixed fraction of their wealth, regardless of their wealth or beliefs.\(^{18}\) Hence, since the optimal consumption is independent of the future expected

\(^{18}\)The intuition lies in the Cobb-Douglas nature of the preferences. It is well-known that agents with such preferences allocate their wealth to different components of their consumption bundle in fixed fractions, determined by the parameters of the utility function. In my model, the discount rate (\( \beta \)) and survival rate (\( \delta \)) govern the households’ saving ratio.
returns, the investors optimally maximize their expected log-wealth, or equivalently their expected log-return, in the next period.\textsuperscript{19}

Part (c) of Lemma 1 shows that the price-dividend ratio is constant. The reason is that according to Lemma 1(b), the economy’s total consumption and wealth should be at the same proportion of that of the investors. Having said that, we can derive the price-dividend ratio as follows.

\[
\int_0^1 W_i^t di + W_N^t = p_t + d_t = \frac{1}{1 - \beta} d_t
\]

Lemma 1(c) has further implications. First, it implies that the price-dividend ratio does not depend on the future dividend prospect. In other words, the price of the tree contains no information about the future growth rates and the only price that partially reflects the informed investors’ information is the risk-free rate.\textsuperscript{20} It also implies that the return on the risky asset is always proportional to the growth rate. Therefore, the fundamental choice that the investors face is to what extent they want to expose their savings to the growth rate $g_{t+1}$. This optimal exposure generally depends on the risk-free rate and one’s posterior about the future growth. Lemma 2 provides the closed-form solution to the optimal portfolio problem.

\textbf{Lemma 2.}

\textit{Suppose type-}k\textit{ investors, }$k \in [0, 1] \cup \{N\}$, \textit{have posterior belief }$q_k^t \in (0, 1)$. \textit{Furthermore, let}

\textsuperscript{19}Such myopic behavior is directly resulted from the logarithmic preferences over consumption. In particular, we can infer from Merton (1973) that the investors with logarithmic preferences (more specifically, when IES is one) do not hedge against the changes in the investment opportunity set. Therefore, they only maximize the (subjective) expected log-return, period by period.

\textsuperscript{20}Note that the main goal here is to study the impact of asymmetric information on wealth inequality and the model is not suited for studying the asset pricing implications of wealth inequality. This question is explored in earlier studies, such as Gollier (2001) and Gomez (2019). In fact, my model examines the interplay between the wealth distribution and information dissemination and it provides reasonable implications about the interaction, as discussed in Section 4.3.
\[ \mu^k_t \equiv \frac{p_t x^k_{t+1}}{p_t x^k_{t+1} + s_t} \] be the fraction of their saving invested in the risky asset at time \( t \). Then\(^{21}\)

\[ \mu^k_t = 1 + q_t^k \frac{1}{\beta \delta R^F_t e^{-g_t} - 1} + (1 - q_t^k) \frac{1}{\beta \delta R^P_t e^{-g_h} - 1} \] (9)

Lemma 2 shows that the share of the risky asset in the investors’ portfolio linearly changes with their beliefs about the next period’s growth. As we see next, a direct implication of this result, combined with the market clearing conditions, is that all investors can infer the wealth-weighted average belief of the other investors based on the equilibrium prices.

### 4.3 The Equilibrium Posterior of the Uninformed Investors

In this section, I discuss the dissemination of information from the informed investors to uninformed investors through the equilibrium prices. As the main result, I show what determines the informativeness of the prices is the ratio between the wealth share of the informed and naive investors. Therefore, the gap between the informed and uninformed investors’ belief shrinks as either a larger fraction of the investors become informed, i.e. smaller \( \lambda_t \), or the informed investors become wealthier. A direct consequence of this result is that more inequality likely results in more informative prices and lower risk premium, consistent with the empirical findings of Bai, Philippon, and Savov (2016). Furthermore, toward the end of this section, I solve for the risk-premium and discuss its interaction with the wealth distribution.

To solve for the uninformed investors’ posterior, first, I extract the following identity from the market clearing conditions.

\(^{21}\)The optimal portfolio is discontinuous at the extreme certainty points \( q^k_t \in \{0, 1\} \). The reason is, as long as each state has a positive probability, the investors do not take extreme positions in order to avoid reaching negative wealth values. In fact, an investor with a negative wealth gets an extremely low \((-\infty)\) terminal payoff as he cannot borrow, due to the borrowing constraint, and hence, cannot consume.
\[ 1 = \int_0^\lambda \mu_i w_i^t di + \int_1^\lambda \mu_i w_i^t di + \mu_t^N w_t^N \]  

(10)

Equation 10 states that the weighted average of the shares invested in the risky asset should add up to one, which is a direct consequence of the fact that the total savings in the economy should be equal to the value of the tree.

By substituting \( \mu_i^k \)'s in (10) with the expression in (9), and after some rearrangements, we get:

\[
(q_t^U w_t^U + q_t^I w_t^I + q_t^N w_t^N) \frac{1}{\beta \delta \hat{R}_t e^{-g_t} - 1} + ((1 - q_t^U) w_t^U + (1 - q_t^I) w_t^I + (1 - q_t^N) w_t^N) \frac{1}{\beta \delta \hat{R}_t e^{-g_t} - 1} = 0
\]

(11)

The important implication of (11) is that all investors can infer the wealth-weighted average of the beliefs, i.e. \( \bar{q}_t \equiv w_t^U q_t^U + w_t^I q_t^I + w_t^N q_t^N \) from the risk-free rate \( \hat{R}_t \). In fact, based on Equation 9, a hypothetical investor with posterior belief \( \bar{q}_t \) should have \( \mu \) of one, meaning he invests all of his saving in the risky asset, or equivalently, does not borrow or lend. For the rest of the paper, I refer to this hypothetical investor as the “representative investor”. An alternative interpretation of \( \bar{q}_t \) is that if the investors are replaced by a large representative investor, his belief should be \( \bar{q}_t \) in order to generate the same vector of prices.

As just mentioned, the belief of the representative investor can be inferred from the vector of prices. Therefore \( w_t^I q_t^I + w_t^N q_t^N \), and consequently, \( w_t^I z_t^I + w_t^N z_t^N \), are observable to the uninformed investors. Note that both \( w_t^I \) and \( w_t^N \) can be derived from \( \mathcal{F}_t^P \), that is no equilibrium object used in \( w_t^I q_t^I + w_t^N q_t^N \). This point is formalized in Lemma 3.

**Lemma 3.**
For any \((z^I_t, z^N_t, \mathcal{F}^P_t)\), \(w^I_t z^I_t + w^N_t z^N_t\) can be inferred from the equilibrium prices. Moreover, all prices are measurable with respect to the \(\sigma\)-algebra induced by \((w^I_t z^I_t + w^N_t z^N_t, \mathcal{F}^P_t)\).

Building on this result, Lemma 4 provides a closed-form expression for the uninformed investors’ posterior, as a function of \(z^I_t, z^N_t\) and the publicly observable objects.

**Lemma 4.**

\[
q_{t}^U = q(g_t) + z_{t}^U,
\]

where

\[
z_{t}^U = \frac{1}{2} \left[ \max \{-\bar{z}^I, z^I_t + \frac{w^N_t}{w^I_t} (z^N_t - \bar{z}^N)\}, \min \{\bar{z}^I, z^I_t + \frac{w^N_t}{w^I_t} (z^N_t + \bar{z}^N)\} \right]
\] (12)

Lemma 4 provides the uninformed investors’ posterior as a function of the wealth distribution at \(t\) and exogenous signals \(z^I_t\) and \(z^U_t\). To generate insights from (12), consider a case that \(\frac{w^N_t}{w^I_t}\) is sufficiently small, so that

\[
-\bar{z}^I \leq z^I_t + \frac{w^N_t}{w^I_t} (z^N_t - \bar{z}^N) \leq z^I_t + \frac{w^N_t}{w^I_t} (z^N_t + \bar{z}^N) \leq \bar{z}^I
\] (13)

In this case, the uninformed investors’ posterior boils down to \(q_{t}^U = q^I_t + \frac{w^N_t}{w^I_t} z^N_t\). We see that the difference between the informed and uninformed investors’ posterior is proportionate to the wealth ratio between the naive and informed investors, \(\frac{w^N_t}{w^I_t}\). That said, we can think of the ratio as a measure of noise-to-signal ratio in the vector of prices. Therefore, the prices become more informative about the future growth rate as a larger fraction of the investors become informed or the wealth share of the informed investors increases.

Therefore, the wealth distribution plays an important role in the informativeness of the prices: The larger is the total wealth share of informed investors, the prices are more informative about the future growth rate. It is intuitive because the informed investors all...
trade in the direction that the informative signal implies. As such, as they hold a larger fraction of the total wealth, they push the prices more strongly, making the prices more reflective of $z_t^f$ and hence, more informative about the future growth prospects.

4.4 Risk Premium

The last step for the equilibrium characterization is to solve for the risk-free rate in the economy. By inspecting (11), we see

$$ R_t^F = (\beta \delta)^{-1} \frac{1}{\bar{q}_t e^{-g_h} + (1 - \bar{q}_t)e^{-g_l}} $$

(14)

Consequently, we can derive the risk premium:

$$ \bar{E}_t \left[ \frac{p_{t+1} + d_{t+1}}{p_t} - R_t^F \right] = \bar{q}_t (1 - \bar{q}_t) \left( \frac{e^{g_h - g_l}}{\bar{q}_t e^{-g_h} + (1 - \bar{q}_t)e^{-g_l}} \right)^2 $$

(15)

where expectation $\bar{E}_t$ is taken with respect to the representative investor’s belief. One can see that the risk premium, computed in (15), is concave in $\bar{q}_t$. Therefore, as the inequality increases and the wealth becomes more concentrated in the hand of the most sophisticated investors, the representative belief becomes more precise and the risk-premium decreases. This result lends an informational justification for the negative relationship between the wealth share at top percentiles and risk-premium, documented by Gomez (2019).

Equation 15 also implies the risk premium decreases as the informative signal becomes more informative. Two forces push down the risk premium: First, the direct channel that a more informative signal reduces the investors’ uncertainty about the future growth rate, at least in average, leading to more accurate representative beliefs. The second channel is
about its impact on the distribution of beliefs. In fact, a more informative signal reduces the belief gap between the informed and uninformed investors. The reason being that the change widens the belief gap between the sophisticated and naive investors, decreasing the naive investors’ wealth share, and consequently, noise in the prices. Therefore, a more informative signal reduces the asymmetric information among the sophisticated investors and increases their wealth share, which leads to more accurate representative beliefs.

5 Evolution of the Wealth Distribution

This section contains the main results relating the information environment and wealth distribution. Lemma 5 and Corollary 1 describe the evolution of wealth shares, given the posteriors and the growth state \( (g_t) \). Then, I provide some characterization of the stationary distribution in Proposition 1, where I discuss the determinants of the thickness of the right tail. Finally, I discuss the model’s predictions about the composition of the wealthiest and distribution of expected returns.

Lemma 5.

*The wealth share of the investor in dynasty \((f, k)\) \((f \in [0, 1] \text{ and } k \in [0, 1] \cup \{N\})\) at \( t+1 \), if alive at \( t + 1 \), is:*

\[
\log w_{f,t+1}^k - \log w_{f,t}^k = \begin{cases} 
\log \frac{q_k^t}{\bar{q}_t} & \text{if } g_{t+1} = g^h \\
\log \frac{1-q_k^t}{1-\bar{q}_t} & \text{if } g_{t+1} = g^l
\end{cases} \quad \forall \ 1 \leq l \leq t \tag{16}
\]

Lemma 5 specifies the dynamics of an investor’s wealth share, as a function of his posterior belief \( q_k^t \), the representative belief \( \bar{q}_t \) and the realized growth rate \( g_t \). The right hand side in (16) specifies the log-ratio of the probabilities assigned to the realized state by the investor.
and representative investors. Therefore, Equation 16 states that an investor’s wealth share increases if his posterior belief is more consistent with the realized growth rate, compared to the representative belief. For instance, if $g_{t+1} = g^h$, the wealth share of the investor increases if $q^k_t > \bar{q}_t$, that is, at $t$, the investor was more optimistic than the representative investor about having a high growth rate at $t + 1$, and hence, allocated a larger fraction of his saving to the risky asset.

There are a few additional remarks on Equation 16. First, note that no equilibrium price is used in (16), which substantially simplifies our analysis, both analytically and numerically. In addition, one can show that Equation 16 is applicable for any belief process and we do not need to restrict to the set of beliefs formed by Bayes’ rule. Putting differently, the proof of Lemma 5 works for any distribution of beliefs and does not require the investors being Bayesian, as are here. It is effectively what I do in Section 7.4.1, where I construct the model-implied wealth dynamics with beliefs constructed based on Michigan Survey of Customers. Moreover, one can utilize this equation to study the distributional implication of non-Bayesian belief formation processes, such as diagnostic beliefs (Bordalo, Gennaioli, and Shleifer (2018)) and extrapolative beliefs. Nevertheless, such analysis are beyond the scope of this paper and relegated to future studies.

Now, equipped with the processes describing the wealth dynamics, we are ready to explore how the information environment impacts the wealth distribution and its dynamics.

### 5.1 Stationary Distribution

In this section, I study the stationary distribution of wealth shares and provide some characterizations of the stationary distribution of wealth. In particular, I mostly discuss the tail of the distribution, which captures the extent of top inequality. Before the characteri-
zations, we first need to verify a stationary distribution indeed exists.

To verify the existence, we first need to introduce the Markov chain specifying the dynamics of the model. In particular, I show the distribution of wealth shares, combined with the growth state, constitute a Markov chain. Therefore, our state variables are:

\[ \Omega_t \equiv \{ \{ w^i_t \}_{i \in [0,1]}, w^N_t, g_t \} \in \Lambda \equiv L^1 \times (0,1) \times \{ g^h, g^l \} \]  

(17)

In (17), \( L^1 \) is the space of all measurable functions over \([0,1]\) with a bounded integral. Note that the state variables here are infinite dimensional. I denote the random mapping that relates the time-\( t+1 \) state \((\Omega_{t+1})\) to time-\( t \) state \((\Omega_t)\) by \( Q(\Omega_t, B) \). In fact, it represents the probability of \( \Omega_{t+1} \) belonging to Borel subset \( B \in 2^\Lambda \), conditional on the time-\( t \) state \( \Omega_t \). Therefore a stationary distribution is a distribution over the set of distribution of wealth shares and the growth state.\(^{22}\) Corollary 1 can be used to specify the transition process.

Corollary 1.

Suppose \( \kappa = (1 - \delta)\tau \). Then, the wealth share in the hand of type-\( k \) investors evolves according to the following equation of motion:

\[
w^{k}_{t+1} = \begin{cases} 
  w^{k}_t g^k_t (1 - \kappa) + \kappa & \text{if } g_{t+1} = g^h \\
  w^{k}_{t} \frac{1 - g^k_t}{1 - \bar{q}_t} (1 - \kappa) + \kappa & \text{if } g_{t+1} = g^l 
\end{cases} \quad \forall k \in [0,1] \cup \{ N \}
\]  

(18)

Note that in Equation 18, \( q^k_t \) and \( \bar{q}_t \) are random variables, which depend on the state

\(^{22}\)Probability measure \( \mu : \mathcal{P}(\Lambda) \rightarrow [0,1] \) is a stationary distribution of the Markov process if for every Borel subset of \( \Lambda \), like \( B \), we have:

\[
\mu(B) = \int_{\Omega} Q(\Omega, B)d\mu(\Omega)
\]
variable \( \Omega_t \) and i.i.d random variables \( \tilde{\lambda}_t \), \( \tilde{z}_t^f \) and \( \tilde{z}_t^N \). Therefore, the transition equation induces a random mapping between \( \Omega_t \) and the distribution of wealth shares at \( t + 1 \). After specifying the Markov chain and its transition process, we are ready to provide our existence result.

**Lemma 6.**

The wealth share of each dynasty \((f, k)\) has a stationary distribution, only depending on type \( k \in [0, 1] \cup \{N\} \).

Suppose the unconditional distribution of wealth shares for dynasty \((f, i)\), with type \( i \in [0, 1] \), has CDF \( G^i(w) = \text{Prob}(w_{f,t}^i < w) \). Then, the “empirical” distribution of wealth shares is defined as:

\[
G(w) = \int_0^1 G^i(w) di
\]

Note that \( G(w) \) is the unconditional fraction of the sophisticated investors with a wealth share less than \( w \). The distribution is called empirical since it is the distribution that an econometrician, with no knowledge of the types, would estimate for the population of sophisticated investors. Proposition 1 provides a characterization of the tail of the empirical distribution. In addition to the distribution, it also discusses the composition of types among the extremely wealthy and how it is different from the overall type composition.

**Proposition 1.**

In any stationary distribution:

a) If \( \tau > 0 \) and \( \delta < 1 \), CDF \( G(\cdot) \) has a thick right tail and its tail parameter is the unique positive solution of the following equation:
Therefore, we have
\[
\lim_{T \to \infty} \mathbb{E}[(\prod_{t=1}^{T} \left( \frac{(q_t^I)^{\gamma+1}}{\bar{q}_t} + \frac{(1 - q_t^I)^{\gamma+1}}{1 - \bar{q}_t} \right))^{\frac{1}{T}}] = (\delta + (1 - \delta)(1 - \tau)^\gamma)^{-1}
\]

(19)

Therefore, we have \( \lim_{w \to \infty} \frac{1 - G(w)}{w^{-\gamma}} \in (0, \infty) \), for some \( \gamma > 1 \).

\( b) \) [Type Composition in the Tail] Suppose \( f(1) > 0 \) and the types are private and random type \( \tilde{i} \) is uniformly distributed in \([0,1]\), then for any \( i < 1 \) we have:

\[
\lim_{w \to \infty} P(\tilde{i} > i | w_{f,t} > w) = 1
\]

(20)

Proposition 1(a) shows that under very general conditions, the empirical distribution has a thick right tail and the thickness is governed by the extent of asymmetric information between the most informed investors and the representative investor. To see this, note that the left hand side in (25) represents a measure of unconditional expected belief gap between the most sophisticated investors and the representative investor. In fact, loosely speaking, to compute the expected belief gap, a geometric variation of the law of large numbers is applied. Therefore, the information environment impacts the thickness parameter \( \gamma \) only through the extent to which the most sophisticated investors have beliefs more accurate than the representative investor. Therefore, this simple measure can be used to gauge the impact of a change in information environment on wealth inequality among the wealthiest.

Building on this result, Corollary 2 relates the tail parameter to the excess return of the most informed investors.

**Corollary 2.**

Let \( r_{f}^I \) be the log-return of the most informed investors (sophisticated investors with type

\( ^{23} \)In fact, \( \gamma \) is the infimum all values like \( \hat{\gamma} \) for which there exists \( A(\hat{\gamma}) > 0 \) such that \( G(w) > 1 - A(\hat{\gamma})w^{-\hat{\gamma}} \). Abusing the notation, I define \( A \equiv A(\gamma) \).
Then, \( \gamma \) is the unique positive solution to the following equation:

\[
\lim_{T \to \infty} E\left[ e^{\gamma \sum_{t=1}^{T} (r_t^I - g_t + \log \beta)} \right] \frac{1}{T} \times (\delta + (1 - \delta)(1 - \tau)^{\gamma}) = 1
\]  

Corollary 2 shows that the tail parameter is related to \( r_t^I - g_t \): The return of the most informed investors minus the growth rate. The intuition is that \( r_t^I - g_t \) captures how faster the wealth of the most informed grows compared to the whole economy. Note that this result resembles the well-known \( r - g \) rule put forward by ?, with only one exception. In Piketty’s rule, the return is the average return on capital, while here the return is the return of the most informed investors on their saving. That said, Corollary 2 underscores the role of asymmetric information in the top inequality and augments Piketty’s rule to account for such heterogeneities in the population.

The proposition also indicates that the tax rate \( \tau \) and the death rate \( 1 - \delta \) also play a counter-balancing role in the top inequality. Consistent with intuition, an increase in the tax rate or death probability reduce the inequality, as they make the redistribution larger and more frequent.

Part (b) in Proposition 1 specifies the composition of the wealthiest. In fact, it states that the fraction of investors with type less than \( i \) and wealth share exceeding \( w \) goes to zero as \( w \) goes to infinity. In other words, as we focus on a smaller set of the top wealthiest, more sophisticated investors are more likely to remain and less sophisticated ones asymptotically vanish from the set. Therefore, the main message is that the tail is populated with highly sophisticated investors. It explains the empirical observation that the individuals with a high cognitive ability overrepresent among the wealthiest investors.

According to (20), what distinguishes the investors and ranks them in the wealth dis-
tribution is their sophistication level $i$, not age. In other words, even though there are plenty of less sophisticated but arbitrarily old investors that accumulate wealth for a long-time, without getting hit by the death shock, they have a small chance to get to the top percentiles.\footnote{A key reason for this result could be the strong persistence in the types. When the types are highly persistent, almost all less sophisticated families will be overtaken and fall substantially behind eventually in the long-run. Therefore, we can expect a less extreme result to emerge when there are movements in the types.}

Now, I discuss two implications of Proposition 1 for the distribution of expected returns in Proposition 2.

**Proposition 2.**

\begin{itemize}
    \item[a)] The expected return is increasing in wealth share, that is $\mathbb{E}[r_{t+1}|w_t = w]$ is increasing in $w$.
    \item[b)] In any stationary distribution, the expected return of the most informed is bounded:
    \[ \mathbb{E}[r^I_t - g_t] < -\log(1-\delta)(1-\tau) - \log\beta \]
\end{itemize}

Proposition 2(a) implies that the distribution of expected returns is increasing. The intuition is that the wealthier investors are more likely to be more informed, who get a higher expected return due to their information advantage. This result provides an explanation for the positive relationship between wealth and average returns, empirically documented by Fagereng, Guiso, Malacrino, and Pistaferri (2016) and Bach, Calvet, and Sodini (2016).

Part (b) in Proposition 2 implies that the expected return of the most informed investors in any stationary distribution is bounded by some parameters unrelated to the information environment. A direct implication is that an increase in the informativeness of signal
Figure 4: Expected return on savings in the expected distribution \( F(\lambda) = \lambda^3 \).

\( z^I_t \), while temporarily boost their return, their expected return eventually converges to a bounded level. In other words, the extent of return heterogeneity across the investors is limited, regardless of the information environment. The intuition is that as the wealth share of the informed investors increases, so does their impact on the prices, making it more informative. Therefore, as more information disseminated through the prices, the expected excess return associated with the information decreases. In short, the wealth distribution becomes more and more skewed toward the more informed investors, up to an equilibrium point at which their expected return balances out with their death and tax rates.

6 Endogenous Information Acquisition

In general, economic agents are heterogeneously informed about the future returns mostly due to the costly nature of information. The cost is partly pecuniary, like that of obtaining access to datasets, or the cost of education, which is paid to earn the skills needed to process available signals (Lusardi, Michaud, and Mitchell (2017)). In this section, I endo-
genize the types by allowing the newborns to choose their type at a cost. Therefore, the investors choose their level of sophistication before they start trading. It can be thought of the decision to acquire permanent human capital to increase the expected returns on savings.

In short, I show that wealth inequality, captured by the tail parameter, decreases as the cost of information acquisition decreases. A direct implication of this finding is that an inequality-reducing policy is to subsidize education. In fact, wealth inequality is negatively related with college tuitions among OECD countries, which lends support to this finding. Another important implication is that the emergence of novel, but highly expensive, ways for information production could also contribute to the rise of inequality, as only a few afford to directly or indirectly (though investment delegations) benefit from them. I formalize this point in the remainder of this section.

6.1 A Model of Wealth Dynamics With Costly Information Acquisition

In this section, I modify the setup in two aspects. First, I assume there are only two types of sophisticated investors, instead of a continuum of types: One type is the informed investors, who always learn the realization $z^I_t$, and the other type is the uninformed ones, who never learn the realization and need to imperfectly infer the signal from the equilibrium prices. Secondly, as mentioned earlier, I assume all newborns choose their type before they start trading. Particularly, they can become an informed investor for their entire lifetime by paying cost $(\beta \delta)^{-1} \chi p_t$ out of their endowed wealth, where $\chi > 0$. They remain uninformed if refrain from the payment. Then, all the payments are evenly distributed among all

\footnote{For details, see article “A debate is under way about the cost of higher education”, The Economist, July 18th 2019}
the newborns, in order to keep the total wealth of the economy unchanged.Lemma 7 characterizes the newborns’ type acquisition decision.

**Lemma 7.**

A newborn with inherited wealth $w_{f,t} = (1 - \tau)\bar{w}_{f,t} + \tau$ becomes informed if

$$\frac{1}{1 - \beta\delta} \log(w_{f,t} - \chi + h_t\chi) + \sum_{m=1}^{\infty} \frac{(\beta\delta)^m}{1 - \beta\delta} \mathbb{E}_t[q_{t+m-1}^I \log \frac{q_{t+m-1}^I}{q_{t+m-1}^U} + (1 - q_{t+m-1}^I) \log \frac{1 - q_{t+m-1}^I}{1 - q_{t+m-1}^U}] \geq \frac{1}{1 - \beta\delta} \log(w_{f,t} + h_t\chi)$$

(22)

where $h_t$ is the fraction of newborns who decide to become informed at $t$.

Lemma 7 illustrates the trade-off a newborn faces in his information acquisition decision. On one hand, becoming informed increases the expected return on savings and increases his expected utility. On the other hand, there is a fixed cost associated with becoming informed, which is justified only for wealthy enough newborns. In fact, there is a threshold value $\bar{w}_t$ such that a newborn becomes informed iff $w_{f,t} \geq \bar{w}_t$. By inspecting (22), we see an increase in the cost of becoming informed, $\chi$, increases the threshold $\bar{w}_t$, and consequently, tends to reduce the fraction of informed investors. As a result, a higher $\chi$ widens the belief gap between the the informed investors and representative investor, who takes the weighted-average belief of all investors. Due to this increase in the asymmetric information, the wealth inequality also generally increases with $\chi$. Proposition 3 formalizes this point.

**Proposition 3.**

Denote $\gamma(\chi)$ is the tail parameter of the empirical distribution corresponding to cost parameter $\chi$. Furthermore, suppose the following condition holds:

---

26This assumption is made for tractability reasons. Particularly, if the costly information acquisition affects the total wealth level, it would also impact the current prices, which complicates the analysis.
\[
\mathbb{E}[q_t^I \log \frac{q_t^f}{q(g_t)} + (1-q_t^I) \log \frac{1-q_t^I}{1-q(g_t)}] + (1-\delta) \log (1-\tau) > 0
\] (23)

where \( q(g_t) = P(g_{t+1} = g^h | g_t = q^j), \quad j \in \{h, l\} \). Then, \( \gamma(\chi) \rightarrow 1 \), as \( \chi \) goes to infinity.

Proposition 3 states that as the cost of information acquisition increases, the inequality also boundlessly increases, provided the informative signal is sufficiently informative. Note that \( \gamma = 1 \) is the lowest possible value for the tail parameter and designates the highest level of inequality. The main force leading to this result is the relationship between \( \chi \) and the extent of asymmetric information between the informed investors and representative investor. As the cost of information acquisition increases, a smaller and smaller fraction of the investors opt to be informed. Therefore, the equilibrium prices continue to reflect less of the informative signals, which levers up the expected return of the informed investors and the speed at which their wealth diverges from the uninformed investors, while the redistributive power of the taxation is fixed by parameter \( \tau \). Therefore, the result indicates a positive relationship between top inequality and cost of information acquisition.

7 Discussion

7.1 Investment Delegation

Thus far, I have assumed that the investors cannot delegate their investment decisions to other, potentially more informed, investors. In this section, I relax this assumption and by so doing, I examine the role of wealth management industry, a $74 trillion industry\(^{27}\), in wealth inequality. In short, I find the inequality has an inverse U-shaped relationship with

\(^{27}\) according to Boston Consulting Group: Global Asset Management 2019: Will These '20s Roar?, July 2019
the size of managed funds.

To study investment delegation, I consider the following modification of the model. Suppose in addition to the naive investors, there are three types of investors: Informed investors, who represent the fund managers, “delegating” investors, who represent the clients, and uninformed investors, who manage their own wealth. The delegating investors are also uninformed, but they delegate their investment decisions to an informed investor. The benefits are dividend between a client and his fund manager based on a Nash bargaining protocol, where the fund manager receives fraction $\psi \in [0, 1]$ of the surplus. More specifically, if the fund manager delivers return $R^I_t$ and the uninformed investors obtain $R^U_t$, then the clients’ effective return is $R^U_t + (1 - \psi)(R^I_t - R^U_t)$. The rest of the return accrues to the fund manager. $\psi$ captures the competitiveness of the market for wealth management. I denote the measure of informed, delegating and uninformed investors by $b^I, b^D, b^U$. Note that $b^I, b^D, b^U = 1 - b$.

**Proposition 4.**

If $b^I, b > 0$ are sufficiently small, then, holding $b^D + b^U$ fixed, the expected wealth share of the fund managers is maximized for an interior values of $(b^D, b^U)$.

As illustrated in Figure 5, Proposition 4 indicates that an interior value of the clientele size maximizes the expected wealth share of the fund managers. As for the increasing part, it is obvious that the managers benefit from managing a larger wealth share, as they accrue a fraction of the return. However, as they manage a larger wealth share of the economy, they impose a larger price impact, revealing more information to the uninformed investors, and hence, shrinking the return gap between the informed and informed investors. In fact, as the managed wealth share increases, two forces hurt the fund managers’ wealth accumulation: First, The fund managers obtain a lower rent from their management service, due to the reduction in $\mathbb{E}[R^I_t - R^U_t]$. Secondly, their own expected return, $\mathbb{E}[R^I_t]$, also decreases as
more information disseminated through the equilibrium prices. Due to these forces, the
fund managers’ expected wealth share does not always increase with their fund size.

The finding suggests that the growth of wealth management industry could have con-
tributed to the rise of super-rich fund managers, and hence, the rise of wealth inequality.
Nevertheless, further growth might alleviate, not aggravate, the inequality. To reduce in-
equality, the result suggests removing the frictions in accessing profitable funds, such as
the minimum investment requirements, prevalent in the industry.

7.2 Asymmetric Information About Growth-independent States

In the baseline setup, the investors were merely asymmetrically informed about the future
growth rates. However, in practice, some investors and fund managers engage in market
neutral or hedged strategies, minimizing their exposure to aggregate risks. In this section,
I show the results carry over for various forms of asymmetric information and the model
implications are not restricted to the specific form of asymmetric information studied in the baseline.

I expand the model by introducing growth-independent state $\tilde{\varepsilon}_t \in \{0, 1\}$. Particularly, suppose the investors, at period $t$, can trade securities contingent on the realization of $\varepsilon_{t+1}$. As such, two new securities, with zero net-supply, are introduced: a security that pays one unit of consumption good at time $t+1$ if $\varepsilon_{t+1} = 1$, and a security that pays one share of the tree if $\varepsilon_{t+1} = 1$.

The information environment around $\varepsilon_{t+1}$ is as follows.

$$
\begin{align*}
\varepsilon_t & \in \{0, 1\} \\
q^\varepsilon & \in (0, 1) \\
\tilde{y}_t^I & \sim U[-\tilde{y}^I, \tilde{y}^I] \\
\tilde{y}_t^N & \sim U[-\tilde{y}^N, \tilde{y}^N] \\
\tilde{y}^N, \tilde{y}^I & < \min\{1 - q^\varepsilon, q^\varepsilon\} \\
P(\tilde{\varepsilon}_{t+1} = 1 | y_t^I, y_t^N) & = q^\varepsilon + y_t^I \quad \forall t
\end{align*}
$$

(24)

Equation 24 indicates that the only signal informative about the value of $\varepsilon_{t+1}$ is $y_t^I$. However, similar to the baseline case, the Naive investors believe $y_t^N$ is informative about $\varepsilon_{t+1}$.

Furthermore, I assume in every period $\lambda^\varepsilon_t \in (0, 1)$ from CDF $F^\varepsilon : [0, 1] \rightarrow [0, 1]$ is drawn and all sophisticated investors with type $i \geq \lambda^\varepsilon_t$ learn the realization of $y_t^I$. The other sophisticated investors need to make a Bayesian inference based on the equilibrium prices.

Note that, according to (24), $\tilde{\varepsilon}_{t+1}$ and $g_{t+1}$ are independent. The information environment about $g_{t+1}$ is similar to the baseline case.

Proposition 5 extends the result in Proposition 1 to the current case with asymmetric information in multiple dimensions.

**Proposition 5.**
Suppose $\tau > 0$ and $\delta < 1$ and let $P^I(\cdot)$ and $\bar{P}(\cdot)$ represent the belief of informed investors, respectively, about the future states. Then, the empirical distribution has a thick right tail, with tail parameter $\gamma$ satisfying:

$$
\lim_{T \to \infty} \mathbb{E}[(\Pi^T_{t=1} \sum_{g^j \in \{g^h, g^l\}, \epsilon \in \{0, 1\}} \frac{P^I(\tilde{g}_{t+1} = g^j, \tilde{\epsilon}_{t+1} = \epsilon)^{\gamma+1}}{P(\tilde{g}_{t+1} = g^j, \tilde{\epsilon}_{t+1} = \epsilon)^{\gamma}})]^\frac{1}{\gamma} = (\delta + (1 - \delta)(1 - \tau)^\gamma)^{-1}
$$

(25)

The key message of Proposition 5 is that the form of asymmetric information does not matter for the equilibrium level of inequality. Either about the growth state or other growth-independent states, asymmetric information about the future payoff relevant states impacts the wealth distribution in the same way and has the same implications. Another implication of this result is that highly informed investors, even in a specific segment of the market, might become super-rich, provided they have enough information advantage and operate in a liquid enough market.

### 7.3 Quantitative Assessment

In this section, I empirically assess the role of asymmetric information in wealth inequality. Clearly, asymmetric information impacts wealth inequality through the investors’ heterogeneous choice of portfolio. Therefore, it has implications for both portfolio dynamics and degree of wealth inequality, which I assess here based on data. In particular, I use data on risky asset holdings for the top 0.1% to match the first two moments of the share of risky assets in their financial wealth.

Table 1 provides the parameter and target values.\textsuperscript{28} The last row exhibits a relatively

\textsuperscript{28}SCF is the Survey of Consumer Finance.
small turnover between risky and riskless assets, suggesting that wealthier investors have
a limited advantage in forecasting the future growth rates. However, as follows, I show
that the numbers are consistent with the case that wealthier investors do most of their
rebalancing among risky assets and less between risky and riskless assets. To do so, I
consider the modified model in Section 7.2, which expands the security space and allows
for the presence of growth-independent payoff relevant states.

Table 2 provides the results for the selected parameter values. Assuming only the top
0.1% are informed, we see a set of parameters can generate the same expected top wealth
share and generate patterns in the portfolio dynamics arguably similar to data, in terms
of average and volatility. The average fraction in the model is smaller than data, which
indicates the model overestimates the risk-aversion of the top 0.1%. Interestingly, we see
the model requires a low degree of asymmetric information about the growth state, while
a high degree of asymmetric information regarding the growth-independent state to match
the data.

The key message of this section is that the empirical patterns on the extent of wealth
inequality and portfolio dynamics are consistent with the model predictions. The model
Table 2: Information-related parameters and outcome values. It is assumed the top 0.1% are the informed investors. $\mu^{IF}_t$ is the fraction of their financial assets invested in the riskless asset.

suggestions that the patterns observed in data correspond to a low degree of asymmetric information regarding the future growth rates and a high degree of asymmetric information about some growth-independent states.

7.4 Empirical Relationships between Belief Heterogeneity and Wealth Inequality

There is a drastic heterogeneity in macroeconomic expectations, as demonstrated by several surveys of the general public and even those of professional forecasters (Mankiw, Reis, and Wolfers (2003)). Undoubtedly, the extent of this heterogeneity has a first order impact on the distribution of returns, and hence, the distribution of wealth. Equipped with the tools developed here, in this section, I quantify the extent of belief heterogeneity and evaluate their impact on the extent of wealth inequality and its dynamics.

More specifically, I construct the evolution of beliefs for different groups of population in the US, based on the rich dataset of Michigan Survey of Consumer and study their implication...
for the wealth distribution. In Section 7.4.1, I study the dynamic implications by utilizing Equation 18 and show the dynamics predicted by the model has a remarkable consistency with the estimated dynamics in the wealth shares over the past decades. In Section 7.4.2, I use Equation 25 to find the tail parameter implied by the beliefs and the model and show it lies in the reasonable range estimated in the empirical studies.

### 7.4.1 Model-implied Evolution of Wealth Shares

My model delivers equation 18 that specifies the evolution of wealth shares as a function of the cross-section of beliefs and the realized growth rates. Therefore, we can find the model-implied evolution of the wealth distribution by feeding the beliefs obtained form the survey into the equation. However, since the data does not have information about the respondents’ wealth, I use their income as the proxy.

Figure 6 plots the model-implied and the actual time-series of wealth ratio between the top 1% and bottom 90%. For the model-implied series, I use the average belief among the top 1% and the bottom 90% in the distribution of the respondents’ income. Furthermore, I use the estimated wealth ratio at 1978 as the initial value. The time-series of the wealth ratio is taken from Saez and Zucman (2016). Finally, I set $\kappa = 0.005$.

Note that Figure (6) exhibits a significant co-movement between the model-implied and the actual wealth ratio, despite the fact that the model only allows for two states and the belief measures are arguably noisy. This observation is particularly important because the earlier models fall short in explaining the fast dynamics of the wealth distribution (Gabaix, Lasry, Lions, and Moll (2016)), while the figure suggests that the belief heterogeneity might, at least partly, explain the dynamics. Note that the key distinction between my model

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29 A more complete description of the survey is provided in Section 2.
and the previous random growth models is that they ignore the role of aggregate shocks and hence, they mostly study the role idiosyncratic shocks in return or income in the wealth inequality. In contrast, there is no idiosyncratic shock in my model and I show in Proposition 6 that theoretically there is no limit on the speed of dynamics when the investors are asymmetrically informed. Therefore, the figure provides a suggestive evidence that the belief heterogeneity might have a first order effect on the dynamics of the wealth inequality.

Figure 6: Model implied vs. Actual wealth ratio between top 1% and bottom 90%, based on Michigan Survey Data for 1978-2012. The actual time-series of the wealth ratio is from Saez and Zucman (2016). To estimate the beliefs, the income proxy is used. The implied evolution is based on equation (16) for $\kappa = 0.005$.

7.4.2 Belief Heterogeneity and Implied Tail Parameter

Data suggests wealthier households tend to have more accurate beliefs about future macroeconomic conditions. Figure 2 and 3 provide supportive evidence by documenting a negative relationship between the expectation errors and two proxies of wealth levels: income level
and total investment in stock market. Furthermore, Das, Kuhnen, and Nagel (2017) find that households with higher income levels have macroeconomic expectations closer to the professional forecasts, while low-income households have a larger bias. To understand the impact of such belief heterogeneity and differences in belief accuracy on wealth inequality, I use Equation 25 to estimate the tail parameter implied by the belief data.

To use the equation, we need to find estimates for the correct probabilities $q_I^t$ (conditional probabilities given the available information), the wealthiest average belief $q_{t^{Top5}}$ and the representative belief $\bar{q}_t$. I assume the most accurate belief is held by the top 5%, which gives a lower bound on the extent of wealth inequality. Note that I am not assuming the wealthiest use all available information for their predictions, thus I allow for $q_I^t \neq q_{t^{Top5}}$. I construct the belief of the top 5% similar to the previous part. The representative belief is constructed by the income-weighted average of beliefs in Michigan Survey. Since the wealth inequality is larger than the income inequality in data, I weight the beliefs with incomes to the power of 1.5 ($INCOME^{1.5}$). Following Das, Kuhnen, and Nagel (2017), I construct the the true probabilities based on the Survey of Professional Forecasters. More specifically, I assume the probability of having a high growth (exceeding the historical mean) is equal to the fraction of the forecasters that predict a one-year-ahead growth rate above its historical mean.

Figure 7 exhibits the relationship between the professional forecasters’ belief with the belief of the top 5% and the representative belief, for years between 1978-2019 (overall 165 quarters). We see that the beliefs of the top 5% are more sensitive to the professional forecasts, compared to the weighted average beliefs. This is another indication of the fact that the wealthier investors, who are more likely to be among the top earners, have relatively more accurate beliefs.
Figure 7: X-axis: Professional forecasters’ belief (1978q1-2019q1). It is the fraction of the forecasters that predicted the one-year-ahead growth rate would be more than the historical mean. Y-axis: The average belief of the top 5% in the income distribution (circles) and the income-weighted average of all beliefs (crosses). The lines represent the local regression lines.

Now, we are ready to estimate the tail parameter. The following equation provides an estimate for the tail parameter, given the constructed beliefs.

\[
[\Pi_T^{T}(q_t^{\text{Top5}}) (\frac{q_t^{\text{Top5}}}{q_t})^\gamma + (1 - q_t^{\text{Top5}})(\frac{1 - q_t^{\text{Top5}}}{1 - q_t})^\gamma]]^{\frac{1}{\gamma}} (\delta + (1 - \delta)(1 - \tau)^\gamma) = 1
\]  

(26)

In (26), \(T = 165\) is the number of periods in the dataset (1978q1-2019q1), \(\delta = 0.96\) is calibrated to a 30-year expected investment period and \(\tau = 0.4\) is used as is the federal marginal estate tax (above $1 million) in 2019. Based on these numbers, the model-implied tail parameter is \(\hat{\gamma} = 1.31\), which is smaller than the empirical estimations of 1.48-1.55 (Vermeulen (2018); Klass, Biham, Levy, Malcai, and Solomon (2007)). It means the model predicts a slightly higher inequality in the tail, compared to data.
The estimation is arguably subject to different estimation errors and the choice of the parameter values. However, the key take-away from this exercise is that the extent of belief heterogeneity observed in data can generate the level of skewness observed in data under a reasonable set of parameters. In other words, this observation is a first evidence that the belief heterogeneity and asymmetric information are likely to be among the main contributors to the current level of top inequality. Therefore, it calls for policies improving the financial knowledge of the public and reducing the information inequality around the existing investment opportunities.

8 Conclusion

I study a general equilibrium model with a continuum of long-lived and heterogeneously informed investors, who trade risky and risk-less claims of a Lucas tree in a financial market. I provide the equations fully specifying the joint evolution of the beliefs and wealth distribution. By employing those equations, I show that the expected distribution of wealth shares features a thick right-tail. As a key distinguishing feature of the informational channel, I show that the tail is populated by the best-informed investors, best representing fund managers, who are known by their sophistication and advantage in access to information.

My work has important policy implications in the areas of consumer finance and wealth management. My results put forward two ways for reducing the inequality via improving household’s saving decisions. First, households should be assisted to make more informed investment decisions by providing comprehensible information about the risk and return prospects of different investment opportunities. Doing so increases the households’ confidence in entering riskier asset classes and could alleviates wealth inequality. Secondly, my
results indicate the importance of investment delegations in reducing the inequality. That said, pension managers, who manage a large fraction of households’ wealth (36% according to Smith, Zidar, and Zwick (2019)), play a key role in reducing the inequality. Therefore, the authorities should ensure pension managers exert the necessary due diligence in their investment decisions and their incentives are best aligned to that of households in wealth accumulation.

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Appendix A: Proofs of Lemmas and Propositions

A1. Proof of Lemma 1

(a,b) As mentioned earlier, the optimal strategy is scale-invariant, that is investors with the same type and different wealth levels consume and save in the same proportion. Since the investors have log-utility, the value function should be logarithmic in wealth level. Therefore, it is left to prove the coefficient in front of \( \log W_t \) in the value function is \( \frac{1}{1 - \beta \delta} \).

I do so by guess and verification. Let \( R_t^k \) be the random return on the optimal portfolio. Therefore, the optimality of the consumption implies:

\[
V_t^k(W_t; F_t) = \max_{c_t} \log c_t + \beta \delta \mathbb{E}_t^k \left[ \frac{1}{1 - \beta \delta} \log((W_t - c_t) R_t^k) + a^k(F_{t+1}) \right]
\]

\[
= \max_{c_t} \log c_t + \frac{\beta \delta}{1 - \beta \delta} \log(W_t - c_t) + \beta \delta \mathbb{E}_t^k \left[ \frac{1}{1 - \beta \delta} \log R_t^k + a^k(F_{t+1}) \right]
\]

Note that the last term in (27) is independent of \( W_t \). Therefore, the first order condition for \( c_t \) implies:

\[
\frac{1}{c_t} = \frac{\beta \delta}{1 - \beta \delta} \frac{1}{W_t - c_t} \Rightarrow c_t = (1 - \beta \delta)W_t
\]

By substituting \( c_t = (1 - \beta \delta)W_t \) in (27), we can easily verify the claim.

c) The market clearing condition for the good market implies:

\[
d_t = c_t^N + \int_0^1 c_i^N \, di = (1 - \beta \delta)(W_t^N + \int_0^1 W_i^N \, di)
\]

\[
= (1 - \beta \delta)((p_t + d_t)(x_i^N + \int_0^1 x_i^N \, di) + s_i^N + \int_0^1 s_i^N \, di) = (1 - \beta \delta)(p_t + d_t)
\]
After a few rearrangements, we get:

\[ p_t = \frac{\beta \delta}{1 - \beta \delta} d_t \]

### A2. Proof of Lemma 2

According to Lemma 1, the investors maximize the expected log-wealth in period \( t + 1 \).\(^{30}\)

We can rewrite the optimization problem as follows:

\[
\max_{\mu} \mathbb{E}_t^k \left[ \log \left( \mu \frac{d_{t+1} + p_{t+1}}{p_t} + (1 - \mu) R_t^F \right) \right] \tag{28}
\]

According to Lemma 1(c), we can rewrite the optimization problem (28) as follows:

\[
\max_{\mu} q_t^k \left\{ \log e^{g^h} (\mu + (1 - \mu) \beta \delta e^{-g^h} R_t^F) \right\} + (1 - q_t^k) \left\{ \log e^{g^l} (\mu + (1 - \mu) \beta \delta e^{-g^l} R_t^F) \right\} \tag{29}
\]

Due to the strict concavity of the objective function, we only need to solve for \( \mu \) that satisfies the first condition. One can verify that is the case for \( \mu_t^k \) specified in (9).

### A3. Proof of Lemma 3

Clearly \( q_t^I \ w_t^I + q_t^N \ w_t^N \) lies in the uninformed investors’ information set. Since \( q(g_t) = q_t^I - z_t^I = q_t^N - z_t^N \) and \( w_t^I + w_t^N = 1 - w_t^U \), \( z_t^U \equiv w_t^I z_t^I + w_t^N z_t^N \) should also be in the in the uninformed investors’ information set, e.g. it is inferable from the equilibrium prices.

\(^{30}\)To see this, note that the investors solve:

\[
\max_{x_{t+1}, y_{t+1}} \log(1 - \beta \delta) W_t^k + \mathbb{E}_t^k \left[ \frac{\beta \delta}{1 - \beta \delta} \log W_{t+1}^k + \beta \delta a^h(\mathcal{F}_t) \right] \nonumber
\]

which is equivalent to maximizing \( \mathbb{E}_t^k \left[ \log W_{t+1}^k \right] \).
Similarly, both \( q^I_t w^I_t + q^N_t w^N_t \) and \((1 - q^I_t)w^I_t + (1 - q^N_t)w^N_t \) are inferable from \( z^U_t \). Finally, \( R^F_t \) is the unique solution to (11). \( p_t \) is measurable with respect to \( F^P_t \). Therefore, both equilibrium prices are measurable with respect to \( F^P_t \cup \{ z^U_t \} \).

**A4. Proof of Lemma 4**

Note that:

\[
q^U_t = \mathbb{E} \left[ \mathbb{I}_{\{g_{t+1}=g^h\}} \left| F^P_t, z^U_t = w^I_t z^I_t + w^N_t z^N_t \right. \right] = \mathbb{E} \left[ \mathbb{E} \left[ \mathbb{I}_{\{g_{t+1}=g^h\}} \left| z^I_t \right. \right. \right] \left| F^P_t, z^U_t \right] = \mathbb{E} \left[ z^I_t \left| F^P_t, z^U_t \right. \right]
\]

\[
\int_{-\bar{z}_t}^{\bar{z}_t} \mathbb{E} \left[ \mathbb{I}_{\{g_{t+1}=g^h\}} \left| \frac{1}{w^I_t}(z^U_t - w^I_t z) \in [-z^N_t, \bar{z}^N] \right. \right] \frac{1}{w^I_t(z^U_t - w^I_t z)} dz = \frac{1}{2} \left\{ \max \left\{ -\bar{z}^I_t, z^I_t + \frac{w^N_t}{w^I_t}(z^N_t - \bar{z}^N_t) \right\}, \min \left\{ z^I_t, z^I_t + \frac{w^N_t}{w^I_t}(z^N_t + \bar{z}^N_t) \right\} \right\}
\]

I apply the law of iterative expectations in the first line and the Bayes’ rule in the second line of the equations above.

**A5. Proof of Lemma 5**

The evolution of wealth for type-\( k \) investors with exposure to the risky asset \( \mu^k_t \) is given by:

\[
W^k_{t+1} = \beta \delta W^k_t \left( \mu^k_t \frac{d_{t+1}}{p_t} + \frac{p_{t+1}}{p_t} \right) + (1 - \mu^k_t) R^F_t
\]

Therefore, by applying Lemma 1(c), the evolution of the wealth share is:

\[
w^k_{t+1} = w^k_t \left( \mu^k_t + (1 - \mu^k_t) \beta \delta R^F_t e^{-g_{t+1}} \right)
\]
Now, by some rearranging and substituting (9) for $\mu^k_t$, we have:

$$w^k_{t+1} = w^k_t + (\mu^k_t - 1)(1 - \beta \delta R^F_t e^{-g_{t+1}})w^k_t$$

$$= w_t[1 + \left(\frac{q^k_t}{\beta \delta R^F_t e^{-g^h - 1} + \kappa} + \frac{1 - q^k_t}{\beta \delta R^F_t e^{-g^l} - 1}\right)(1 - \beta \delta R^F_t e^{-g_{t+1}})]$$

(30)

When $g_{t+1} = g^h$, we have:

$$\frac{w^k_{t+1}}{w^k_t} = 1 - q^k_t \beta \delta R^F_t (e^{-g^h} - 1) - 1 + q^k_t = q^k_t \frac{\beta \delta R^F_t (e^{-g^h} - e^{-g^l})}{\beta \delta R^F_t e^{-g^l} - 1}$$

(31)

Note that Equation 31 holds for any agent, including the representative investor with belief $\bar{q}_t \equiv w^N_t q^N_t + w^I_t q^I_t + w^U_t q^U_t$. Furthermore, the wealth share of this representative agent is one. Equations 11 and 9 imply that such representative agent always chooses $\mu_t = 1$ and Equation 30 implies that the wealth share of this representative agent is constant at one. Therefore, if we plug the belief and the wealth of the representative agent into (31), we get:

$$\frac{\beta \delta R^F_t (e^{-g^h} - e^{-g^l})}{\beta \delta R^F_t e^{-g^l} - 1} = 1$$

Therefore, we can rewrite (31) as follows:

$$\frac{w^k_{t+1}}{w^k_t} = \frac{q^k_t}{\bar{q}_t}$$

For $g_{t+1} = g^l$, we can similarly show:

$$\frac{w^k_{t+1}}{\kappa + (1 - \kappa)w^k_t} = \frac{1 - q^k_t}{1 - \bar{q}_t}$$

The specification in the Lemma directly follows from the equations above.
A6. Proof of Proposition 4

Suppose $Ew(\lambda)^I$ and $Ex(\lambda)^U$ are expected wealth shares of the informed and informed investors in an economy with fraction $\lambda$ uninformed. The expected wealth share of the fund managers is:

$$b^I Ew(b^U)^I + (1 - b - b^I - b^U) \psi(Ew(b^U)^I - Ew(b^U)^U)$$

For $b^U = 0$ and sufficiently small $b$, we know $Ew(b^U)^I - Ew(b^U)^U$ can be arbitrarily small. Moreover, $Ew(b^U)^I$ is increasing in $b^U$. Also, for sufficiently small $b^I$, it is easy to check that the expression is decreasing in $b^U$ at $b^U = 1 - b - b^I$.

A7. Proof of Proposition 5

First, I revise the wealth dynamics provided by Lemma 5 for the modified setup in Lemma 8. The remaining steps are similar to the proof of Proposition 1.

Lemma 8.
The wealth share of investor from family $f \in [0, 1]$ and with type $k \in [0, 1] \cup \{N\}$, if alive at both $t$ and $t + 1$, in the modified setup (in Section 7.2), is given by:

$$\log w^k_{f,t+1} - \log w^k_{f,t} = \frac{P_k^k(g_{t+1}, \varepsilon_{t+1})}{P(g_{t+1}, \varepsilon_{t+1})}$$

(32)

The proof strategy is similar to that of Lemma 5.\textsuperscript{31}

\textsuperscript{31}Similar to Blume and Easley (1992), one can show Equation 32 holds if the number of payoff relevant states does not exceed the number of independent assets when economic agents have logarithmic preferences.
A8. Proof of Proposition 6

Note that:

\[
\mathbb{E}[\log \frac{w^{i_1}_{f_1,t+1}}{w^{i_2}_{f_2,t+1}}] = \mathbb{E}[\log \frac{w^{i_1}_{f_1,t}}{w^{i_2}_{f_2,t}} + q^I_t \log \frac{q^I_t}{q^U_t} + (1 - q^I_t) \log \frac{1 - q^I_t}{1 - q^U_t}]
\]

\[
= \log \frac{w^{i_1}_{f_1,t}}{w^{i_2}_{f_2,t}} + \mathbb{E}[q^I_t \log \frac{q^U_t}{q^I_t} + (1 - q^I_t) \log \frac{1 - q^U_t}{1 - q^I_t} | \lambda_t \in (i_1, i_2)](F(i_2) - F(i_1))
\]

From Lemma 4, we can see \(q^U_t | q^I_t, w^{i_1}_{f_1,t}, w^{i_2}_{f_2,t} \) is uniformly distributed in a subset of \([q(g_t) - \bar{z}^I, q(g_t) + \bar{z}^I] \) \((g_t = g^I)\) and, depending on the realizations, it could have a mass point at \(q(g_t)\). To simplify the expositions, suppose the distribution does not have a mass point.\(^{32}\)

Therefore, let \(q = q^U_t | q^I_t, w^{i_1}_{f_1,t}, w^{i_2}_{f_2,t} \sim U[q, \bar{q}]\). In the next step, I find an upper bound for the expression below:

\[
\mathcal{H}(q^*; q, \bar{q}) \equiv \mathbb{E}[q^* \log \frac{q}{q^*} + (1 - q^*) \log \frac{1 - \bar{q}}{1 - q^*} | \bar{q} \sim U[q, \bar{q}]]
\]

By taking the expectation, we get:

\[
\mathcal{H}(q^*; q, \bar{q}) = \frac{1}{\bar{q} - q} [q^* (\bar{q} \log \bar{q} - q \log q - (\bar{q} - q)) + (1 - q^*) (1 - q \log (1 - q) - (1 - \bar{q}) \log (1 - \bar{q}) - (\bar{q} - q))] - q^* \log q^* - (1 - q^*) \log (1 - q^*)
\]

Now, let us use the Taylor series to approximate \(\mathcal{H}(q^*; q, \bar{q})\) around \(a^*\) up to the third order.

After some simplifications, we get:

\(^{32}\)The proof steps are exactly the same for the case with a mass point in the distribution
\[ H(q^*; q, \bar{q}) = -\frac{1}{6}(q^* + \frac{1}{1-q^*})((\bar{q}-q^*)^2 + (q^* - \bar{q})^2 + (\bar{q}-q^*)(q^* - \bar{q})) + o((\max\{\bar{q}-q^*, q^* - \bar{q}\})^3) \] 

(36)

According to Lemma 4, the minimum possible value that the bracket above takes is \((\frac{w^N_t}{w^I_t} z^N)^2\). Furthermore, note that according to Corollary 1, \(w^N_t \geq \kappa b\) for all \(t \geq 1\). Therefore, \(\frac{w^N_t}{w^I_t} \geq \frac{w^N_t}{1-w^N_t} > \frac{\kappa b}{1-\kappa b}\). The proof is completed by substituting these expressions in (33).

A9. Proof of Lemma 6

I prove the lemma in two steps.\(^{33}\) First, I prove the lemma for the case that there is only a finite number of types among the sophisticated investors. Then, using this result, I construct a sequence of approximating distributions of wealth shares for every investor by restricting \(\lambda_t\) to take value from \(\frac{j}{M}\), where \(M\) is a positive integer, and taking \(M\) to infinity. This first step yields a set of approximating distributions of wealth shares for each type of investors \(k \in [0, 1) \cup \{N\}\). Then, in the second step, I show the approximating distributions have a limit, which yields the stationary distributions.

**Proof for the case with finite sophisticated type:** Suppose \(\lambda_t\) only takes value from finite set \(\{0, \frac{1}{M}, \ldots, \frac{M-1}{M}\}\), for some positive integer \(M\). Furthermore, suppose the probability the threshold type being \(\frac{j}{M}\), \(0 \leq j \leq M-1\), is \(F(\frac{j+1}{M}) - F(\frac{j}{M})\). Therefore, the probability that investor \(i \in [0, 1]\) becomes informed is \(1 - F(\frac{\lfloor M \rfloor}{M})\). I will take \(M\) to infinity in the next step.

In this case the state variable is finite-dimensional and the transition kernel can be derived

\(^{33}\)Note that \(\Lambda\) is not totally ordered, therefore we cannot directly apply the existence and uniqueness theorem of Hopenhayn and Prescott (1992).
similar to (18). In particular, in this case, the state of state variables is as follows:

\[
\Omega^M_i = \{\{\tilde{w}_j^i\}_{1 \leq j \leq M}, \tilde{w}_N^i(M), g_t\} \in \Lambda^M = [0, 1]^{M+1} \times \{g^l, g^h\}
\]

\[
\tilde{w}_j^i = \int_{\frac{j-1}{M}}^{\frac{j}{M}} w_i \, di
\]

where \(\tilde{w}_j^i\) is the total wealth share in the hand of group \(j\) and \(\tilde{w}_N^i(M)\) is the wealth share of the naive investors under this information structure. Denote the corresponding transition kernel by \(Q^M(\cdot, \cdot)\). Note that \(Q^M(\cdot, \cdot)\) is defined on the distributions over compact set \(\Lambda^M\). Due to continuity of the mapping with respect to the distribution, the process also satisfies the Feller property. Therefore, it has a stationary distribution.

For the next step, denote the implying the stationary distribution for the wealth share of the investors with type \(k \in [0, 1] \cup \{N\}\) by \(\zeta^M_i\).

**Existence of stationary distribution for every** \(k \in [0, 1] \cup \{N\}\). The goal in this step is to show the sequence \(\zeta^M_i\) is converging to some distribution \(\zeta^*_i\). To show this, we first note that the expectation of all of these distributions is bounded by \(\frac{1}{1-i}\); because the expected wealth share for type-\(i\) investors, \(\int w d\zeta^M_i(w)\), is weakly increasing in \(i\), as the investors with a higher \(i\) receive a more informative signal in Blackwell sense. The uniformly boundedness of the expectations implies the sequence is tight. Therefore, according to Prohorov’s theorem, a subsequence of this collection of distributions, \(\{\zeta^M_i\}_{M=1}^\infty\), should converge to some distribution \(\zeta^*_i\). Therefore, \(\zeta^*_i\) is a stationary distribution for the converging sequence of type-\(i\) wealth shares, which is \(w_i^I\). It completes the proof.

Note that this proof can be applied to the distribution of wealth shares for any subset of investors or the joint distributions. As a result, the joint distribution of the wealth share of the informed investors \(w_i^I\) and naive investors also follows a stationary distribution. It
is enough to show the beliefs also have a stationary distribution.

A10. Proof of Proposition 1

Lemma 9.

For any \( i \in [0, 1] \), we have
\[
E[q_i^t \log(\frac{q_i}{q_t}) + (1 - q_i^t) \log(\frac{1 - q_i}{1 - q_t})] \leq -\log(1 - \kappa).^{34}
\]

Proof. To see the argument, note that Corollary 1 implies:
\[
\log w_{i,t+1} - \log w_{i,t} > (\log(\frac{q_i}{q_t}))\mathbb{1}_{(g_{t+1} = g^h)} + (\log(\frac{1 - q_i}{1 - q_t}))\mathbb{1}_{(g_{t+1} = g^l)} + \log(1 - \kappa) \quad \forall \ i \in [0, 1]
\]

Therefore, if the inequality in Lemma 9 does not hold for some \( i \in (0, 1) \), then the wealth share of the investors with type above \( i \) would boundlessly increase and hence, it would exceed one, which is in contradiction with \( \int_0^1 w_{i,t} di \) being bounded.

The wealth share of family \((f, i)\) follows the following Markov process:
\[
w_{f,t+1}^i = \begin{cases} w_{f,t}^i \frac{q_i}{q_t} \quad \text{w.p.} \ \delta \\ b_i^k (1 - \tau) w_{f,t}^i + \tau \quad \text{w.p.} \ 1 - \delta \end{cases}
\]

where
\[
b_i^k = \begin{cases} \frac{q_i}{q_t} & g_{t+1} = g^h \\ \frac{1 - q_i}{1 - q_t} & g_{t+1} = g^l \end{cases}
\]

Lemma 6 implies that we can write the process in the form of \( w_{f,t+1}^i = B_i^t w_{f,t}^i + A_i^t \), where \((B_i^t, A_i^t)\) follows a Markov process. In Corollary 9, we see that \( E[\log B_i^t] \leq 0 \). Therefore

\[34 \kappa \equiv (1 - \delta) \tau \]
it is a Kesten process \( \text{Kesten} (1973) \) and its stationary distribution has a thick right tail if \( P(B_t > 1^i) > 0 \) (Roitershtein et al. (2007)), with the tail parameter \( \gamma(i) \) that satisfies:

\[
\lim_{T \to \infty} \left( E[\Pi_{t=1}^T B_t^{\gamma(i)}] \right)^{\frac{1}{T}} = \lim_{T \to \infty} E[\Pi_{t=1}^T (b_t^{\gamma(i)})]^{\frac{1}{T}} (\delta + (1 - \delta)(1 - \tau)^{\gamma(i)} = 1
\]

\[
\Rightarrow \lim_{T \to \infty} E[q_t^i \left( \frac{q_t^i}{q_t} \right)^{\gamma(i)} + (1 - q_t^i)\left( \frac{1 - q_t^i}{1 - q_t} \right)^{\gamma(i)}]^{\frac{1}{T}} (\delta + (1 - \delta)(1 - \tau)^{\gamma(i)} = 1
\]

, where in (39), I used the fact that \( (\tilde{\lambda}_t, z_t^1, z_t^N) \) are drawn independently across the periods and \( q_t^h = 1 - q_t^i \). For \( \gamma(i) \) that satisfies (39) we have \( P(w_{j,t} > w) > Q(i)w^{-\gamma(i)} \), for some \( Q(i) > 0 \) and sufficiently large \( w \).

Note that \( \gamma(i) \) is decreasing in \( i \), because for any \( \gamma > 1 \) and \( i_2 > i_1 \in [0, 1] \), we have:

\[
E[q_t^i \left( \frac{q_t^i}{q_t} \right)^{\gamma} + (1 - q_t^i)\left( \frac{1 - q_t^i}{1 - q_t} \right)^{\gamma}] > E[q_t^i \left( \frac{q_t^i}{q_t} \right)^{\gamma} + (1 - q_t^i)\left( \frac{1 - q_t^i}{1 - q_t} \right)^{\gamma}]
\]

\[
\Rightarrow E[(b_t^i)^{\gamma}] = F(i_2)E[q_t^i \left( \frac{q_t^i}{q_t} \right)^{\gamma} + (1 - q_t^i)\left( \frac{1 - q_t^i}{1 - q_t} \right)^{\gamma}] + (1 - F(i_2))E[q_t^i \left( \frac{q_t^i}{q_t} \right)^{\gamma} + (1 - q_t^i)\left( \frac{1 - q_t^i}{1 - q_t} \right)^{\gamma}]
\]

\[
> F(i_1)E[q_t^i \left( \frac{q_t^i}{q_t} \right)^{\gamma} + (1 - q_t^i)\left( \frac{1 - q_t^i}{1 - q_t} \right)^{\gamma}] + (1 - F(i_1))E[q_t^i \left( \frac{q_t^i}{q_t} \right)^{\gamma} + (1 - q_t^i)\left( \frac{1 - q_t^i}{1 - q_t} \right)^{\gamma}]
\]

, where the first inequality is achieved from the fact that \( q_t^i \left( \frac{q_t^i}{q_t} \right)^{\gamma} + (1 - q_t^i)\left( \frac{1 - q_t^i}{1 - q_t} \right)^{\gamma} \) is a convex function of \( \gamma \) and the distribution of \( q_t^i \) is a mean-preserving spread of that of \( q_t^U \).

Therefore:

\[
1 - G^i(w) = \int_0^1 P(w_{j,t} \geq w)di > \int_0^1 Q(i)w^{-\gamma(i)}di \geq \bar{Q}w^{-\gamma(1)} \quad \exists \bar{Q} > 0
\]

where the last inequality is obtained from the fact that \( \gamma(1) \leq \gamma(i) \), for \( i \in [0, 1] \), and Equation 5 in Gabaix (2009). Therefore, \( 1 - G(w) \sim w^{-\gamma(1)} \), which completes the proof.
Part b. By using the derivations in the previous part and applying Bayes’ rule on the
unconditional distributions, we get:

\[ P(\tilde{i} < i | w_{f,t}^i > w) = \frac{\int_0^i P(w_{f,t}^i > w)di'}{\int_0^1 P(w_{f,t}^i > w)di'} < \frac{L_1w^{-\gamma(i)}}{L_2w^{-\gamma(1)}} \] (40)

,where \( L_1 \) and \( L_2 \) are some positive real numbers. Since \( \gamma(1) < \gamma(i) \), the statement can be
proven by taking \( w \) in (40) to infinity.

A11. Proof of Proposition 2

Proof of Part (a)

Note that according to Lemma 1(b), 
\[ R_t^i = \frac{W_{i+1}}{\beta W_t} = \beta^{-1}e^{\gamma t+1}b_t^i \]
where \( b_t^i \) is defined in (38).
Therefore, we only need to show \( \mathbb{E}[\log b_t^i] \) is increasing and convex in \( i \). We can see that
the derivative is the following:

\[ \frac{d}{di} \mathbb{E}[\log b_t^i] = \mathbb{E}[q_t^I \log \frac{q_t^I}{q_t^U} + (1 - q_t^I) \log \frac{1 - q_t^I}{1 - q_t^U} | \lambda_t = i] \] (41)

The function inside the expectation in (41) is convex in \( q_t^U \) and the minimum is achieved
at \( q_t^I \). Therefore, it is strictly positive.

Now, I show it is also increasing in \( i \). Since the dissemination of information decreases as \( \lambda_t \)
increases, \( \mathbb{E}[|q_t^I - q_t^U| | \Omega_t, \lambda_t] \) also increases in \( \lambda_t \). Therefore, for any \( \Omega_t \in \Lambda, \mathbb{E}[q_t^I \log \frac{q_t^I}{q_t^U} + (1 - q_t^I) \log \frac{1 - q_t^I}{1 - q_t^U} | \Omega_t, \lambda_t = i] \) is increasing in \( i \). By appealing to the law of iterative expectations
with respect to \( \Omega_t \), we see the derivative in (41) is increasing in \( i \).

Proof of Part (b)

Note that the exponential function is convex and \( \gamma > 1 \). Therefore, applying the Jensen
inequality to (21), we get:

\[
exp(\gamma E[r^I_t - g_t + \log \beta])(\delta + (1 - \delta)(1 - \tau)\gamma) < 1
\]

Therefore, by taking log from both sides and noting \(\gamma \geq 1\):

\[
\gamma E[r^I_t - g_t + \log \beta] + \gamma \log(1 - \delta) + \gamma \log(1 - \tau) < E[r^I_t - g_t + \log \beta] + \log(\delta + (1 - \delta)(1 - \tau)\gamma) < 0
\]

Since \(\gamma > 0\), it completes the proof.

**A12. Proof of Proposition 3**

Similar to the baseline case, the tail is governed by the informed investors. Denote the stationary distribution for a given \(\chi\) by \(G_\chi(\cdot)\). Consider the contrary and suppose the tail parameter converges to some \(\gamma^* > 1\). Therefore, for sufficiently small values of \(\varepsilon > 0\), there exists positive number \(A(\varepsilon)\) such that:

\[
1 - G_\chi(w) < A(\varepsilon)w^{-(\gamma^* - \varepsilon)} \quad (42)
\]

Inequality 42 implies that the fraction of investors affording to pay the information cost \(\chi\) goes to zero as \(\chi\) goes to infinity. That said, the wealth share of the informed investors share should converge to zero. Therefore, the representative belief converges to \(q(g_t)\), almost surely. Due to the convergence, according to condition (23), there exists a sufficiently large \(\chi\) and positive number \(A\) such that

\[
\mathbb{E}_t[q^I_t \log \frac{q^I_t}{q_t} + (1 - q^I_t) \log \frac{1 - q^I_t}{1 - q_t}] + (1 - \delta) \log(1 - \tau) > A > 0
\]

A-12
Note that we have the following inequality for the wealth share of an informed investor, like $f$:

$$\mathbb{E}[\log w_{f,t+1}] - \log w_{f,t} > \delta \mathbb{E}_t[\log \frac{q^I_t(g_{t+1})}{q_t(g_{t+1})}] + (1 - \delta) \mathbb{E}_t[\log \frac{q^I_t(g_{t+1})}{q_t(g_{t+1})}] + (1 - \delta) \log(1 - \tau)$$

$$= \mathbb{E}_t[q^I_t \log \frac{q^I_t}{q_t} + (1 - q^I_t) \log \frac{1 - q^I_t}{1 - q_t} + (1 - \delta) \log(1 - \tau) > A$$

It means for such $\chi$, the wealth share of the informed investors boundlessly increases, which is a contradiction.

**A13. Proof of Proposition 8**

For the proof, we only need to incorporate the bequest motive into the baseline value function, specified in (27). Similar to the proof of Proposition 1, we can guess and verify the value function is logarithmic in wealth and is in the following form:

$$V^k_\phi(W_t; \mathcal{F}_t) = \frac{1 + \beta(1 - \delta)^\phi}{1 - \beta\delta} \log W_t + a^k_\phi(\mathcal{F}_t)$$

(43)

The optimal consumption can be similarly solved and verified that $c_t = \frac{1 - \beta\delta}{1 + \beta\delta(1 - \delta)}$. Therefore, similar to the baseline case, the investors optimally consume a fixed fraction of their wealth in every period. Moreover, due to the logarithmic form of the value function, the investors optimally maximize the expected log-return of their investment, similar to the baseline case. Therefore, a change in $\phi$ does not affect the composition of portfolios, and as a result, does not impact the information dissemination through the prices. Therefore, by following the procedure carried out in the baseline case, we can find the wealth and
belief dynamics and prove part (b).

**A14. Proof of Proposition 9**

Note that the tail parameter is the solution to the following equation:

$$\lim_{T \to \infty} \mathbb{E}[(\frac{q_{f,t}(g_{t+1})}{\bar{q}(g_{t+1})})^\gamma]^\frac{1}{\gamma} (\delta + (1 - \delta)(1 - \tau)) = 1$$

(44)

where \(q_{f,t}\) is the probability that the investor from family \(f\) assigned on the realized growth \(g_{t+1}\) at time \(t\).\(^{35}\) A higher persistence in types means that consecutive terms in sequence \(\{q_{f,t}\}_{t=1}^{\infty}\) have a higher correlation. Therefore, one can show that the expectation of the consecutive terms in the sequence, \(\mathbb{E}(\prod_{j=0}^{T} q_{f,t+j}(g_{t+j+1}))\), is increasing in \(\rho\) by applying law of iterated expectation and noting the wealth share of the informed investors increases with \(\rho\).

A higher \(\rho\) implies that the informed investors can save more and, eventually, claim a larger fraction of the wealth, which implies the representative belief becomes more accurate. It leads to an increase in \(\mathbb{E}(\prod_{j=0}^{T} q_{f,t+j}(g_{t+j+1}))\) for every \(T\). However, if the increase is so large that \(\mathbb{E}(\prod_{j=0}^{T} q_{f,t+j}(g_{t+j+1}))\) decreases, it means the expected wealth share of the sophisticated investors should decrease in \(\rho\), or equivalently, the wealth share of the naive investors should increase with \(\rho\), which is not possible.

**A15. Proof of Lemma 7**

The expected utility of a newborn from choosing to be informed at time \(t\) is:

\(^{35}\)Note that, in this case, the investors are homogeneous, ex-ante.
\[ V_t^I(w_{f,t}) = \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \log((1 - \beta \delta) \frac{d_{t+j}}{1 - \beta \delta} (w_{f,t} - \chi + h_t \chi) \prod_{j'=0}^{j-1} \frac{q_{t+j'}^{I+1}(g_{t+j' + 1})}{q_{t+j'}(g_{t+j' + 1})}) \]

\[ = \sum_{j=0}^{\infty} (\beta \delta)^j \mathbb{E}_t [\log d_{t+j}] + \frac{1}{1 - \beta \delta} \log (w_{f,t} - \chi + h_t \chi) \]

\[ + \sum_{m=1}^{\infty} \frac{(\beta \delta)^m}{1 - \beta \delta} \mathbb{E}_t[q_{t+m-1}^I \log \frac{q_{t+m-1}^I}{q_{t+m-1}} + (1 - q_{t+m-1}^I) \log \frac{1 - q_{t+m-1}^I}{1 - q_{t+m-1}} | \Omega_t] \]

(45)

,where \( q_{t+j'}^{I+1}(g_{t+j' + 1}) \) denotes the probability that the informed investors assigned at \( t + j' \) to the realized growth rate at \( t + j' + 1 \). \( \bar{q}_{t+j'}(g_{t+j' + 1}) \) is similarly defined. Similarly, the expected payoff from remaining uninformed is:

\[ V_t^U(w_{f,t}) = \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \log((1 - \beta \delta) \frac{d_{t+j}}{1 - \beta \delta} (w_{f,t} + h_t \chi) \prod_{j'=0}^{j-1} \frac{q_{t+j'}^{U+1}(g_{t+j' + 1})}{q_{t+j'}(g_{t+j' + 1})}) \]

\[ = \sum_{j=0}^{\infty} (\beta \delta)^j \mathbb{E}_t [\log d_{t+j}] + \frac{1}{1 - \beta \delta} \log (w_{f,t} + h_t \chi) \]

\[ + \sum_{m=1}^{\infty} \frac{(\beta \delta)^m}{1 - \beta \delta} \mathbb{E}_t[q_{t+m-1}^U \log \frac{q_{t+m-1}^U}{q_{t+m-1}} + (1 - q_{t+m-1}^U) \log \frac{1 - q_{t+m-1}^U}{1 - q_{t+m-1}} | \Omega_t] \]

(46)

Inequality (22) can be verified by comparing the equations above.

### A16. Proof of Proposition 7

To prove the proposition, I show for any sequence of growth rate realizations \( G = (g_{t+1}^*, g_{t+2}^*, \ldots, g_{t+N}^*) \), all investors assign the same price to the security that pays \( \varepsilon \) at begging of \( t + N \) following \( G \), under the allocation provided for the baseline case. In particular, we need to
show:

\[
(\beta \delta)^N E^k \left[ \frac{u'(c(G))}{c_t} \mathbb{I}_{\{(g_{t+1}, g_{t+2}, \ldots, g_{t+N}) = G\}} \right] = (\beta \delta)^N E^{k'} \left[ \frac{u'(c(G))}{c_t} \mathbb{I}_{\{(g_{t+1}, g_{t+2}, \ldots, g_{t+N}) = G\}} \right] \quad \forall k, k' \in [0, 1] \cup N
\]

(47)

where \(c(g_{t+1}, g_{t+2}, \ldots, g_{t+K})\) denotes the consumption under the baseline allocation after the realization \((g_{t+1}, g_{t+2}, \ldots, g_{t+K})\) and \(u(\cdot) \equiv \log(\cdot)\). In the baseline, the allocations between any two consecutive periods are optimal, given the information available at the time of consumption and portfolio decision. Therefore, for any \(1 \leq j \leq N - 1\) and any type of investor \(k \in [0, 1] \cup \{N\}\), we have:

\[
\beta \delta \frac{u'(c(\ldots, g_{t+j}^*, g_{t+j+1}^*))}{u'(c(\ldots, g_{t+j}^*))} P_{t+j}^k (\tilde{g}_{t+j+1} = g_{t+j+1}^*) = 1
\]

(48)

Now, by multiplying all of these Euler equations, we get:

\[
(\beta \delta)^N \frac{u'(c(G))}{u'(c(g_{t+1}^*))} \prod_{j=1}^{N-1} P_{t+j}^k (\tilde{g}_{t+j+1} = g_{t+j+1}^*) = 1
\]

(49)

After taking time \(t\) expectation, we have:

\[
(\beta \delta)^N \frac{u'(c(G))}{u'(c(g_{t+1}^*))} \prod_{j=1}^{N-1} P (\tilde{g}_{t+j+1} = g_{t+j+1}^* | g_{t+j}^*) = 1
\]

(50)

By multiplying the last equation with the Euler equation at time \(t\), we have:

\[
(\beta \delta)^N \frac{u'(c(G))}{u'(c_t)} P_t^k (\tilde{g}_{t+1} = g_{t+1}^*) \prod_{j=1}^{N-1} P (\tilde{g}_{t+j+1} = g_{t+j+1}^* | g_{t+j}^*) = (\beta \delta)^N \frac{u'(c(G))}{u'(c_t)} P_t^k (G) = 1
\]

(51)
Equation 51 shows that all investors assign the same price to the security that pays contingent on $\mathcal{G}$. Since this security is in zero net supply, it would not be traded. Therefore, the dynamic incompleteness imposed in the baseline does not change the allocations.
Appendix B: Additional Results

B1. Dynamic Implications of Asymmetric Information

Overall, three elements of the information environment impact the wealth distribution. First, the precision of available signals about the future states. Second, the allocation of available signals among the economic agents, capturing how accessible, or even understandable, they are for the investors. Third, the non-fundamental noises that enter into the prices and hamper the dissemination of information through the prices. In this section, I investigate the role of these three elements in wealth distribution.

To this end, I generate some economic insights by analyzing the wealth gaps resulted from information gaps, or sophistication differences, between pairs of investors. I do this by combining Lemma 4, which characterizes the distribution of posteriors, with Lemma 5, which characterizes the wealth dynamics as a function of the posteriors. Combining these two results, Proposition 6 provides a helpful characterization of the dynamics of the wealth gaps, for a given information environment. Then, I employ the lemma to study how the tail parameter is affected by the these changes in the information environment.

Proposition 6.

Suppose \( q^i_t \leq 1 - \frac{nb}{1-\bar{z}} \frac{z^N}{N} \) with probability one and \( q(g_t) \) is the conditional probability given the growth state \( (g_t = g^j) \). Furthermore, suppose the sophisticated investors in families \( (f_1, i_1) \) and \( (f_2, i_2) \) have wealth shares \( w_{f_1, t}^{i_1} \), \( i_2 > i_1 \), and \( w_{f_2, t}^{i_2} \) at \( t \) respectively. Then, provided neither of them receive the death shock at \( t+1 \), we have:

\[
- \frac{1}{3} \left( \frac{w_t^N}{\int_{i_2} w_t^N \, di} \right)^2 \leq \frac{\mathbb{E}_t \left[ \log \frac{w_{f_1, t+1}^{i_1}}{w_{f_2, t+1}^{i_2}} \right] - \log \frac{w_{f_1, t}^{i_1}}{w_{f_2, t}^{i_2}}}{(\log (q(g_t) + 2^{i_1})(1-q(g_t) + 2^{i_2}))(1-q(g_t) - 2^{i_1})(1-q(g_t) - 2^{i_2}))} \leq - \frac{1}{12} \left( \frac{w_N^t}{\int_{i_2} w_N^t \, di} \right)^2 \quad (52)
\]
Inequality 52 provides a lower and upper bound for the speed of wealth divergence among the sophisticated investors. We see the wealth ratio between a less informed investor (here \((f_1, i_1)\)) and a more informed investor (here \((f_2, i_2)\)) decreases in expectation, as long as they survive. Moreover, the expected amount of the decrease depends on all three factors mentioned earlier: Informativeness of the available signal, captured by \(\hat{z}^I\), the allocation of the informative signal, captured by CDF \(F(\cdot)\) and the non-fundamental noise, captured by \(\hat{z}^N\). Furthermore, one can show that the bounds in (52) are bounded for any \(i_1\) and \(i_2\) sine \(w_N t > \kappa b\) and \(w_I t > \kappa\) for any \(i \in [0, 1]\).

Now, I discuss these channels in turn.

Regarding the role of the informative signal, inequality 52 implies that a sufficiently large increase in the informativeness of the signal widens the wealth gap, and hence, increases the inequality. We can see this by noting that the expression \(\log \left(\frac{q(g_t) + \hat{z}^I (1 - q(g_t)) + \hat{z}^I}{q(g_t) - \hat{z}^I (1 - q(g_t)) - \hat{z}^I}\right)\) can take any arbitrarily large value, while the bounds in (52) are bounded. This shows as the available information resources become more informative, the information inequality, namely the investors having unequal access to such resources, becomes more pronounced in the wealth inequality. Strikingly, the wealth inequality increases even though a more informative signal also makes the prices more informative, which reduces the belief gap between the informed and uninformed investors.\(^{37}\)

To see the intuition, note that the information and beliefs impact the wealth distribution through the portfolio decisions. Facing less uncertainty, the investors take more aggressive positions and react to their beliefs more strongly when the signal is more informative.

\(^{36}\)Every type of investors have total wealth share of at least \(\kappa\) per unit mass, because the newborns constitute fraction \(1 - \delta\) of each type of investors and each newborn has the wealth of share at least \(\tau\). As a result, the upper bound in (52) is bounded by \(-\frac{1}{12} \left(\frac{\kappa b}{1 - \kappa b}\right)^2\) and the lower bound is bounded by \(-\frac{1}{3} \left(\frac{1 - \kappa (1 - i_2)(1 - b)}{1 - \kappa i_2(1 - b)}\right)^2\).

\(^{37}\)To see this, note that a more informative signal increases the belief gap between the sophisticated and naive investors. Therefore, loosely speaking, the wealth share of the naive investors decreases, which implies \(\frac{w_N}{w_I}\), and hence \(|q^I_U - q^I_I|\), also decrease.
This can be well illustrated in the investors’ optimal portfolio formula, specified in (9). According to the equation, the sensitivity of an investor’s portfolio ($\mu_{kt}$) to his belief ($q_{kt}$) is given by the expression below:

$$\frac{\partial \mu_{kt}}{\partial q_{kt}} = \frac{1}{\beta \delta R_t^F e^{-\gamma_t} - 1} - \frac{1}{\beta \delta R_t^F e^{-\gamma_t} - 1}$$

Note that the return on the risky asset is $\frac{p_{t+1} + d_{t+1}}{p_t} = (\beta \delta)^{-1} e^{\gamma_{t+1}}$. Moreover, as shown earlier, the risk premium also decreases as the signal becomes more informative, leading to an increase in $\frac{\partial \mu_{kt}}{\partial q_{kt}}$. Therefore, the same belief dispersion results in a larger portfolio heterogeneity as the risk premium shrinks. As a result, having superior information matters even more when the prices are already more informative.

Many policies and regulations aim to improve transparency in financial markets and lower the barrier to access information about available investment opportunities. Informing the distributional impact of such policies, inequality 52 states that there is almost a one-to-one relationship between the wealth divergence and extent of asymmetric information, more specifically, the probability that an investor receives strictly superior information than another investor ($F(i_2) - F(i_1)$). In other words, it recommends to reduce inequality, the informative signals should become publicly available and understandable, possibly through subsidizing access to information. Especially, it can massively reduce inequality when distribution $F(\cdot)$ itself is right-skewed, meaning the informative signals are virtually only available to a small group of investors.

Inequality 52 also underscores the role of non-fundamental noises in the wealth distribution. An increase in the magnitude of non-fundamental noise, represented by $\tilde{z}^N$, reduces the price informativeness, and hence, hampers the dissemination of information through the prices. It enlarges the belief gap, resulting in a faster wealth divergence, and consequently,
Finally, inequality 52 implies that the wealth divergence is the fastest among the most informed. We can see this by noting the bounds increase as we focus on the more informed groups. The intuition lies in the way the information is disseminated in the economy: If only few most informed investors learn the realization of the informative signal, which happens when $\lambda_t$ is large, their impact on the prices is less than the case of a smaller $\lambda_t$, in which a larger fraction of the investors learn the realization and all trade according to the same signal realization. Thus, the prices are less informative when $\lambda_t$ is larger. Therefore, conditional on $\lambda_t$ being large, the asymmetric information between the informed and uninformed is also larger, which leads to faster wealth dynamics among the most informed investors.

B2. Expanding the Security Space

So far, we have assumed that the markets are dynamically incomplete, meaning that the set of available securities do not span the whole space of potential contingent transfers. For instance, a log-term risk-free bond cannot be manufactured with the one-period risk-free bonds and the shares of the tree. A natural question is how expanding the set of securities impact the allocations, and more specifically, the wealth distribution. Proposition 7 shows that the expansion has no impact on the wealth distribution.

Proposition 7.

In the baseline setup described in Section 3, suppose the markets are dynamically complete, keeping the information structure unchanged. Then, Equations 16 and 18 still specify the wealth dynamics.

Proposition 7 states expanding the set of securities does not change the wealth distribution
and its dynamics. The intuition is as follows: Standing at time $t$, all investors assign the same probability to the events after time $t + 1$, once it is conditioned on the next period’s growth, $g_{t+1}$. In other words, the only state that they have disagreement about is $g_{t+1}$. In fact, in the proof, I show that all investors price all long-term assets (the ones that their value depends on a subset of $g_{t+2}, g_{t+3}, \ldots$) in the same way since all investors agree on the relative prices between any two consecutive periods. By combining this property in the baseline allocations and the abovementioned point, I conclude that all investors should assign the same price to all long-term assets, given the baseline allocation. Since the long-term assets are in zero net supply, except the tree, they are not traded in equilibrium. Therefore, the allocations remain intact.

### B3. Bequest Motives

An extensive literature highlights the role of bequest motive in wealth inequality (De Nardi (2004)). In this section, I allow the investors to have bequest motive for their intertemporal decisions. In short, I find the wealth dynamics remain unchanged if the investors are homogeneous in their bequest motive, that is all investors trade off their own and their child expected utility similarly. The intuition being that it impacts the consumption and saving decision of all investors in the same way and it does not affect the portfolio compositions.

I modify the utility function in (3) by adding a term $\Phi(\cdot)$ capturing the bequest motive. Equation (53) presents the modified utility function. In fact, $\Phi(W)$ is the utility an investor gets from leaving $W$ units of consumption goods, before tax, to his child.\(^{38}\)

\(^{38}\)Note that there is a one-to-one mapping between before-tax and after-tax inherited wealth.
\[ U(\{c_{f,t}^k\}_{t=s_1}^{s_2-1}, W_{s_2}) = \sum_{t=s_1}^{s_2-1} \beta^{t-s_1} \log c_{f,t}^k + \beta^{s_2-s_1} \Phi(W_{s_2}) \quad k \in [0, 1] \cup \{N\} \]  

(53)

To maintain tractability, I assume \( \Phi \) is logarithmic in the bequest wealth, i.e. \( \Phi(W) = \phi \log W \). The following proposition shows how the bequest motives changes the consumption and saving behaviors and wealth dynamics.

**Proposition 8.**

**a)** For every \( t \geq 1 \), all investors consume fraction \( \frac{1-\beta\delta}{1+\beta\delta(1-\delta)} \) of their wealth, i.e.

\[ c_{f,t}^k = \frac{1-\beta\delta}{1+\beta\delta(1-\delta)} W_{f,t}^k \quad \text{where } f \in [0, 1] \text{ and } k \in [0, 1] \cup \{N\}. \]

**b)** The distribution of the beliefs and evolution of wealth shares, specified in (16), are the same for all real values of \( \phi \).

Proposition 8(a) shows that the bequest motive changes the saving behavior of all investors uniformly. It means as \( \phi \) increases, all investors scale their savings at the same proportion. This increase in demand for saving pushes up the asset prices, and subsequently, lowers the expected returns. Overall, this general equilibrium effect fully offsets the higher demand for saving, which renders \( \phi \) irrelevant for the wealth distribution. This point is formalized in Part (b) of the proposition.

Note that Proposition 8 states the bequest motive has no role in wealth distribution when there is no heterogeneity in this motive for saving. However, a heterogeneity in the bequest motive can exacerbate or ameliorate wealth inequality, which is relegated to future studies.
B4. Moving Types

Data suggests billionaires’ descendants are not as good as their fathers in wealth creation and maintenance. In fact, there is a tendency to mean in the average returns that different family generations get and this mean-reversion has amplified over the past decades. For example Kaplan and Rauh (2013) find that only one-third of the richest individuals in Forbes 400 list at 2011 have grown wealthy, compared to two-third at 1988. Relatedly, Fagereng, Guiso, Malacrino, and Pistaferri (2016) find an economically small intergenerational correlation in returns to financial wealth and net worth. These findings suggest that the investment sophistication is imperfectly transmitted to the next generations.

Motivated by these findings, in this section, I analyze a modification of the model in which the newborns imperfectly inherit their parents type. Particularly, suppose there are only two types of sophisticated investors: informed and uninformed, similar to Section 6. However, a newborn inherits the type of his parent with probability \( \rho \in (0, 1) \). Therefore, the child of an informed investor is also informed with probability \( \rho \) and becomes uninformed with probability \( 1 - \rho \). The following proposition shows the wealth inequality is larger for higher values of \( \rho \).

**Proposition 9.**

*The tail parameter \( \gamma(\rho) \) is at least weakly decreasing in \( \rho \).*

Higher persistence in the sophistication types increases the inequality since it amplifies the compounding effect, that is the ones with a higher return today are more likely to get a higher return tomorrow, which further increases the inequality. This result is another indication of the importance of education in reducing inequality. Chetty, Friedman, Saez, Turner, and Yagan (2017) find that after students attending the same college land in relatively similar income percentiles, regardless of their parents’ income status. It underscores
the role of higher education in social mobility.