Online Appendix for “Limited Attention: Implications for Financial Reporting”

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1 Examples of strategic complementarity

In this subsection, I study an IPO setting and a bank-run setting using numerical methods. These two settings are examples of complementarity among investors’ decisions. In both settings, the information structure is the same as in Section 4 of the main model. I show that the findings in the main text carry over to these two settings.

1.1 An IPO game

In this subsection, I consider an IPO problem similar to the setting in Frankel et al. (2019). A firm is raising capital from a continuum of investors (uniformly distributed on the [0, 1] interval). We use $k_i \in \{0, 1\}$ to denote investor $i$’s subscription decision, where 1 means subscribing and 0 means not subscribing. The aggregate subscription is $K \equiv \int k_idi$.

The IPO works in the following way. The firm sells $s$ units of its shares in return for $t$ units of capital. The IPO price is thus $p = \frac{t}{s}$ per share. If a fraction $K$ of investors subscribes to the IPO, firm value will be $\theta + rK$, where $r > 0$ captures the strength of complementarity. Each subscribing investor contributes her capital in return for a $\frac{1}{p}$ share of the firm. Thus, investor $i$’s net gain from subscribing is

$$u_i = \frac{1}{p}(\theta + rK) - 1. \quad (1)$$

Based on (1), an investor $i$ will subscribe to the IPO if and only if $E[u_i] \geq 0$, or equivalently,

$$k_i = \begin{cases} 1, & \text{if } E[\theta] + rE[K] \geq p. \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

From (2), we can clearly see how complementarity plays a role in this IPO game. In particular, each investor will form expectations not only about $\theta$, the fundamental, but also about $K$, the aggregate subscribing fraction. Furthermore, fixing the fundamental
\( \theta \), the higher the aggregate subscription \( K \) is, the more likely it is that investor \( i \) will subscribe. Investors’ decisions are therefore complements, with \( r \) capturing the strength of complementarity.

### 1.1.1 Investors’ strategy in regime S

In regime \( S \), each investor \( i \) processes the summary with noise and receives \( z_i = \theta + \gamma + \epsilon_i \). Standard results in the global game literature (e.g., Morris and Shin (2001)) imply that investors will follow a threshold strategy: investor \( i \) will subscribe if and only if her signal \( z_i \) is above some threshold \( q_S \).

We now solve for the threshold \( q_S \). An investor receiving \( z_i = q_S \) should be indifferent between investing and not investing, which implies \( E(\theta|z_i = q_S) + r \Pr(z_j \geq q_S|z_i = q_S) = p \). By symmetry, we know \( \Pr(z_j \geq q_S|z_i = q_S) = \frac{1}{2} \). Hence, we must have \( E(\theta|z_i = q_S) = p - \frac{r}{2} \) or equivalently, the following proposition.

**Proposition 1** In regime \( S \), each creditor withdraws if and only if \( z_i \leq q_S \), where

\[
q_S \equiv \frac{p - \frac{r}{2}}{C^2(\sigma^2 + \gamma^2)}.
\]

### 1.1.2 Investors’ strategy in regime SD

We make the following conjecture about investors’ attention allocation strategy in regime SD.

**Conjecture:** If \( C \) is sufficiently large, an equilibrium exists in which all investors will divide their attention evenly between the two details. Furthermore, investor \( i \) chooses \( k_i = 0 \) if \( x_i + y_i < q_{SD} \), and \( k_i = 1 \) if \( x_i + y_i \geq q_{SD} \), where \( q_{SD} \equiv \frac{p - \frac{r}{2}}{1 - \frac{r}{C}} \).

This conjecture is verified numerically. In Figure 1 below, I plot investor \( i \)'s incentive to deviate to the summary as a function of \( C \) when \( \sigma^2 = 1.5 \), \( r = 1 \), and \( p = 1 \). The plot indeed shows that the larger \( C \) is, the lower the incentive to deviate. Hence, when \( C \) is sufficiently large, we should expect an equilibrium where all investors divide their
attention evenly between the two details.

Figure 1: Deviation incentive as a function of $C$ when $\sigma^2 = 1.5$, $r = 1$, and $p = 1$

1.1.3 Trade-off between the summary and the details

We use $W_{SU}$ and $W_{DB}$ to denote investors’ welfare when they devote their attention to the summary and divide their attention evenly between the details, respectively. Based on the results in the main model, we make the following conjectures.

Conjecture: $W_{SU} - W_{DB}$ increases in $r$.

In Figure 2 below, I plot $W_{SU} - W_{DB}$ as a function of $r$. The plot indeed shows that the larger $r$ is, the larger $W_{SU} - W_{DB}$ is. Furthermore, $W_{SU} - W_{DB}$ changes sign around $r = 0.7$. Figure 2 implies that the summary is more likely to dominate details as coordination becomes more important.
**Conjecture:** $W_{SU} - W_{DB}$ decreases in $C$.

In Figure 3 below, I plot the difference between $W_{SU}$ and $W_{DB}$ as a function of $C$. The plot indeed shows that the larger $C$ is, the smaller $W_{SU} - W_{DB}$ is. Furthermore, $W_{SU} - W_{DB}$ changes sign around $C = 2.55$. Figure 3 implies that the summary is more likely to dominate details as capacity becomes smaller.
1.2 Bank run

In this subsection, I consider a bank-run setting similar to Morris and Shin (2000). There is a bank and a continuum of creditors on \([0, 1]\). The bank has access to a long-term project that transforms 1 unit of deposit to \(\theta\) units on \(t = 2\). However, the project is illiquid in the sense that if a fraction \(l\) of the creditors withdraw on \(t = 1\), the return of the project on \(t = 2\) will decrease to \(\theta - \delta l\), where \(\delta > 0\) captures the intensity of strategic complementarity among creditors. If a creditor withdraws on \(t = 1\), she is guaranteed to get one unit back. Hence, using \(u_i\) to denote creditor \(i\)'s utility, we have

\[
\begin{align*}
    u_i &= \begin{cases} 
        1, & \text{in the case of early withdrawal.} \\
        \theta - \delta l, & \text{otherwise}
    \end{cases}
\end{align*}
\]

Based on (3), creditor \(i\) will stay if and only if \(E[\theta - \delta l] \geq 1\). We use \(l_i \in \{0, 1\}\) to denote investor \(i\)'s withdrawal decision, where 1 means withdrawing and 0 means
staying. Therefore,
\[ l_i = \begin{cases} 
0, & \text{if } E[\theta] - \delta E[l] \geq 1, \\
1, & \text{otherwise} 
\end{cases} \quad (4) \]

From (4), we can clearly see how complementarity plays a role in this bank-run setting. In particular, each creditor will not only form expectations about \( \theta \), the fundamental, but also about \( l \), the aggregate withdrawing fraction. Furthermore, the higher \( l \) is, the more likely it is that investor \( i \) will withdraw. Creditors’ decisions are therefore complements, with \( \delta \) capturing the strength of complementarity.

1.2.1 Creditors’ strategy and payoff in regime S

In regime \( S \), because of limited attention, each creditor \( i \) processes the summary with noise and receives \( z_i = \theta + \gamma + \epsilon_i \). I follow the global game literature (e.g., Morris and Shin (2001)) and consider the following threshold strategy: creditor \( i \) will stay if and only if her signal \( z_i \) is above some threshold \( q_S \).

We now solve for the threshold \( q_S \). A creditor receiving \( z_i = q_S \) should be indifferent between staying and withdrawing, which implies \( E(\theta|z_i = q_S) - \delta \Pr(z_j < q_S|z_i = q_S) = 1 \). By symmetry, we know that \( \Pr(z_j < q_S|z_i = q_S) = \frac{1}{2} \). Hence, we must have \( E(\theta|z_i = q_S) = 1 + \frac{\delta}{2} \) or equivalently, the following proposition:

**Proposition 2** In regime \( S \), each creditor withdraws if and only if \( z_i \leq q_S \), where
\[ q_S \equiv \frac{1 + \frac{\delta}{2}}{\frac{1}{C^2}(\sigma^2 + 1)} . \]

1.2.2 Creditors’ strategy and payoff in regime SD

We make the following conjecture about investors’ attention allocation strategy in regime SD.

**Conjecture:** If \( C \) is sufficiently large, an equilibrium exists in which all creditors divide their attention evenly between the two details. Furthermore, in this equilibrium, creditor \( i \) withdraws if and only if \( x_i + y_i \leq q_{SD} \), where \( q_{SD} \equiv \frac{1 + \frac{\delta}{2}}{1 - \frac{1}{C^2}} \).
The conjecture is verified numerically. In Figure 4 below, I plot creditor $i$’s incentive to deviate to the summary as a function of $C$, when $\sigma^2_\gamma = 1.5$ and $\delta = 1$. The plot indeed shows that the larger $C$ is, the lower the incentive to deviate. Hence, when $C$ is sufficiently large, we expect an equilibrium where all creditors divide their attention evenly between the two details.

Figure 4: Deviation incentive as a function of $C$ when $\sigma^2_\gamma = 1.5$ and $\delta = 1$

1.2.3 Trade-off between the summary and the details

We use $W_{SU}$ and $W_{DB}$ to denote investors’ welfare when they devote their attention to the summary and divide their attention evenly between the details, respectively. Based on the results in the main model, we make the following conjectures.

**Conjecture:** $W_{SU} - W_{DB}$ increases in $\delta$.

In Figure 5 below, I plot $W_{SU} - W_{DB}$ as a function of $\delta$. The plot indeed shows
that the larger $\delta$ is, the larger $W_{SU} - W_{DB}$ is. Furthermore, $W_{SU} - W_{DB}$ changes sign around $\delta = 0.7$. Figure 5 implies that the summary is more likely to dominate details as coordination becomes more important.

Figure 5: $W_{SU} - W_{DB}$ as a function of $\delta$ when $\sigma_{\gamma}^2 = 1.5$ and $C = 2$

Conjecture: $W_{SU} - W_{DB}$ decreases in $C$.

In Figure 6 below, I plot the welfare difference $W_{SU} - W_{DB}$ as a function of $C$. The plot indeed shows that the larger $C$ is, the smaller $W_{SU} - W_{DB}$ is. Furthermore, $W_{SU} - W_{DB}$ changes sign around $C = 2.4$. Figure 6 implies that the summary is more likely to dominate details as capacity becomes smaller.
2 An alternative way of modeling the information loss in the summary

In the baseline model, I model the information loss in the summary by adding noise to the summary that is independent of details. In this section, I use an alternative method of modeling the information loss in the summary. Consider the following motivating example. A firm’s total revenue consists of foreign and domestic revenue. Domestic revenue is more persistent and more informative about the firm’s prospects. However, the summary measure, total revenue, puts equal weights on both revenues. Hence, there is noise in the summary measure but the noise is correlated with details. I formally model this situation in this section and show that the main findings of the baseline model continue to hold.
I now assume the fundamental $\theta = \theta_x + t\theta_y$, where $t > 1$ captures the relative importance of the second detail. $\theta_x$ and $\theta_y$ can be interpreted as foreign and domestic revenue, respectively. The investor utility is the same as before and is given by $u = \theta k - \frac{1}{2}k^2$.

The key friction of the model is that the firm cannot directly report $\theta_x + t\theta_y$. Instead, it can only report an equally weighted summary or details or both.

In Regime S (summary), the reporting system reports the total revenue: $I_S = \{\theta_x + \theta_y\}$. The investor processes the summary with noise, and receives $z = \theta_x + \theta_y + \epsilon$, where $\epsilon \sim N(0, \sigma^2)$. The mutual entropy constraint is

$$I(z, \theta_x + \theta_y) \leq \kappa \iff 1 + \frac{\sigma_x^2 + \sigma_y^2 + \sigma_\gamma^2}{\sigma^2} \leq C^2.$$  \hspace{1cm} (5)

In Regime D (details), the reporting system reports both foreign and domestic revenues: $I_D = \{\theta_x, \theta_y\}$. The investor receives two signals about each detail: $x = \theta_x + \epsilon_1$, $y = \theta_y + \epsilon_2$, where $\epsilon_1 \sim N(0, \sigma_1^2)$ and $\epsilon_2 \sim N(0, \sigma_2^2)$ are independent noises. The mutual entropy constraint is:

$$I(\{x, y\}, \{\theta_x, \theta_y\}) \leq \kappa \iff (1 + \frac{\sigma_x^2}{\sigma_1^2})(1 + \frac{\sigma_y^2}{\sigma_2^2}) \leq C^2.$$  \hspace{1cm} (6)

2.1 Analysis

I first examine investors’ attention allocation decisions. In regime S (summary), investors only have the total revenue to attend to and therefore will devote all their capacity to it. In regime D (details), investors will allocate their attention between the two components. $\theta_y$ will attract more attention because it is more relevant. This is summarized in the following proposition.

**Proposition 3*** In regime D, investors will pay more attention to $\theta_y$ than $\theta_x$. When $C > t$, investors will choose $\sigma_1^2 = \frac{t}{C-t}$ and $\sigma_2^2 = \frac{1}{Ct-1}$. When $C < t$, investors will devote all their attention to $\theta_y$ and ignore $\theta_x$.***
Proposition 3 confirms the findings of Mackowiak and Wiederholt (2009), who show that firms pay more attention to the more important macroeconomic condition.

The next proposition examines the trade-off between the summary and details.

**Proposition 4** When \( t > 1 + \sqrt{2} \), details always dominate the summary. When \( 1 < t < 1 + \sqrt{2} \), the summary dominates details if and only if \( C < C^{**} \equiv \sqrt{\frac{-t^4 + 6t^2 - 1}{(t-1)^4}} + \frac{2t}{(t-1)^2} \).

When \( t \) is large so that the summary is very imprecise, investors always prefer the details because the summary is less informative about the fundamental than detail \( \theta_y \). When \( t \) is small so that the summary is precise, investors prefer the summary over details when their capacity is small. Proposition 4 echoes the findings of Proposition 1 in the main text.

### 3 Endogenizing the investor’s capacity choice

In this appendix, I endogenize the investor’s capacity \( C \) by introducing a cost of capacity, which captures both the monetary cost and the opportunity cost of investing in information processing. For example, to understand financial statements, an investor needs to spend time and money to study financial accounting. Another example is that at the individual level, to analyze a set of financial statements, attention needs to be drawn from other activities (e.g., watching TV, reading a book, exercising, cooking).

The goal of this section is to examine how endogenous capacity affects the ranking of the two regimes (summary and details).

The model set-up is the same as the baseline model in Section 2 of the main text except that capacity \( C \) must be acquired at cost \( c(C) = h(C-1) \) where the parameter \( h > 0 \) captures the magnitude of the cost. To focus on the incremental effect of cost of capacity, I assume that the details are symmetric so that \( \sigma_x = \sigma_y = 1 \). I also assume

\[\text{Recall that the minimum level of } C \text{ is } 1. \text{ Thus } C - 1 \text{ captures the additional capacity that the investor acquires.}\]
that $\sigma^2_\gamma < \sigma^2_\theta = 2$ to focus on the case where the summary is more precise than one
detail.

The next lemma characterizes the investor’s capacity choice in regimes S (defined as $C_S$) and D (defined as $C_D$).

**Lemma 1** When $h > \frac{4}{2+\sigma^2_\gamma}$, the investor acquires no capacity in regimes S and D.

When $h \in (1, \frac{4}{2+\sigma^2_\gamma})$, the investor acquires no capacity in regime D and chooses $C_S = \frac{2^{2/3}}{\sqrt[3]{h(\sigma^2_\gamma + 2)}}$ in regime S.

When $h < 1$, the investor chooses $C_S = \frac{2^{2/3}}{\sqrt[3]{h(\sigma^2_\gamma + 2)}}$ in regime S and $C_D = \frac{1}{\sqrt{h}}$ in regime D.

We now compare the investor’s expected utility in the two regimes.

**Proposition 5** There exists a threshold $\bar{h} \in (0, 1)$ such that:

1) When $h \in (0, \bar{h})$, the details dominate the summary.

2) When $h \in (\bar{h}, \frac{4}{2+\sigma^2_\gamma})$, the summary dominates the details.

3) When $h \in (\frac{4}{2+\sigma^2_\gamma}, +\infty)$, the investor’s expected utility is the same in the two regimes.

The main insight of Proposition 5 is similar to that of the baseline model in the main
text. Specifically, providing more data in accounting reports can benefit investors only
when the cost of acquiring capacity is small (i.e., when capacity is a cheap resource).

When capacity is expensive, providing more data will result in (weakly) less information
being extracted by investors. The incremental insight of Proposition 5 is that when
capacity is too expensive, the investor will give up acquiring capacity in both regimes.
4 Supplementary proofs for claims in the main text

4.1 Supplementary proof for Lemma 2

We know from the main text that
\[ u_i(C^2 - 1, p_{22}) = \frac{(r - 1)^2 \left(C^2 + p_{22} \left(\frac{C^6}{(p_{22}+1)(C^2(1-r)+r)} - 1\right) - 1\right)}{C^2} + \frac{1}{2} (r - 1)^2 \left(\frac{1}{C^2} + p_{22} \left(\frac{1}{C^2} - \frac{C^4}{(p_{22}+1)(C^2(1-r)+r)}\right) - 1\right). \]

We now determine the value of \( p_{22} \in [0, C^2 - 1] \) that maximizes \( u_i(C^2 - 1, p_{22}) \). The first-order condition \( \frac{du_i(C^2 - 1, p_{22})}{dp_{22}} = 0 \) leads to
\[
\frac{du_i(C^2 - 1, p_{22})}{dp_{22}} = \frac{(r - 1)^2 \left(-C^4 (p_{22} + 1)^2 (r - 1)^2 + 2C^2 (p_{22} + 1)^2 (r - 1)r + C^6 - (p_{22} + 1)^2 r^2\right)}{2C^2 (p_{22} + 1)^2 (r - C^2(r - 1))^2} = 0.
\]

Therefore, \( \frac{du_i(C^2 - 1, p_{22})}{dp_{22}} \) has the same sign as \( g(p_{22}) = -p_{22}^2 \left(C^4(r - 1)^2 - 2C^2(r - 1)r + r^2\right) - p_{22} (2C^4(r - 1)^2 - 4C^2(r - 1)r + 2r^2) - C^4(r - 1)^2 + 2C^2(r - 1)r + C^6 - r^2. \) We now analyze the sign of \( g(p_{22}) \). First, because \( C > 1 \) and \( 0 < r < 1 \), \( g(p_{22}) \) is a quadratic function that opens downwards. It has two roots: \( x_1 \equiv \frac{(C+1)(C^2-Cr+r)}{C^2-C^2r-2r} \) and \( x_2 \equiv \frac{-C^2r-C^3+C^2r^2}{C^2r-C^2-2r} \).

Because \( C > 1 \) and \( 0 < r < 1 \), \( x_1 \) is always negative and is therefore not a candidate for the maximum point of \( u_i(C^2 - 1, p_{22}) \). Furthermore, \( 0 < x_2 < C^2 - 1 \) if and only if \( 0 < r < \frac{C}{C+1} \). Therefore, if \( 0 < r < \frac{C}{C+1} \), we know that on \( [0, C^2 - 1] \), \( g(p_{22}) \) first increases in \( p_{22} \) and then decreases in \( p_{22} \). In other words, \( u_i(C^2 - 1, p_{22}) \) is maximized at an interior point. If \( \frac{C}{C+1} < r < 1 \), \( g(p_{22}) \) always increases in \( p_{22} \). In this case, \( u_i(C^2 - 1, p_{22}) \) is maximized at \( p_{22} = C^2 - 1 \). To sum up, \( u_i(C^2 - 1, p_{22}) \) is maximized at \( p_{22} = C^2 - 1 \) if and only if \( \frac{C}{C+1} < r < 1 \).
4.2 Proof of Lemma 5

Recall that from the main text that

\[ u_{SU} = \frac{2C^2 (C^2 - 1) (r - 1)^2}{(\sigma_\gamma^2 + 2) (r - C^2 (r - 1))^2}, \]  
\[ u_{DB} = \frac{(C - 1) C (r - 1)^2}{(C(1 - r) + r)^2}, \]  
\[ u_{DX} = u_{DY} = \frac{C^2 (C^2 - 1) (r - 1)^2}{2 (r - C^2 (r - 1))^2}. \]

(7) (8) (9)

First, \( \frac{u_{SU}}{u_{DX}} = \frac{4}{\sigma_\gamma^2 + 2} \). Because \( \sigma_\gamma^2 < 2 \), we know \( u_{SU} > u_{DX} \). This proves i).

In addition, \( \frac{d(u_{SU})}{dr} = \frac{4C^2 (C^2 - 1)(C(1-r)+r)}{(\sigma_\gamma^2 + 2)(C(1-r)+r)^3} > 0 \). The inequality holds because \( C > 1 \) and \( 0 < r < 1 \). This proves ii).

We now compare \( u_{SU} \) and \( u_{DB} \) by computing their ratio:

\[ h(r) \equiv \frac{u_{SU}}{u_{DB}} = \frac{2C(C + 1)(C(-r) + C + r)^2}{(\sigma_\gamma^2 + 2) (r - C^2 (r - 1))^2}. \]

We already know that \( h'(r) > 0 \). Furthermore, \( h(1) = \frac{2C(C+1)}{\sigma_\gamma^2 + 2} \) and \( h(0) = \frac{2(C+1)}{C(\sigma_\gamma^2 + 2)} \).

Because \( \sigma_\gamma^2 < 2 \) and \( C > 1 \), \( h(1) > 1 \). In addition, \( h(0) > 1 \) if and only if \( C < \frac{2}{\sigma_\gamma^2} \).

Therefore, when \( C < \frac{2}{\sigma_\gamma^2} \), \( u_{SU} > u_{DB} \). When \( C > \frac{2}{\sigma_\gamma^2} \), there exists a unique \( r^* \in (0,1) \) such that \( u_{SU} > u_{DB} \) if and only if \( r > r^* \), where \( r^* \) satisfies \( h(r^*) = 1 \). Solving this equation leads to

\[ r^* = \frac{C^2 \sigma_\gamma^2}{(C - 1)(C\sigma_\gamma^2 + \sigma_\gamma^2 + 2)} - \sqrt{2} \sqrt{\frac{C^3 \sigma_\gamma^2 + 2C^3}{(C - 1)^2(C + 1)(C\sigma_\gamma^2 + \sigma_\gamma^2 + 2)^2}}. \]

(10)

We now compare \( r_0 \equiv \frac{C}{C+1} \) and \( r^* \). We have \( h(r_0) = \frac{8C}{\sigma_\gamma^2 + C\sigma_\gamma^2 + 2C + 2} \). \( h(r_0) > 1 \) because \( \sigma_\gamma^2 < 2 \). Because \( h'(r) > 0 \) and \( h(r^*) = 1 \), we know that \( r^* < r_0 \). This proves iii) and iv). Q.E.D.

4.3 Supplementary proof for Corollary 2

Recall from the main text that \( g(C, \sigma_\gamma^2) \equiv \sqrt{2} \sqrt{C (\sigma_\gamma^2 + 2) ((C(2C^2 + C + 2) + 3) \sigma_\gamma^2 + 2(C + 3)) + 4C(C + 1)^{3/2} \sigma_\gamma^2 (-\sigma_\gamma^2 + C - 2)} \). We now prove \( g(C, \sigma_\gamma^2) > 0 \) for any \( C > 1 \) and
0 < \sigma_\gamma^2 < 2. We expand \( g(C, \sigma_\gamma^2) \) to the following 11 terms.

\[
g(C, \sigma_\gamma^2) = 2\sqrt{2C^3\sigma_\gamma^2}\sqrt{C} (\sigma_\gamma^2 + 2) + \sqrt{2C^3\sigma_\gamma^2}\sqrt{C} (\sigma_\gamma^2 + 2) + 2\sqrt{2C}\sigma_\gamma^2\sqrt{C} (\sigma_\gamma^2 + 2) \\
+ 2\sqrt{2C}\sqrt{C} (\sigma_\gamma^2 + 2) + 3\sqrt{2}\sigma_\gamma^2\sqrt{C} (\sigma_\gamma^2 + 2) + 6\sqrt{2}\sqrt{C} (\sigma_\gamma^2 + 2) + 4C^3\sqrt{C + 1}\sigma_\gamma^2 \\
- 4C^2\sqrt{C + 1}\sigma_\gamma^4 - 4C^2\sqrt{C + 1}\sigma_\gamma^2 - 4C\sqrt{C + 1}\sigma_\gamma^4 - 8C\sqrt{C + 1}\sigma_\gamma^2.
\]

Because \( \sigma_\gamma^2 < 2 \), we know that \( \sqrt{\sigma_\gamma^2 + 2} > \sigma_\gamma^2 \). Applying this inequality, we have

\[
g(C, \sigma_\gamma^2) > 2\sqrt{2C^3\sigma_\gamma^4}\sqrt{C} + \sqrt{2C^3\sigma_\gamma^4}\sqrt{C} + 2\sqrt{2C}\sigma_\gamma^4\sqrt{C} \\
+ 2\sqrt{2C}\sqrt{C} (\sigma_\gamma^2 + 2) + 3\sqrt{2}\sigma_\gamma^4\sqrt{C} + 6\sqrt{2}\sigma_\gamma^2\sqrt{C} + 4C^3\sqrt{C + 1}\sigma_\gamma^2 \\
- 4C^2\sqrt{C + 1}\sigma_\gamma^4 - 4C^2\sqrt{C + 1}\sigma_\gamma^2 - 4C\sqrt{C + 1}\sigma_\gamma^4 - 8C\sqrt{C + 1}\sigma_\gamma^2 \\
= \sigma_\gamma^4(2\sqrt{2C^5/2 + C^{3/2} + 2\sqrt{C} + 3}) - 4C(C + 1)^{3/2} \\
+ \sigma_\gamma^2(2\sqrt{2C + 1}C^{5/2} + \sqrt{2C} + 3\sqrt{2} - 4C\sqrt{C + 1}(C + 2)).
\]

Therefore, to prove \( g(C, \sigma_\gamma^2) > 0 \), it suffices to show that \( h_1(C) \equiv \sqrt{2C^5/2 + C^{3/2} + 2\sqrt{C} + 3} - 4C(C + 1)^{3/2} > 0 \) and \( h_2(C) \equiv 2\sqrt{C}(2\sqrt{C + 1}C^{5/2} + \sqrt{2C} + 3\sqrt{2} - 4C\sqrt{C + 1}(C + 2)) > 0 \).

We first determine the sign of \( h_1(C) \). Note that \( h_1(C) > 0 \) is equivalent to \( h_3(C) \equiv \frac{\sqrt{2C(2C^{5/2} + C^{3/2} + 2\sqrt{C} + 3)}}{4C(C + 1)^{3/2}} > 1 \). We have \( h_3'(C) = \frac{4C^3 + 10C^2 - C - 9\sqrt{C} + 2}{4\sqrt{2C}(C + 1)^{5/2}} \). Hence, \( h_3'(C) > 0 \) because \( C > 1 \). This implies that \( h_3(C) \) takes its minimum value at \( C = 1 \). Therefore, \( h_3(C) > h_3(1) = 1 \), which implies that \( h_1(C) > 0 \).

We next determine the sign of \( h_2(C) \). We have \( h_2'(C) = \frac{4C^3 + 10C^2 - C - 9\sqrt{C} + 2}{4\sqrt{2C}(C + 1)^{5/2}} \). Hence, \( h_2'(C) \) has the same sign as \( h_4(C) = 14C^{5/2} - 12C^{3/2} - 8\sqrt{C} + 3\sqrt{2}\sqrt{C + 1} \). Because \( C > 1 \), \( h_4(C) = 14C^{5/2} - 12C^{3/2} - 8\sqrt{C} + 3\sqrt{2}\sqrt{C + 1} > 2C^{5/2} - 8\sqrt{C} + 3\sqrt{2}\sqrt{C + 1} \). Here, \( h_5(C) = 5C^{3/2} + \frac{3}{\sqrt{2\sqrt{C + 1}}} - \frac{4}{\sqrt{C}} > 0 \) because \( C > 1 \). Thus, \( h_5(C) \) takes its minimum value at \( C = 1 \), which implies that \( h_5(C) > h_5(1) = 0 \) and thus \( h_4(C) > h_5(C) > 0 \). Therefore, \( h_2'(C) > 0 \) and \( h_2(C) \) takes its minimum value at \( C = 1 \). This implies that \( h_2(C) > h_2(1) = 0 \). Q.E.D.
4.4 Supplementary proof for Lemma 3

Recall that $u_{\text{deviation}} = \frac{(C-1)(r-1)^2(C^2(\sigma^2_\gamma - 3r^2 + 2) + r\sigma^2_\gamma)}{C^2(r-1)^2} + \frac{(r+1)^2}{C^2(r-1)^2}$ and $u_{SU} = \frac{2C^2(C^2-1)(r-1)^2}{(r+2)(r-C(r-1))}$. Thus,

$$
\frac{u_{\text{deviation}}}{u_{SU}} = \frac{(\sigma^2_\gamma (C^2(r-1) - 1) - r - 2C^2)^2}{2C^3(C+1)(\sigma^2_\gamma + 2)}.
$$

Hence, $u_{\text{deviation}} < u_{SU}$ if and only if $(\sigma^2_\gamma (C^2(r-1) - 1) - r - 2C^2)^2 < 2C^3(C+1)(\sigma^2_\gamma + 2)$

or equivalently, $f(r) \equiv (\sigma^2_\gamma (C^2(r-1) - 1) - r - 2C^2)^2 - 2C^3(C+1)(\sigma^2_\gamma + 2) < 0$.

$$f(r) = 2(\sigma^2_\gamma - C^2\sigma^2_\gamma) (\sigma^2_\gamma - r\sigma^2_\gamma + 2 + r\sigma^2_\gamma) < 0$$
because $C > 1$ and $0 < r < 1$. In addition $f(1) = (\sigma^2_\gamma + 2C^2)^2 - 2C^3(C+1)(\sigma^2_\gamma + 2) < 0$. $f(0) = C^3(\sigma^2_\gamma + 2)(C\sigma^2_\gamma - 2)$.

Therefore, when $C < \frac{2}{\sigma^2_\gamma}$, $f(0) < 0$ and $u_{\text{deviation}} < u_{SU}$ always holds regardless of $r$.

When $C > \frac{2}{\sigma^2_\gamma}$, $f(0) > 0$. In this case, there exists a unique $r_2$ such that $u_{\text{deviation}} < u_{SU}$ if and only if $r > r_2$, where $r_2$ satisfies $f(r_2) = 0$. Solving this equation leads to

$$r_2 = \frac{C^2\sigma^2_\gamma + 2C^2}{(C^2 - 1)\sigma^2_\gamma} - \sqrt{2} \sqrt{\frac{C^3\sigma^2_\gamma + 2C^3}{(C - 1)^2(C + 1)\sigma^2_\gamma}}. \quad (11)$$

Finally, $f \left( \frac{C}{C+1} \right) = C^2((2C - 2C^2)\sigma^2_\gamma + \sigma^2_\gamma - 4) < 0$ because $\sigma^2_\gamma < 2$ and $C > 1$.

Therefore, $r_2 < r_0 \equiv \frac{C}{C+1}$.

4.5 Supplementary proof for Lemma 4

Recall that $\bar{r} := \frac{C(C^2\sigma^2_\gamma - 2)}{(C^2 - 1)\sigma^2_\gamma}$. We now show $\bar{r} \in (r^*, r_0)$, where $r_0 \equiv \frac{C}{C+1}$ and $r^*$ is given in (10).

First, recall that $r^*$ is the unique solution to $h(r) \equiv \frac{u_{SU}}{u_{DB}} = \frac{2C^2(C+1)(C(r+C)+r)^2}{(r+2)(r-C^2(r-1))^2} = 1$ in the case of $C > \frac{2}{\sigma^2_\gamma}$. Because $\frac{u_{SU}}{u_{DB}}$ increases in $r$ and because $h(\bar{r}) = \frac{C(\sigma^2_\gamma + 2)}{2(C+1)} > 1 = h(r^*)$, $\bar{r} > r^*$.

Second, $\bar{r} - r_0 = \frac{C(\sigma^2_\gamma - 2)}{(C^2 - 1)\sigma^2_\gamma} < 0$ because $C > 1$ and $\sigma^2_\gamma < 2$. Therefore, $\bar{r} < r_0$. Q.E.D.
A Proofs for claims in the online appendix

A.1 Proof of Proposition 3

Recall that when details are disclosed, the investor receives $x = \theta_x + \epsilon_1$ and $y = \theta_y + \epsilon_2$, where $\epsilon_1 \sim N(0, \sigma_1^2)$ and $\epsilon_2 \sim N(0, \sigma_2^2)$. We work with precision and define $p_1 = \frac{1}{\sigma_1^2}$ and $p_2 = \frac{1}{\sigma_2^2}$. Conditional on receiving the signals $x$ and $y$, the investor will choose investment as $k = E[\theta | x, y] = \beta_x x + t \beta_y y$, where $\beta_x = \frac{\sigma_x^2}{\sigma_x^2 + 1/p_1} = \frac{1}{1 + 1/p_1}$, $\beta_y = \frac{\sigma_y^2}{\sigma_y^2 + 1/p_2} = \frac{1}{1 + 1/p_2}$. The investor’s problem is

$$\max_{p_1, p_2} E[u] = E[\theta k - \frac{1}{2} k^2]$$

$$= E[(\beta_x x + t \beta_y y) \theta - \frac{1}{2}(\beta_x x + t \beta_y y)^2]$$

$$= \beta_x E[\theta^2] + t \beta_y E[\theta^2 y] - \frac{1}{2}(\beta_x^2 E[x^2] + t^2 \beta_y^2 E[y^2])$$

$$= \beta_x + t \beta_y - \frac{1}{2}(\beta_x^2 (1 + \frac{1}{p_1}) + t^2 \beta_y^2 (1 + \frac{1}{p_2}))$$

subject to $(1 + p_1)(1 + p_2) \leq C^2$.

The constraint will obviously bind. Hence, we can plug $p_1 = \frac{C^2 - 1 - p_2}{p_2 + 1}$, $\beta_x = \frac{1}{1 + 1/p_1}$, and $\beta_y = \frac{1}{1 + 1/p_2}$ into the objective function. After simplification, $E[u] = \frac{p_2(C^2(t^2 + 1) - 2) + C^2 - p_2^2 - 1}{2C^2(p_2 + 1)}$.

It can be easily verified that when $C < t$, $E[u]$ always increases in $p_2$. When $C > t$, $E[u]$ increases in $p_2$ if and only if $p_2 < Ct - 1$. Hence, when $C < t$, the investor devotes all her attention to the more important detail ($\theta_y$). When $C > t$, the investor chooses $p_1 = C - t$ and $p_2 = Ct - 1$. Q.E.D.

A.2 Proof of Proposition 4

We can plug the investor’s optimal attention-allocation decision into the utility function and compute the investor’s ex-ante expected utility when she focuses on the summary,
both details, and one detail, respectively:

\[ u_{SU} = \frac{(C^2 - 1) (t + 1)^2}{4C^2}, \]
\[ u_{DB} = \frac{Ct^2 + C - 2t}{2C}, \]
\[ u_{DX} = \frac{(C^2 - 1) t^2}{2C^2}, \]
\[ u_{DY} = \frac{(C^2 - 1) t^2}{2C^2}. \]

When \( t > 1 + \sqrt{2} \), it can be easily verified that \( u_{SU} < u_{DB} \) and that \( u_{SU} < u_{DY} \).
Hence, details always dominate the summary.

When \( t < 1 + \sqrt{2} \) and \( C < t \), investors focus on detail \( \theta_y \) in regime D. It can be verified that \( u_{SU} > u_{DY} \) in this case.

When \( t < 1 + \sqrt{2} \) and \( C > t \), investors focus on both details in regime D. It can be verified that \( u_{SU} > u_{DB} \) if and only if \( C < C^{**} = \sqrt{-\frac{t^4 + 6t^2 - 1}{(t-1)^4}} + \frac{2t}{(t-1)^2} \).
Furthermore, we also have \( C^{**} = \sqrt{-\frac{t^4 + 6t^2 - 1}{(t-1)^4}} + \frac{2t}{(t-1)^2} > t \). Therefore, when \( t < 1 + \sqrt{2} \), the summary dominates details if and only if \( C < C^{**} \).
Q.E.D.

A.3 Proof of Lemma 1

We first derive the investor’s capacity choice in regime S. In the main text, we have derived the investor’s expected utility as a function of \( C \): \( u_S(C) = \frac{2(C^2 - 1)}{C^3(\sigma^2_d + 2)} \). The investor therefore chooses \( C \) to maximize \( u_S(C) - h(C - 1) \) subject to the constraint \( C > 1 \).

The first-order derivative is \( u'_S(C) - h = \frac{4}{C^3(\sigma^2_d + 2)} - h \). Hence, when \( h > \frac{4}{2 + \sigma^2_d} \), the derivative is always negative, and the investor chooses \( C_S = 1 \). When \( h < \frac{4}{2 + \sigma^2_d} \), the investor chooses \( C \) such that \( u'_S(C) - h = 0 \), which implies \( C_S = \frac{2^{2/3}}{\sqrt[h]{\sigma^2_d + 2}} \).

We then derive the investor’s capacity choice in regime D. In the main text, we have derived the investor’s expected utility as a function of \( C \): \( u_D(C) = \frac{C - 1}{C} \). The investor
therefore chooses $C$ to maximize $u_D(C) - h(C - 1)$ subject to the constraint $C > 1$.

The first-order derivative is $u'_D(C) - h = \frac{1}{C^2} - h$. Hence, when $h > 1$, the derivative is always negative, and the investor chooses $C_S = 1$. When $h < 1$, the investor chooses $C$ such that $u'_D(C) - h = 0$, which implies $C_D = \frac{1}{\sqrt{h}}$. Q.E.D.

### A.4 Proof of Proposition 5

Plugging the investor’s interior capacity choice into the expected utilities in the two regimes leads to $u_S = \frac{2 - \left(h \left(\frac{\sigma^2}{\gamma} + 2\right)\right)^{2/3}}{\sigma^2 + 2}$ and $u_D = 1 - \sqrt{h}$. It can be easily verified that when $h \in (0, 1)$, there exists a threshold $\tilde{h} \in (0, 1)$ such that $u_S < u_D$ if and only if $h < \tilde{h}$. Furthermore, when $h \in (1, \frac{4}{2 + \sigma^2})$, S dominates D because the investor acquires capacity in S but does not acquire capacity in D. When $h > \frac{4}{2 + \sigma^2}$, the investor does not acquire capacity in both regimes, and hence the expected utility is the same in the two regimes. Q.E.D.

### References


