A Model

A.1 Representative Household

A representative household determines how much to consume and to invest by maximizing the utility function:

$$\sum_{t=0}^{\infty} \beta^t u(C_t),$$

where $C_t$ is consumption at date $t$, and $0 < \beta < 1$ is the discount factor.$^{49}$ The budget constraint is:

$$C_t + K_{t+1} \leq (1 + R_t - \delta)K_t + W_tL_t + \Pi_t, \forall t \geq 0,$$

where $K_t$ is the aggregate capital stock at date $t$, $L_t$ is labor at date $t$, $R_t$ and $W_t$ are the capital rental and wage rates at date $t$, $\delta$ is the depreciation rate, and $\Pi_t$ is total profit from operations of all firms.$^{50}$ Capital is owned by a representative household and rented to firms, and labor is supplied inelastically to the labor market because a representative household does not value leisure: $L_t = L$.

$^{49}$The utility function, $u(C_t)$, is strictly increasing, strictly concave, and continuously differentiable.

$^{50}$I assume $0 \leq K_{t+1} \leq K$ to ensure that the payoff function is bounded.
A.2 Monopolistic Competition with Heterogeneous Firms

A.2.1 Technology

A continuum of intermediate-good producers exists with a fixed measure of 1. Each producer is indexed by \( i \) in a Cobb-Douglas production function:

\[
Y_{it} = K_{it}^{\alpha_1} L_{it}^{\alpha_2}, \quad \alpha_1 + \alpha_2 = 1, \tag{17}
\]

where \( Y_{it} \) is an intermediate good of producer \( i \) at date \( t \), \( K_{it} \) is the capital stock of producer \( i \) at date \( t \), \( L_{it} \) is the labor of producer \( i \) at date \( t \), and \( \alpha_j \) is the output elasticity of the inputs.\(^{51}\)

A.2.2 Market Structure and Revenue

I model the market structure in the economy as monopolistic competition with heterogeneous firms, using a standard constant elasticity substitution (CES) aggregator for a final good:

\[
Y_t = (\int A_{it} Y_{it}^{\theta-1} di)^{\frac{\theta}{\theta-1}}, \quad \theta \in (1, \infty), \tag{18}
\]

where \( Y_t \) is a final good at date \( t \), \( A_{it} \) is the productivity of producer \( i \) at date \( t \), and \( \theta \) is the elasticity of substitution (e.g., Dixit and Stiglitz, 1977; Melitz, 2003; Hsieh and Klenow, 2009). I use a firm and an intermediate-good producer interchangeably.\(^{52}\) A log form of productivity, \( a_{it} \), follows an AR(1) model:

\[
a_{it} = (1 - \rho)\bar{a} + \rho a_{it-1} + \epsilon_{it}, \quad \epsilon_{it} \sim N(0, \sigma^2), \quad \sigma_a^2 = \frac{1}{1 - \rho^2}\sigma^2, \tag{19}
\]

\(^{51}\)Intermediate-good producers have a value-added production function. Their production functions consist of only two inputs—capital and labor—and no intermediate input. I assume the same production function for all intermediate-good producers. Value-added production functions are frequently used because a Leontief gross production function in an intermediate input justifies a value-added production function (e.g., Ackerberg et al., 2015). Another justification of this assumption is that GDP can be calculated by adding up the value-added part of each agent in the economy.

\(^{52}\)A final good represents the composite good for a representative household’s consumption (e.g., Melitz, 2003). I therefore describe a final-good producer’s decision from a representative household’s perspective.
where $\rho$ is a persistence parameter and $\epsilon_{it}$ is the i.i.d. innovation of firm $i$’s productivity at date $t$. A lower-case letter indicates a log form of an upper-case letter in the model if not explicitly stated otherwise. For example, $a_{it}$ is $\log A_{it}$.

Under the market structure, product-market competition is characterized as each firm experiencing a downward-sloping demand function because each firm makes a differentiated product:

$$P_{it} = A_{it} \left( \frac{Y_{it}}{Y_t} \right)^{-\frac{1}{\theta}}. \quad (19)$$

A demand function’s slope reflects the degree of competition that prevails in the product market. For example, the higher elasticity of substitution, $\theta$, means that a representative household is more sensitive to changes in prices of differentiated products, and a firm’s revenue is thus expressed as:

$$P_{it}Y_{it} = Y_t^{\frac{1}{\theta}} A_{it} K_{it}^{\hat{\alpha}_1} L_{it}^{\hat{\alpha}_2}, \quad \hat{\alpha}_j \equiv (1 - \frac{1}{\theta}) \alpha_j, \quad \hat{\alpha} \equiv \hat{\alpha}_1 + \hat{\alpha}_2. \quad (20)$$

Productivity in the model relates closely to accounting figures, such as revenue and profit. Equation (19) indicates that productivity, as a demand shifter, represents how much a representative household is willing to pay for a differentiated product. Productivity thus affects a firm’s revenue directly in equation (20). Defining productivity alternatively as physical productivity, $Y_{it} = A_{it} K_{it}^{\alpha_1} L_{it}^{\alpha_2}$, does not change the implications of the model because $A_{it}$ is the only variation across firms, which determines revenue and profit, in both specifications. Defining $A_{it}$ as the only variation across firms hinders investigating sources of productivity, such as consumer preferences and physical productivity. However, if a firm makes production decisions while maximizing profit regardless of such sources, an understanding of accounting figures is paramount during production decisions.

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53 I define $\hat{\alpha}_j$, instead of $\alpha_j$, for simplicity in subsequent sections.
54 Foster et al. (2008) discuss the difference between revenue and physical productivity.
A.3 Constant Elasticity Substitution Aggregator

A CES aggregator means that the degree to which a representative household substitutes one good for another good when responding to a change in relative prices between these goods is constant. The final good is produced by a competitive firm with perfect information. The first-order condition of a final-good producer’s problem provides a downward-sloping demand function for an intermediate-good producer:

\[ P_{it} = \frac{\theta}{\theta - 1} \left( \int A_{jt} Y_{jt}^{\theta - 1} Y_t^{1 - \theta} \right)^{\theta \over \theta - 1} A_{it} Y_{it}^{-\theta} = A_{it} (Y_{it} / Y_t)^{-\theta}, \]

where \( P_{it} \) is a relative price of intermediate good \( i \) with respect to a final good, \( Y_t \). I verify the constant elasticity of substitution, \( \theta \), using this demand function. The ratios of price and quantity of two different intermediate goods satisfy the equation:

\[ \frac{P_{jt}}{P_{it}} = \frac{A_{jt}}{A_{it}} \left( Y_{it} / Y_{jt} \right)^{1 \over \theta}. \]

I transform this equation into a log form:

\[ \log \frac{P_{jt}}{P_{it}} - \log \frac{A_{jt}}{A_{it}} - \frac{1}{\theta} \log \frac{Y_{it}}{Y_{jt}} = 0. \]

The implicit function theorem suggests that the elasticity of substitution, \( \theta \), is constant:

\[ -\frac{\partial (Y_{it} / Y_{jt})}{\partial (P_{it} / P_{jt})} / (Y_{it} / Y_{jt}) = -\frac{\partial \log (Y_{it} / Y_{jt})}{\partial \log (P_{it} / P_{jt})} = \frac{1}{\theta} = \theta. \]

A.4 Optimal Capital Investment Decisions

Equation (3) implies:

\[ \frac{L_{it}}{K_{it}} = \frac{\hat{\alpha}_2 R_t}{\hat{\alpha}_1 W_t}. \]
Using equation (21), a firm’s maximization problem is reduced to an optimal capital decision:

\[
\max_{K_{it}} Y_t^{\frac{1}{\beta}} E_{it-1}[A_{it}] K_{it}^{\hat{\alpha}_2}(\frac{\hat{\alpha}_2 R_t}{\hat{\alpha}_1 W_t})^{\hat{\alpha}_2} - (1 + \frac{\hat{\alpha}_2}{\hat{\alpha}_1})R_t K_{it}.
\]  

(22)

The first-order condition for an optimal decision problem with respect to capital is as follows:

\[
\hat{\alpha} Y_t^{\frac{1}{\beta}} E_{it-1}[A_{it}] K_{it}^{\hat{\alpha}-1}(\frac{\hat{\alpha}_2 R_t}{\hat{\alpha}_1 W_t})^{\hat{\alpha}_2} = (1 + \frac{\hat{\alpha}_2}{\hat{\alpha}_1})R_t.
\]

The sum of the capital stock of all firms is the same as the aggregate capital stock, \(K_t\):

\[
\int K_{it} \, di = \frac{1}{1-\alpha} (\hat{\alpha} Y_t^{\frac{1}{\beta}} (\frac{\hat{\alpha}_2 R_t}{\hat{\alpha}_1 W_t})^{\hat{\alpha}_2} ((1 + \frac{\hat{\alpha}_2}{\hat{\alpha}_1})R_t)^{-1})^{\frac{1}{1-\alpha}} \int E_{it-1}[A_{it}]^{\frac{1}{1-\alpha}} \, di.
\]

\[
\int \frac{K_t}{E_{it-1}[A_{it}]^{\frac{1}{1-\alpha}}} \, di = \frac{1}{1-\alpha} (\hat{\alpha} Y_t^{\frac{1}{\beta}} (\frac{\hat{\alpha}_2 R_t}{\hat{\alpha}_1 W_t})^{\hat{\alpha}_2} ((1 + \frac{\hat{\alpha}_2}{\hat{\alpha}_1})R_t)^{-1})^{\frac{1}{1-\alpha}} \int E_{it-1}[A_{it}]^{\frac{1}{1-\alpha}} \, di.
\]  

(23)

### A.5 Kalman Filter

The base model expresses a manager’s expectation process by using a state-space representation of a manager’s information structure as an aspect of productivity. The state and observation equations are expressed as follows:

\( a_{it} = (1 - \rho)\bar{a} + \rho a_{it-1} + \epsilon_{it}, \)

\( X_{it-1} = H a_{it-1} + U + \eta_{it-1}, \)

\[
X_{it} = \begin{bmatrix} a_{it}^{ef'} \\ a_{it}^{ae'} \\ s_{it} \end{bmatrix}, \quad H = \begin{bmatrix} 1 \\ 1 \\ \rho \end{bmatrix}, \quad U = \begin{bmatrix} -\frac{\sigma_{it}^2}{2} \\ -\frac{\sigma_{it}^2}{2} \\ (1-\rho)\bar{a} \end{bmatrix}, \quad \eta_{it} = \begin{bmatrix} a_{it}^{ef''} \\ a_{it}^{ae''} \\ \epsilon_{it+1} + a_{it}^s \end{bmatrix}.
\]
\[
\begin{bmatrix}
\epsilon_{it} \\
\eta_{it-1}
\end{bmatrix}
\sim N\left(\begin{bmatrix} 0 \\
0 \\
0
\end{bmatrix}, \Omega = \begin{bmatrix}
\sigma^2 & 0 & 0 \\
0 & \sigma^2_{cf} & 0 \\
0 & 0 & \sigma^2_{ae}
\end{bmatrix}\right),
\]

where \(a^{cf'}_{it} = a^{cf}_{it} + \frac{\sigma^2_{cf}}{2}\) and \(a^{ae''}_{it} = a^{ae}_{it} + \frac{\sigma^2_{ae}}{2}\). In equation (24), a manager forms an expectation of current productivity, \(a_{it}\), by observing a vector of signals, \(X_{it-1}\). This paper assumes that the error terms in \(\eta_{it}\) are uncorrelated with each other.

A manager forms a best estimate of current productivity using the Kalman filter, because it is an optimal process for predicting current productivity when the data-generating process is linear and normal (e.g., Kalman, 1960; Gourieroux and Monfort, 1997, pp. 575-585). A key mechanism is Bayes’ theorem, which is used to derive the conditional distribution of current productivity. The Kalman filter updates information and forecasts future states in three stages. First, managers form their expectations about signals, \(X_{it-1}\), based on their expectation of historical productivity.

\[
a_{it-1} | I_{it-2} \sim N\left(E_{it-2}[a_{it-1}], V_{it-2}[a_{it-1}]\right),
\]

\[
X_{it-1} | I_{it-2} \sim N\left(HE_{it-2}[a_{it-1}] + U, V_{it-2}[a_{it-1}]HH' + R\right).
\]

Second, the unexpected part of signals updates managers’ expectations of historical productivity, \(E_{it-1}[a_{it-1}]\):

\[
a_{it-1} | I_{it-1} \sim N\left(E_{it-1}[a_{it-1}], V_{it-1}[a_{it-1}]\right),
\]

\[
I_{it-1} = I_{it-2} \cup \{a^{cf}_{it-1}, a^{ae}_{it-1}, s_{it-1}\},
\]

\[
E_{it-1}[a_{it-1}] = E_{it-2}[a_{it-1}] + G_{it-1}(X_{it-1} - HE_{it-2}[a_{it-1}] - U),
\]

\[
G_{it-1} = V_{it-2}[a_{it-1}]H'(V_{it-2}[a_{it-1}]HH' + R)^{-1},
\]

\[
V_{it-1}[a_{it-1}] = (1 - G_{it-1}H)V_{it-2}[a_{it-1}].
\]
It is worthy of mentioning that managers use three pieces of information at date \( t - 1 \), and all the information available to managers at date \( t - 2 \). Other information, \( s_{it-1} \), which is a signal about \( a_{it} \), is still useful to update \( E_{it-1}[a_{it-1}] \) because, for example, a higher signal, \( s_{it-1} \), suggests that \( a_{it-1} \) was more likely to be higher due to the AR(1) process. Finally, managers form their expectations of current productivity, \( a_{it} \), using updated information and other information, \( s_{it-1} \):

\[
a_{it} | \mathcal{I}_{it-1} \sim N(E_{it-1}[a_{it}], V_{it-1}[a_{it}]),
\]

\[
E_{it-1}[a_{it}] = (1 - \rho)\pi + (\rho - SR^{-1}H)E_{it-1}[a_{it-1}] + SR^{-1}(X_{it-1} - U),
\]

\[
\begin{align*}
V_{it-1}[a_{it}] &= (\rho - SR^{-1}H)^2V_{it-1}[a_{it-1}] + \sigma^2 - SR^{-1}S' \quad \text{(Uncertainty about historical productivity)} \\
                 &= (\rho - SR^{-1}H)^2(1 - V_{it-2}[a_{it-1}]H'(V_{it-2}[a_{it-1}]HH' + R)^{-1}H)V_{it-2}[a_{it-1}] \\
                 &\quad + \sigma^2 - SR^{-1}S' \quad \text{(Uncertainty about a shock to current productivity)}
\end{align*}
\]

On the right-hand side in the above equation, \( V_{it-2}[a_{it-1}] \) represents the variance of \( a_{it-1} \) conditional on the information available at date \( t - 2 \). On the left-hand side, \( V_{it-1}[a_{it}] \) represents the variance of \( a_{it} \) conditional on the information available at date \( t - 1 \). According to Proposition 13.2 in Hamilton (1994), both \( V_{it-2}[a_{it-1}] \) and \( V_{it-1}[a_{it}] \) converge to \( \bar{V} \) as \( t \) approaches an infinity because the AR(1) process is stationary. The stationary covariance matrix satisfies the following:

\[
\begin{align*}
\bar{V} &= (\rho - SR^{-1}H)^2(\bar{V} - \bar{V}H'(\bar{V}HH' + R)^{-1}H\bar{V}) + \sigma^2 - SR^{-1}S', \\
\bar{G} &= \bar{V}H'(\bar{V}HH' + R)^{-1}.
\end{align*}
\]

I initiate this filtering by setting up \( E_{i0}[a_{i1}] \) and \( V_{i0}[a_{i1}] \). I simulate \( a_{i1} \) such that \( a_{i1} \) follows the unconditional distribution of productivity. \( E_{i0}[a_{i1}] \) is given to satisfy \( V_{i0}[a_{i1}] = \bar{V} \). Thus \( V_{it-1}[a_{it}] \) and \( G_{it-1} \) are constant as \( V_{it-1}[a_{it}] = \bar{V} \) and \( G_{it-1} = \bar{G} \).
Equation (25) defines the relation between the quality of information and the conditional variance of current productivity. The equation is rearranged as follows.

\[
V = \rho^2 \left( \frac{\sigma^2}{\sigma^2 + \sigma_s^2} \right)^2 \frac{\sigma_{ae}^2 \sigma_{cf}^2 (\sigma^2 + \sigma_s^2)}{\det(B)} + \frac{\sigma^2 \sigma_s^2}{\sigma^2 + \sigma_s^2},
\]

\[
B \equiv \bar{V} H H' + R,
\]

\[
det(B) = \bar{V} (\sigma_{ae}^2 + \sigma_{cf}^2) (\sigma^2 + \sigma_s^2) + \sigma_{ae}^2 \sigma_{cf}^2 (\sigma^2 + \sigma_s^2 + \rho^2 \bar{V}) \geq \sigma_{ae}^2 \sigma_{cf}^2 (\sigma^2 + \sigma_s^2).
\]

The implicit function theorem then implies

\[
\frac{dV}{d\sigma_{ae}^2} = \frac{\bar{V}^2 \sigma_{cf}^4 (\sigma^2 + \sigma_s^2)^2}{M} \geq 0,
\]

\[
M \equiv \left( \frac{\sigma^2 + \sigma_s^2}{\rho \sigma_s^2} \right)^2 \det(B)^2 - \sigma_{ae}^4 \sigma_{cf}^4 (\sigma^2 + \sigma_s^2)^2.
\]

In addition:

\[
\frac{d^2V}{d\sigma_{cf}^2 d\sigma_{ae}^2} = \frac{\partial}{\partial \sigma_{ae}^2} \frac{dV}{\partial \sigma_{cf}^2} + \frac{\partial}{\partial \sigma_{cf}^2} \frac{dV}{\partial \sigma_{ae}^2} \frac{dV}{\partial \sigma_{cf}^2} \geq 0,
\]

\[
\frac{\partial}{\partial \sigma_{ae}^2} \frac{dV}{\partial \sigma_{cf}^2} = \frac{\bar{V}^2 2 \sigma_{cf}^2 (\sigma^2 + \sigma_s^2)^2}{M^2} \det(B) \left( \frac{\sigma^2 + \sigma_s^2}{\rho \sigma_s^2} \right)^2 \bar{V} \sigma_{ae}^2 (\sigma^2 + \sigma_s^2) \geq 0,
\]

\[
\frac{\partial}{\partial \sigma_{cf}^2} \frac{dV}{\partial \sigma_{ae}^2} = \frac{\bar{V}^2 \sigma_{cf}^4 \sigma_{ae}^2 (\sigma^2 + \sigma_s^2)^2}{M^2} \det(B) - \sigma_{ae}^4 \sigma_{cf}^4 (\sigma^2 + \sigma_s^2)^2 \geq 0,
\]

\[
\frac{dV}{d\sigma_{cf}^2} = \frac{\bar{V}^2 \sigma_{ae}^4 (\sigma^2 + \sigma_s^2)^2}{M} \geq 0.
\]

The model expresses a manager’s expectation process with accrual reversal by using a state-space representation of the manager’s information structure as an aspect of productiv-
ity. The state and observation equations are expressed as follows:

\[ a_{it} = (1 - \rho)\bar{a} + \rho a_{it-1} + \epsilon_{it}, \]

\[ a_{it}^{c'f'} = a_{it} + a_{it}^{c'f} - a_{it-1} = a_{it} + a_{it}^{c'f''} - a_{it-1}, \]

\[ a_{it}^{ae'} = a_{it} + a_{it}^{ae} - a_{it-1} = a_{it} + a_{it}^{ae''} - a_{it-1}, \]

\[ s_{it} = a_{it+1} + a_{it}^s, \]

\[ z_{it} = H_1 z_{it-1} + U_1 + \eta_{it}, \]

\[ X_{it-1} = H_2 z_{it-1} + U_2 + \epsilon_{it-1}, \]

\[ z_{it} = \begin{bmatrix} a_{it} \\ a_{it}^{c'f''} \\ a_{it-1}^{c'f''} \\ a_{it}^{ae''} \\ a_{it-1}^{ae''} \end{bmatrix}, H_1 = \begin{bmatrix} \rho & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}, U_1 = \begin{bmatrix} (1 - \rho)\bar{a} \\ 0 \\ 0 \\ 0 \end{bmatrix}, \eta_{it} = \begin{bmatrix} \epsilon_{it} \\ a_{it}^{c'f''} \\ a_{it}^{ae''} \end{bmatrix}, \]

\[ X_{it} = \begin{bmatrix} a_{it}^{c'f'} \\ a_{it}^{ae'} \\ s_{it} \end{bmatrix}, H_2 = \begin{bmatrix} 1 & 1 & -1 & 0 & 0 \\ 1 & 0 & 0 & 1 & -1 \\ \rho & 0 & 0 & 0 & 0 \end{bmatrix}, U_2 = \begin{bmatrix} 0 \\ 0 \\ (1 - \rho)\bar{a} \end{bmatrix}, \eta_{it} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \]
\[ \begin{bmatrix} \eta_{it} \\ \upsilon_{it-1} \end{bmatrix} \sim \mathcal{N}\left( \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \mathbf{\Omega} = \begin{bmatrix} R_1_{5\times5} & S_{5\times3} \\ S'_{3\times5} & R_2_{3\times3} \end{bmatrix} \right) \]

As in the base model, a manager forms a best estimate of current productivity by using the Kalman filter. The \(5 \times 5\) stationary covariance matrix satisfies the following:

\[ \mathbf{V} = \left( \mathbf{H}_1 - \mathbf{SR}_2^{-1}\mathbf{H}_2 \right) \left( \mathbf{V} - \mathbf{V}\mathbf{H}_2^{-1}\mathbf{H}_2' + \mathbf{R}_2^{-1}\mathbf{H}_2\mathbf{V} \right) \left( \mathbf{H}_1 - \mathbf{SR}_2^{-1}\mathbf{H}_2 \right)' + \mathbf{R}_1 - \mathbf{SR}_2^{-1}\mathbf{S}' \]

A.6 Aggregate Productivity

Two integrals are simplified as follows:

\[ \begin{bmatrix} a_{it} \\ E_{it-1}[a_{it}] \end{bmatrix} \sim \mathcal{N}\left( \begin{bmatrix} \bar{a} \\ \bar{a} \end{bmatrix}, \begin{bmatrix} \sigma_a^2 & \sigma_a^2 - \mathbf{V} \\ \sigma_a^2 - \mathbf{V} & \sigma_a^2 - \bar{V} \end{bmatrix} \right) \]

\[ \log \int A_{it}(E_{it-1}[A_{it}])^{\frac{1}{1-\hat{\alpha}}} di = \log \int \exp\left(a_{it} + \frac{\hat{\alpha}}{1-\hat{\alpha}} \log E_{it-1}[A_{it}] \right) di 
\]

\[ = \log \int \exp\left(a_{it} + \frac{\hat{\alpha}}{1-\hat{\alpha}} E_{it-1}[a_{it}] + \frac{1}{2} \frac{\hat{\alpha}}{1-\hat{\alpha}} \mathbf{V} \right) di 
\]

\[ = \frac{1}{1-\hat{\alpha}} \bar{a} + \frac{1}{2} \sigma_a^2 + \frac{1}{2} \left( \frac{\hat{\alpha}}{1-\hat{\alpha}} \right)^2 \left( \sigma_a^2 - \bar{V} \right) 
\]

\[ + \frac{\hat{\alpha}}{1-\hat{\alpha}} \left( \sigma_a^2 - \bar{V} \right) + \frac{1}{2} \frac{\hat{\alpha}}{1-\hat{\alpha}} \bar{V}. \]
\[
\log \int (E_{it-1}[A_{it}])^{\frac{1}{\hat{\alpha}}} di = \log \int \exp\left(\frac{1}{1-\hat{\alpha}} \log E_{it-1}[A_{it}]\right) di \\
= \log \int \exp\left(\frac{1}{1-\hat{\alpha}} E_{it-1}[a_{it}] + \frac{1}{2 \hat{\alpha}} \epsilon d_{it}\right) di \\
= \frac{1}{1-\hat{\alpha}} \hat{\alpha} + \frac{1}{2} \left(\frac{1}{1-\hat{\alpha}}\right)^2 (\sigma_a^2 - \bar{V}) + \frac{1}{2} \frac{1}{1-\hat{\alpha}} \bar{V}
\]

A.7 Rental Rate of Capital

To characterize aggregate output and capital fully, I first characterize the rental rate of capital and the wage rate. In a steady state, the Euler equation for a representative household implies:\textsuperscript{55}

\[ 1 = \beta (1 - \delta + R). \tag{26} \]

A representative household distributes its income to aggregate consumption and aggregate investment while considering the capital rental rate, \( R \); the discount factor, \( \beta \); and the depreciation rate, \( \delta \). In a steady state, the functional form of the utility function does not affect the rental rate for capital, \( R \), because the marginal utility of consumption today will be identical tomorrow due to constant consumption.

A.8 Wage Rate

First, I derive an optimal decision problem of labor:

\[
\max_{L_{it}} Y^{\frac{1}{\hat{\alpha}}} E_{it-1}[A_{it}](\frac{\hat{\alpha}_1 W}{\hat{\alpha}_2 R})^{\hat{\alpha}_1} L_{it}^{\hat{\alpha}} - (1 + \frac{\hat{\alpha}_1}{\hat{\alpha}_2})W L_{it}.
\]

The first-order condition of the maximization problem above is as follows:

\[
\hat{\alpha} Y^{\frac{1}{\hat{\alpha}}} E_{it-1}[A_{it}](\frac{\hat{\alpha}_1 W}{\hat{\alpha}_2 R})^{\hat{\alpha}_1} L_{it}^{\hat{\alpha} - 1} = (1 + \frac{\hat{\alpha}_1}{\hat{\alpha}_2})W,
\]

\textsuperscript{55}A steady-state equilibrium means that the aggregate variables are stable, suggesting that the left-hand side of equation (26) is \( \frac{u'(C)}{u''(C)} = 1 \). The Euler equation and transversality condition are the necessary and sufficient conditions for a representative household’s optimal decisions (Acemoglu, 2009, p.212). In a steady state, the transversality condition is readily satisfied: \( \lim_{t \to \infty} \beta'(1 - \delta + R)u'(C)K = 0 \).
Equation (27) characterizes an optimal labor decision of a firm. Second, the labor-market-clearing condition implies the following:

\[
L = (\hat{\alpha}_2 Y^{\frac{1}{\hat{\alpha}_1}} E_{it-1}[A_{it}] (\hat{\alpha}_1 \hat{\alpha}_2 \hat{\alpha}_1) \hat{\alpha}_1 W^{\hat{\alpha}_1 - 1})^{\frac{1}{1-\hat{\alpha}}}. \tag{27}
\]

Equation (27) characterizes an optimal labor decision of a firm. Second, the labor-market-clearing condition implies the following:

\[
L = (\hat{\alpha}_2 Y^{\frac{1}{\hat{\alpha}_1}} E_{it-1}[A_{it}] (\hat{\alpha}_1 \hat{\alpha}_2 \hat{\alpha}_1) \hat{\alpha}_1 W^{\hat{\alpha}_1 - 1})^{\frac{1}{1-\hat{\alpha}}},
\]

where

\[
W = (\hat{\alpha}_2 (\hat{\alpha}_1 \hat{\alpha}_2 \hat{\alpha}_1) \hat{\alpha}_1 L^{\hat{\alpha} - 1})^{\frac{1}{1-\hat{\alpha}_1}} (E_{it-1}[A_{it}]^{-1} \hat{\alpha}_2 \hat{\alpha}_1 W^{\hat{\alpha}_1 - 1} Y^{\frac{1}{\hat{\alpha}_1}})^{\frac{1}{1-\hat{\alpha}_1}}
\]

The wage rate can be expressed as a function of aggregate output, given parameter values in a steady-state equilibrium:

\[
w = \frac{1}{1 - \hat{\alpha}_1} \log(\hat{\alpha}_2 (\hat{\alpha}_1 \hat{\alpha}_2 \hat{\alpha}_1) \hat{\alpha}_1 L^{\hat{\alpha} - 1}) + \frac{1}{1 - \hat{\alpha}_1} (\bar{\alpha} + \frac{1}{2} \sigma_\alpha^2 - \frac{1}{2} \chi_\alpha^2 + \frac{1}{2} \bar{V} + \frac{1}{2} \sigma_\theta^2).
\]  

In equation (28), a steady-state equilibrium wage, \( w \), relates positively to aggregate output because demand for labor determines an equilibrium wage, given that aggregate labor is supplied inelastically.\(^{56}\)

**B Estimation**

**B.1 Simulated Method of Moments**

The intuition behind the method of moments is to find a value of \( \Psi \) that minimizes the difference between empirical moments and analytical, or simulated, moments (Hansen, 1982). Estimation consists of multiple steps; Strebulaev and Whited (2012) explain the method-

\(^{56}\)Online appendices I.A and I.B of David et al. (2016) explain equation (28).
ology. First, I calculate the empirical moments, \( m(D) \), using data from Compustat and Compustat Global data. Second, I simulate data by guessing parameter values and then calculate simulated moments. I simulate \( a_{it}, a_{it}^{cf'}, a_{it}^{ae'}, \) and \( s_{it} \) for \( N \) firms over 200 periods, given that \( \Psi = \{ \rho, \sigma^2, \sigma_{cf}^2, \sigma_{ae}^2, \sigma_s^2 \} \), \( S \) times.\(^{57}\) I calculate optimal investment decisions using Kalman filtering with a steady-state-limit filter. Determining how to simulate \( k_{it} \) based on this information is important. According to available information sets for managers, including \( a_{it}^{cf'}, a_{it}^{ae'}, \) and \( s_{it} \), managers form expectations optimally as in online appendix A.5. The mean squared errors of these expectations are the same as \( \nabla \). To keep \( V_{it}[a_{it}] = \nabla \) over the entire period, I use a stationary covariance matrix for every period and every firm by setting initial expectations with \( V_{i1}[a_{i1}] = \nabla \). To mitigate the influence of initial values on estimation, I use the last three periods to calculate one cross-sectional observation of \( Z_{i|\Psi} \). I use this observation to calculate simulated moments, \( m(Z_{i|\Psi}) \). I do not specify aggregate variables to simulate productivity and investment because I mainly use a difference specification. \( \Delta a_{it}^{cf'}, \Delta a_{it}^{ae'}, \) and \( \Delta i_{it} \) depend only on \( \Psi, \theta, \alpha_1, \) and \( \alpha_2 \). Third, I choose parameter values to minimize the weighted difference between empirical moments and simulated moments:

\[
\hat{\Psi} = \arg\min_{\Psi} \Gamma(\Psi) = \arg\min_{\Psi} g(D, \Psi)' \times [\Sigma(1 + \frac{N}{NS})]^{-1} \times g(D, \Psi),
\]

where \( \Sigma \) is the covariance matrix of \( m(D) \). I use Knitro as an optimization function (e.g., Hastings and Shapiro, 2013). To avoid local minima, I use simulated annealing as an optimization function first to set up an initial value because simulated annealing is a global optimization function. An influence function is used to estimate \( \Sigma \) (e.g., Strebulaev and Whited, 2012), assuming that errors can be correlated across firms in an industry but are independent across industries. I use the Delta method to estimate the covariance matrix of

\(^{57}\) \( S \) is 10 if \( N \) is less than 10,000; \( S \) is 1 if \( N \) is greater than 10,000 to ensure that \( S \cdot N \) is greater than 10,000, which is standard (Dejong and Dave, 2011, p.296).
parameter estimates including $\nabla$. To satisfy equation (29), the SMM assumes that $\Psi$ exists.

The asymptotic distribution of $\hat{\Psi}$ follows a normal distribution:

$$\sqrt{N}(\hat{\Psi} - \Psi_0) \to N(0, \mathbb{W}),$$

$$\mathbb{W} = (1 + \frac{N}{NS})(Q\Sigma^{-1}Q')^{-1},$$

$$Q' = \frac{\partial g(D, \Psi)}{\partial\Psi}.$$

$\hat{Q}$ is numerically calculated using $\hat{\Psi}$. To examine model fit, I conduct a general test of the overidentifying restrictions of the model because I have five parameters to estimate and seven moment conditions in the base model:

$$J = \sqrt{Ng}(D, \hat{\Psi})' \times [\hat{\Sigma}(1 + \frac{N}{NS})]^{-1} \times \sqrt{Ng}(D, \hat{\Psi}) \to \chi^2(2).$$

The $J$ statistic tests the null hypothesis that parameter values exist that satisfy the moment conditions. The sensitivity of parameter estimates to moment conditions in Andrews et al. (2017) is calculated as:

$$\hat{\Lambda} = -(\hat{Q}\hat{\Sigma}^{-1}\hat{Q}')^{-1}\hat{Q}\hat{\Sigma}^{-1}$$

I compute the standardized sensitivity, shown in Figure 4, using the estimated standard deviation of parameter estimates and moment conditions following Andrews et al. (2017) and Zakolyukina (2018).

### B.2 Counterfactual Analysis

The most important step in conducting a counterfactual analysis is to estimate the size of a hypothetical informational friction. The key parameters are $\rho, \sigma, \sigma_{ae}, \sigma_{cf}$, and $\sigma_s$ in equation (8). To calculate a hypothetical informational friction, I use the estimated parameters except for $\sigma_{ae}$. For the counterfactual analysis in Table 6, I use an infinity as a value for $\sigma_{ae}$, such
that practically firms have only two informative signals, cash flows and other information. The size of the hypothetical informational friction, 0.0185, indicates that having the current form of accrual systems in addition to cash accounting reduces the size of the informational friction by 0.0016 in the United States. Using equations (9) and (11), the decrease of 0.16% in informational frictions is translated into an increase of 0.49% in aggregate productivity, or 0.16% multiplied by 3, and an increase of 0.73% in aggregate output, or 0.16% multiplied by 4.5, assuming that $\theta$ is 6 and $\alpha_1$ is one third.

C Robustness Tests

C.1 Accounting Properties and Other Information

As a robustness test, I estimate the influence of accrual accounting on resource allocation and aggregate productivity while relaxing the accounting-property assumptions. Although the accounting-property assumptions provide a natural starting point, firm characteristics and accounting practices are likely to affect these assumptions and, in turn, to influence estimation. Nikolaev (2019) explains these possibilities and elucidates how to generalize his model to incorporate them. For growing firms, an increase in productivity might relate to negative timing errors: growing firms are likely to invest more in working capital, capital, and labor (McNichols, 2000). These positive working-capital investments might generate a negative relationship between productivity shocks and timing errors because an increase in cash flows might be smaller than an increase in productivity.

Estimation errors might also correlate with productivity shocks and timing errors (e.g., Healy, 1985; Basu, 1997; Gerakos and Kovrijnykh, 2013). Accrual accounting systems might not eliminate all timing errors, resulting in a relationship between productivity shocks and estimation errors in the same direction as the relationship between productivity shocks and timing errors. Managers’ income-smoothing incentives might lead to a negative relationship between productivity shocks and estimation errors (e.g., Gerakos and Kovrijnykh, 2013).
Accrual accounting systems might underreact to productivity shocks if managers are willing to smooth accounting earnings over time; another reason for underreaction might be conservatism (e.g., Basu, 1997; Watts, 2003a,b). For contracting purposes, accrual accounting systems defer the recognition of positive news, such as positive revaluations of assets, resulting in a positive relationship between timing errors and estimation errors.

To evaluate the influence of these cases on estimation results, I use two specifications of accounting systems. First, I explore the effects on estimation of the correlation between productivity shocks and errors. The alternative specification of accounting systems is:

\[
a_{ct}^{cf'} = a_{it} + \xi_{cf} \epsilon_{it} + a_{ct}^{cf}, \quad a_{it}^{ae'} = a_{it} + \xi_{ae} \epsilon_{it} + a_{it}^{ae},
\]

where \(\xi_{cf}\) and \(\xi_{ae}\) reflect the relationship of productivity shocks with cash flows and with accounting earnings, and \(\epsilon_{it}\) represents the innovation in productivity. Table A.1, Panel A, shows that both timing and estimation errors relate negatively to productivity shocks. However, the negative relationship is smaller for accounting earnings than for cash flows in all three countries. The estimated parameter values in Table A.1, Panel A, suggest that the role of accrual accounting information is more important than in the benchmark case in equations (6) and (7).

Second, I investigate how the correlation between timing errors and estimation errors changes estimation. The alternative specification of accounting systems is:

\[
a_{ct}^{cf'} = a_{it} + a_{it}^{ce} + a_{ct}^{cf}, \quad a_{it}^{ae'} = a_{it} + a_{it}^{ce} + a_{it}^{ae},
\]

where \(a_{it}^{ce}\) expresses common errors in cash flows and accounting earnings that are unrelated to productivity shocks. Table A.1, Panel B, shows that the magnitude of common errors in cash flows and accounting earnings is small. Estimations are robust to this alternative

---

58 Table A.1 uses four additional moments: \(\text{corr}(a_{it}^{cf'}, a_{it-2}^{cf})\), \(\text{corr}(a_{it}^{ae'}, a_{it-2}^{ae})\), \(\text{cov}(\Delta a_{it}^{cf}, \Delta a_{it-1}^{ae})\), and \(\text{cov}(\Delta a_{it}^{ae'}, \Delta a_{it-1}^{cf})\). Table A.1 estimates parameters by searching for limited sets of reasonable parameter values: \(\xi_{cf}\) and \(\xi_{ae}\) are greater than -0.5 and less than 0.5.
Finally, I estimate the influence of accrual accounting on resource allocation and aggregate productivity while relaxing the independent error-term assumption. I add estimation errors to other information because stock prices are part of other information and incorporate information about accounting earnings.

\[ s_{it} = a_{it+1} + a_{it} + 0.5\rho a_{it}^{ae}. \]

The estimates in Table A.1, Panel C, indicate that the importance of other information has slightly increased: the high correlation between accounting earnings and capital investment might imply the importance of other information, due to a common error in both information sources for firms’ production decisions, instead of simply more noise in other information. The impact of accrual accounting on aggregate productivity has been reduced from 0.49% to 0.36% in the United States, for example, but the estimates are still economically significant.

### C.2 Elasticity of Substitution and Value Added

I evaluate the sensitivity of estimation to the elasticity of substitution, \( \theta \), and to a measure of value added. I use \( \theta \) both to calculate productivity and to conduct counterfactual analyses, with common values of this parameter ranging from 3 to 10 in the literature. Table A.2, Panels A and B, show that the estimated parameter values are robust to values of \( \theta \) in all three countries. However, the estimated effect of accrual accounting information on aggregate productivity and output increases as the elasticity of substitution increases, because of the competition effect. This result relates closely to equations (9) and (11), the equilibrium results in the model. The influence of accrual accounting information on aggregate productivity ranges from 0.3% to 3.2% in the United States, China, and India.

Alternatively, the value added for accounting earnings can be calculated as 50% of sales to exclude costs of intermediate inputs from sales, following David et al. (2016).
A.2, Panel C, suggests that the influence of accrual accounting information on aggregate productivity ranges from 0.7% to 2.0% in the United States, China, and India.\footnote{Considering a difference in the ratio of value added to sales across industries does not notably change the empirical results in the United States.}

C.3 Different Industry Compositions and Time Periods

I evaluate the effect of industry composition and time periods on cross-country results. Table A.3 shows that concentrating only on manufacturing firms and averaging estimated parameter values over 10 years slightly reduces the difference in information structures among countries. The quality of accrual accounting is still highest in the United States, but the estimated influence of accrual accounting on aggregate productivity is larger in China and India than in the United States. Overall, estimates for accrual accounting systems across countries are unlikely to be driven solely by disparate industry composition and a sample period.

C.4 Industry Analysis

I conduct an industry analysis to validate the estimation method. Extant studies demonstrate that both cash flows and accounting earnings are less accurate as measures of firm performance when an operating cycle is long (e.g., Dechow, 1994; Dechow and Dichev, 2002).\footnote{Operating cycle_{it} = \frac{(AR_{it}+AR_{i,t-1})/2}{Sales_{it}/360} + \frac{(Inv_{it}+Inv_{i,t-1})/2}{COGS_{it}/360}, where AR_{it} is accounts receivable and Inv_{it} is inventory.} A long cycle implies that cash collections and payments are more likely to be misaligned with the timing of business transactions, and makes matching the benefits and costs of business transactions difficult for accrual accounting systems. Estimates of the size of timing and estimation errors in Figure A.1, Panels A and B, are consistent with extant findings. Using the same identification assumptions, Nikolaev (2019) also documents that the size of timing and estimation errors increases at the firm level as an inventory cycle becomes longer.\footnote{Nikolaev also finds that volatile stock prices are positively correlated with the size of timing and estimation errors.}
tend this argument by evaluating whether the importance of accruals increases or decreases with an operating cycle. Figure A.1, Panel C, shows that accrual accounting systems reduce informational frictions, $\bar{V}$, more in an industry with a longer operating cycle; this signifies that large timing errors in cash flows elevate the role of accounting earnings even though accounting earnings also exhibit large estimation errors. This result accords with Dechow et al. (1998); accruals are superior to cash flows in predicting future cash flows, especially during a long operating cycle.
Table A.1: Different Specifications of Information Properties

This table shows how estimates of the influence of accrual accounting on aggregate productivity and output change under different specifications of information properties. In Panel A, \( a_{it}^{cf} = a_{it} + \xi_{it}^{cf} \epsilon_{it} + a_{it}^{cf} \) and \( a_{it}^{ae} = a_{it} + \xi_{it}^{ae} \epsilon_{it} + a_{it}^{ae} \). \( \xi_{it}^{cf} \) and \( \xi_{it}^{ae} \) specify the relation of productivity shocks with cash flows and accounting earnings. In Panel B, \( a_{it}^{cf} = a_{it} + a_{it}^{ce} + a_{it}^{cf} \) and \( a_{it}^{ae} = a_{it} + a_{it}^{ce} + a_{it}^{ae} \). \( a_{it}^{ce} \) represents common errors in cash flows and accounting earnings that are unrelated to productivity shocks. In Panel C, \( s_{it} = a_{it+1} + a_{it}^{s} + 0.5 \rho a_{it}^{ae} \). Productivity, \( a_{it} \), follows an AR(1) model: \( a_{it} = (1 - \rho) a_{it-1} + \rho a_{it} + \epsilon_{it} \). \( \rho \) is the persistence of productivity. \( \sigma^2 \) is the volatility of innovation in productivity. \( s_{it} \) is other information about productivity: \( s_{it} = a_{it+1} + a_{it}^{s} \). \( \sigma_{cf}^2, \sigma_{ae}^2, \) and \( \sigma_{s}^2 \) are the variance of noise in cash flows, accounting earnings, and other information, respectively. \( \nabla \) is a summary measure of informational frictions. To estimate the influence of accrual accounting systems on aggregate productivity and output, I first calculate a hypothetical conditional variance of current productivity, \( \tilde{\nabla} \), based on a counterfactual value of the quality of accounting earnings and the estimated values of the other parameters. Second, I use the difference between \( \nabla \) and \( \tilde{\nabla} \) to exploit the following equations: \( \frac{da}{d\nabla} = -\frac{1}{2} \theta \) and \( \frac{dy}{d\nabla} = -\frac{1}{2} \theta + \frac{1}{1-a_1} \). \( a \) is the aggregate productivity. \( y \) is the aggregate output. The sample firms are public firms in the United States, China, and India in 2012. I demean variables controlling for a year fixed effect. I exclude the top and bottom 2% extreme observations for variables.

<table>
<thead>
<tr>
<th></th>
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<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>US</td>
<td>0.0801</td>
<td>0.0296</td>
<td>-0.1728</td>
<td>-0.1575</td>
<td>-0.0027</td>
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<tr>
<td></td>
<td>China</td>
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<td>-0.1698</td>
<td>-0.0013</td>
</tr>
<tr>
<td></td>
<td>India</td>
<td>0.1754</td>
<td>0.0605</td>
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<td>-0.0527</td>
<td>-0.0070</td>
</tr>
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</table>

<table>
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<tr>
<th>Panel B: Common Error</th>
<th>Country</th>
<th>Volatility of timing error</th>
<th>Volatility of estimation error</th>
<th>Volatility of common error</th>
<th>Impact on info. frictions</th>
<th>Impact on agg. productivity</th>
</tr>
</thead>
<tbody>
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<td>0.0158</td>
<td>0.0000</td>
<td>-0.0016</td>
<td>-0.0016</td>
</tr>
<tr>
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<td>0.0603</td>
<td>0.0001</td>
<td>-0.0074</td>
<td>-0.0074</td>
</tr>
<tr>
<td></td>
<td>India</td>
<td>0.1714</td>
<td>0.0806</td>
<td>0.0007</td>
<td>-0.0046</td>
<td>-0.0046</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Panel C: Other Information</th>
<th>Country</th>
<th>Volatility of timing error</th>
<th>Volatility of estimation error</th>
<th>Volatility of noise in other info.</th>
<th>Impact of info. frictions</th>
<th>Impact on agg. productivity</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.2189</td>
<td>-0.0012</td>
<td>-0.0012</td>
</tr>
<tr>
<td></td>
<td>China</td>
<td>0.2088</td>
<td>0.0703</td>
<td>0.3461</td>
<td>-0.0063</td>
<td>-0.0063</td>
</tr>
<tr>
<td></td>
<td>India</td>
<td>0.1848</td>
<td>0.0853</td>
<td>0.3195</td>
<td>-0.0034</td>
<td>-0.0034</td>
</tr>
</tbody>
</table>
Table A.2: Elasticity of Substitution and Value Added

This table shows sensitivity tests with respect to the elasticity of substitution and a measure of value added. In Panels A and B, the elasticity of substitution, $\theta$, is 4 and 8 respectively. In Panel C, the value added for accounting earnings, $VA_{it}$, is calculated as 50% of sales. $\sigma_{cf}^2$ and $\sigma_{ae}^2$ are cash-flow-based and accounting-earnings-based productivity. $\sigma_{cf}$ and $\sigma_{ae}$ are the standard deviations of noise in cash flows and accounting earnings. Productivity, $a_{it}$, follows an AR(1) model: $a_{it} = (1 - \rho)\bar{a} + \rho a_{it-1} + \epsilon_{it}$. $\rho$ is the persistence of productivity. $\sigma^2$ is the volatility of innovation in productivity. Cash flows and accounting earnings are transformed into imperfect measures of productivity: $a_{it}^{cf} + \text{Constant} = va_{it}^{cf} - \hat{\alpha}_k$ and $a_{it}^{ae} + \text{Constant} = va_{it}^{ae} - \hat{\alpha}_k$. $s_{it}$ is other information about productivity: $s_{it} = a_{it+1} + a_{it}^s$. $\sigma_s^2$ is the variance of noise in other information respectively. $\overline{V}$ is a summary measure of informational frictions. To estimate the influence of accrual accounting systems on aggregate productivity and output, I first calculate a hypothetical conditional variance of current productivity, $\hat{V}$, based on a counterfactual value of the quality of accounting earnings and the estimated values of the other parameters. Second, I use the difference between $\overline{V}$ and $\hat{V}$ to exploit the following equations: $\frac{da}{dV} = \frac{-1}{2} \theta$ and $\frac{dy}{d\theta} = -\frac{1}{2} \frac{1}{1-\alpha}$. $a$ is the aggregate productivity.

The sample firms are public firms in the United States, China, and India in 2012. I demean variables controlling for a year fixed effect. I exclude the top and bottom 2% extreme observations for variables.

<table>
<thead>
<tr>
<th>Country</th>
<th>Volatility of timing error</th>
<th>Volatility of estimation error</th>
<th>Impact on info. frictions</th>
<th>Impact on agg. productivity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma_{cf}$</td>
<td>$\sigma_{ae}$</td>
<td>$\Delta \bar{V}$</td>
<td>$\Delta \bar{a}$</td>
</tr>
<tr>
<td>US</td>
<td>0.0863</td>
<td>0.0432</td>
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<td>0.30%</td>
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<tr>
<td>China</td>
<td>0.2090</td>
<td>0.0714</td>
<td>-0.0070</td>
<td>1.40%</td>
</tr>
<tr>
<td>India</td>
<td>0.1854</td>
<td>0.0846</td>
<td>-0.0048</td>
<td>0.97%</td>
</tr>
</tbody>
</table>

Panel B: $\theta = 8$

<table>
<thead>
<tr>
<th>Country</th>
<th>Volatility of timing error</th>
<th>Volatility of estimation error</th>
<th>Impact on info. frictions</th>
<th>Impact on agg. productivity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma_{cf}$</td>
<td>$\sigma_{ae}$</td>
<td>$\Delta \bar{V}$</td>
<td>$\Delta \bar{a}$</td>
</tr>
<tr>
<td>US</td>
<td>0.0864</td>
<td>0.0449</td>
<td>-0.0017</td>
<td>0.66%</td>
</tr>
<tr>
<td>China</td>
<td>0.2095</td>
<td>0.0738</td>
<td>-0.0081</td>
<td>3.24%</td>
</tr>
<tr>
<td>India</td>
<td>0.1845</td>
<td>0.0815</td>
<td>-0.0057</td>
<td>2.28%</td>
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</table>

Panel C: Value Added

<table>
<thead>
<tr>
<th>Country</th>
<th>Volatility of timing error</th>
<th>Volatility of estimation error</th>
<th>Impact on info. frictions</th>
<th>Impact on agg. productivity</th>
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<tbody>
<tr>
<td></td>
<td>$\sigma_{cf}$</td>
<td>$\sigma_{ae}$</td>
<td>$\Delta \bar{V}$</td>
<td>$\Delta \bar{a}$</td>
</tr>
<tr>
<td>US</td>
<td>0.1053</td>
<td>0.0426</td>
<td>-0.0023</td>
<td>0.69%</td>
</tr>
<tr>
<td>China</td>
<td>0.2094</td>
<td>0.0626</td>
<td>-0.0067</td>
<td>2.02%</td>
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<tr>
<td>India</td>
<td>0.2022</td>
<td>0.0686</td>
<td>-0.0066</td>
<td>1.98%</td>
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</table>
This table shows sensitivity tests with respect to different industry compositions, time periods, and investment horizons. In Panel A, the sample firms are public manufacturing firms in the United States, China, and India in 2012. In Panel B, the sample firms are public firms in the United States, China, and India from 2003 to 2012. In Panel C, I use public firms in the United States, China, and India in 2012 as the sample firms and assume the investment horizon as three years. $a_{it}^{cf}$ and $a_{it}^{ae}$ are cash-flow-based and accounting-earnings-based productivity. Productivity, $a_{it}$, follows an AR(1) model: $a_{it} = (1 - \rho)\bar{a} + \rho a_{it-1} + \epsilon_{it}$. $\rho$ is the persistence of productivity. $\sigma^2$ is the volatility of innovation in productivity. $s_{it}$ is other information about productivity: $s_{it} = a_{it+1} + a_{it}^s$. $\sigma^2_{cf}$, $\sigma^2_{ae}$, and $\sigma^2_s$ are the variance of noise in cash flows, accounting earnings, and other information respectively. $\overline{V}$ is a summary measure of informational frictions. To estimate the influence of accrual accounting systems on aggregate productivity and output, I first calculate a hypothetical conditional variance of current productivity, $\tilde{V}$, based on a counterfactual value of the quality of accounting earnings and the estimated values of the other parameters. Second, I use the difference between $\overline{V}$ and $\tilde{V}$ to exploit the following equations: $\frac{da}{d\overline{V}} = -\frac{1}{2} \theta$ and $\frac{dy}{d\overline{V}} = -\frac{1}{2} \theta \frac{1}{1-\alpha}$. $a$ is the aggregate productivity. $y$ is the aggregate output. I demean variables controlling for a year fixed effect. I exclude the top and bottom 2% extreme observations for variables.

### Panel A: Manufacturing Industry

<table>
<thead>
<tr>
<th>Country</th>
<th>Volatility of timing error</th>
<th>Volatility of estimation error</th>
<th>Impact on info. frictions</th>
<th>Impact on agg. productivity</th>
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<tbody>
<tr>
<td></td>
<td>$\sigma_{cf}$</td>
<td>$\sigma_{ae}$</td>
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<td>$\Delta \overline{V}$</td>
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<tr>
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<tr>
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<td>0.2112</td>
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<td>0.0355</td>
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<tr>
<td>India</td>
<td>0.1721</td>
<td>0.0859</td>
<td>0.0351</td>
<td>-0.0042</td>
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### Panel B: 2003-2012

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<tr>
<th>Country</th>
<th>Volatility of timing error</th>
<th>Volatility of estimation error</th>
<th>Impact on info. frictions</th>
<th>Impact on agg. productivity</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma_{cf}$</td>
<td>$\sigma_{ae}$</td>
<td>$\overline{V}$</td>
<td>$\Delta \overline{V}$</td>
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<tr>
<td>India</td>
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### Panel C: Three-year Investment Horizon

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<th>Country</th>
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<th>Volatility of estimation error</th>
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<td>0.0509</td>
<td>-0.0014</td>
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<tr>
<td>China</td>
<td>0.1260</td>
<td>0.0933</td>
<td>0.0897</td>
<td>-0.0036</td>
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<tr>
<td>India</td>
<td>0.1283</td>
<td>0.0727</td>
<td>0.0752</td>
<td>-0.0037</td>
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Figure A.1: Accrual Accounting and Operating Cycle
This figure illustrates the relation between the role of accrual accounting and the length of the operating cycle. Panel A illustrates the relation between the standard deviation of timing errors, $\sigma_{cf}$, and the length of the operating cycle across industries. Panel B illustrates the relation between the standard deviation of estimation errors, $\sigma_{ae}$, and the length of the operating cycle across industries. Panel C illustrates the relation between the influence of accrual accounting on informational frictions, $\nabla$, and the length of the operating cycle across industries. The sample firms are public firms in the United States from 1988 to 2012. I demean variables controlling for a year fixed effect. I exclude the top and bottom 2% extreme observations for variables. I use Fama-French 48 industry classification.
References


