Screening Talent for Task Assignment: Absolute or Percentile Thresholds?

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Online Appendix
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**Lemma A1** Suppose the principal must fill the tasks. Under APTS,

(i) the agent’s (equilibrium) manipulation level is non-increasing as the principal increases the threshold \( x_0 \) provided \( \frac{\partial F(x_0; t, c_t^A)}{\partial x} > 0 \) and \( \frac{\partial^2 F(x_0; t, c_t^A)}{\partial c_t \partial x} \geq 0 \);

(ii) the optimal threshold \( x_0^* \) equates the marginal effect on the talented agent’s likelihood of receiving a high signal with that for the untalented type; and

(iii) moreover, performance manipulation by the talented (untalented) agent increases (decreases) the principal’s expected payoff.

**Proof of Lemma A1.** Part (i). The optimal manipulation level is described by the following first-order condition:

\[
- \left[ \frac{W_{t1} - W_{t2}}{2} \right] \left[ \frac{\partial F(x_0; t, c_t^A)}{\partial c_t} \right] - c_t^A = 0. \tag{B-1}
\]

We can then derive \( \frac{dc_t^A}{dx_0} \) as

\[
\frac{dc_t^A}{dx_0} = - \frac{1}{2} \frac{W_{t1} - W_{t2}}{W_{t1} - W_{t2}} \frac{\partial^2 F(x_0; t, c_t^A)}{(\partial c_t)^2} + 1 \leq 0. \tag{B-2}
\]

The inequality in (B-2) is ensured by (a) \( W_{t1} > W_{t2} \); (b) Assumption 5; and (c) \( \frac{\partial^2 F(x_0; t, c_t^A)}{\partial c_t \partial x} \geq 0 \).

Part (ii). The principal chooses \( x_0 \) to minimize her expected loss. If both agents are either talented or untalented, her expected loss is \( \lambda^2 \Upsilon_T \) or \((1 - \lambda)^2 \Upsilon_U \). Therefore, the choice of \( x_0 \) has no effect on the expected loss. If the two agents have different types, the principal incurs a loss whenever the talented agent produces a low signal and the untalented agent produces a high signal. In this case, the principal’s expected loss is

\[
P_E L (x_0, c_t^A) = \lambda (1 - \lambda) \left[ p_{UH} (c_t^A) + 1 - p_{TH} (c_t^A) \right] \left[ \Upsilon_T + \Upsilon_U \right], \tag{B-3}
\]
where \( c^A_T \) and \( c^A_U \) are the principal’s conjectures of the agents’ manipulation levels which will be realized in equilibrium. Differentiating (B-3) and setting it to zero, the optimal threshold level \( x_0^{**} \) solves:

\[
\frac{-\partial F(x_0^{**}; U, c^A_T)}{\partial x_0} - \frac{\partial F(x_0^{**}; U, c^A_U)}{\partial c_t} \frac{dc^A_t}{dx_0} + \frac{\partial F(x_0^{**}; T, c^A_T)}{\partial x_0} + \frac{\partial F(x_0^{**}; T, c^A_U)}{\partial c_t} \frac{dc^A_T}{dx_0} = 0. \tag{B-4}
\]

With \( c^A_t > 0 \), the choice of threshold not only affects the likelihood of the agent producing a high signal, but also affects his incentive to manipulate. The first and the third terms of (B-4) are the (direct) effects of \( x_0 \) given the untalented and the talented agents’ manipulation levels; whereas the second and the fourth terms are the effects of \( x_0 \) on the agents’ manipulation that (indirectly) affects the agents’ likelihood of producing a high signal. The threshold \( x_0^{**} \) equates its marginal effect for the talented agent with that for the untalented agent. Given Assumption 5, the assumptions \( \frac{\partial F(x_0; c^A_T)}{\partial x_0} > 0 \) and \( \frac{dc^A_t}{dx_0} \leq 0 \) (as in (B-2)), the first two terms in (B-4) are negative and the last two terms are positive, guaranteeing the existence of an interior solution for \( x_0^{**} \).

Part (iii). To see how the choice of \( c^A_t \) affects the principal’s expected payoff, evaluating (B-3) at \( x_0^{**} \) and taking the derivative with respect to \( c^A_t \) yield

\[
\text{Sign} \left( \frac{\partial PEL(x_0^{**}, c^A_t)}{\partial c^A_T} \right) = \text{Sign} \left( -\frac{\partial PEL(x_0^{**}, c^A_t)}{\partial c^A_T} \right) = \text{Sign} \left( \frac{\partial F(x_0^{**}; T, c^A_T)}{\partial c_t} \right) < 0, \tag{B-5}
\]

and,

\[
\text{Sign} \left( \frac{\partial PEL(x_0^{**}, c^A_t)}{\partial c^A_U} \right) = \text{Sign} \left( \frac{\partial PEL(x_0^{**}, c^A_t)}{\partial c^A_U} \right) = \text{Sign} \left( -\frac{\partial F(x_0^{**}; U, c^A_U)}{\partial c_U} \right) > 0. \tag{B-6}
\]

The inequality (B-5) states that \( c^A_T \) decreases expected loss and thus increases the principal’s net payoff. The inequality (B-6) states that \( c^A_U \) increases expected loss and thus decreases
Proposition A1  Under APTS, the untalented agent engages in a higher level of performance manipulation than does the talented agent when (i) $\Gamma_1 \leq \Gamma_1^*$ and $\Gamma_2 > \Gamma_2^*$; or (ii) $\Gamma_1 > \Gamma_1^*$ and $\Gamma_2 \leq \Gamma_2^*$.

Proof of Proposition A1. Part (i). Consider the case in which $\Gamma_1 \leq \Gamma_1^*$ and $\Gamma_2 > \Gamma_2^*$ so that the principal prefers flexible (full) staffing when she observes $\{L, L\}$ ($\{H, H\}$). For a given $x_0$, the type $t$ agent conjectures about his peer’s manipulation levels, $\hat{c}_T^A$ and $\hat{c}_U^A$, and chooses $c_t^A$ to maximize his net expected payoff:

$$
\max_{c_t} U_t^A = p_{TH} \left( c_t^A \right) \left[ \lambda p_{TH} \left( \hat{c}_T^A \right) + (1 - \lambda) p_{UH} \left( \hat{c}_U^A \right) \right] \frac{W_{t1} + W_{t2}}{2} \\
+ \left[ 1 - p_{TH} \left( c_t^A \right) \right] \left\{ \lambda \left[ 1 - p_{TH} \left( \hat{c}_T^A \right) \right] + (1 - \lambda) \left[ 1 - p_{UH} \left( \hat{c}_U^A \right) \right] \right\} W_{t2} \\
+ p_{TH} \left( c_t^A \right) \left\{ \lambda \left[ 1 - p_{TH} \left( \hat{c}_T^A \right) \right] + (1 - \lambda) \left[ 1 - p_{UH} \left( \hat{c}_U^A \right) \right] \right\} W_{t1} \\
+ \left[ 1 - p_{TH} \left( c_t^A \right) \right] \left\{ \lambda p_{TH} \left( \hat{c}_T^A \right) + (1 - \lambda) p_{UH} \left( \hat{c}_U^A \right) \right\} W_{t1} - \frac{(c_t^A)^2}{2}
$$

\(\Rightarrow\)

$$
\max_{c_t} U_t^A = p_{TH} \left( c_t^A \right) \left[ \lambda p_{TH} \left( \hat{c}_T^A \right) + (1 - \lambda) p_{UH} \left( \hat{c}_U^A \right) \right] \frac{W_{t1} + W_{t2}}{2} \\
+ \left[ 1 - p_{TH} \left( c_t^A \right) \right] W_{t2} \\
+ p_{TH} \left( c_t^A \right) \left\{ \lambda \left[ 1 - p_{TH} \left( \hat{c}_T^A \right) \right] + (1 - \lambda) \left[ 1 - p_{UH} \left( \hat{c}_U^A \right) \right] \right\} W_{t1} \\
- \frac{(c_t^A)^2}{2}.
$$

(D-7)

Differentiating (D-7) with respect to $c_t$ and setting it to zero while taking $\hat{c}_T^A$ and $\hat{c}_U^A$ as given yields

$$
\frac{\partial U_t^A}{\partial c_t} = \frac{\partial p_{TH} \left( c_t^A \right)}{\partial c_t} \left\{ \frac{1}{2} \left[ \lambda p_{TH} \left( \hat{c}_T^A \right) + (1 - \lambda) p_{UH} \left( \hat{c}_U^A \right) \right] \right\} W_{t1} \\
+ \frac{\partial p_{TH} \left( c_t^A \right)}{\partial c_t} \left\{ \frac{\lambda p_{TH} \left( \hat{c}_T^A \right) + (1 - \lambda) p_{UH} \left( \hat{c}_U^A \right)}{2} - 1 \right\} W_{t2} - c_t^A = 0
$$
\[\Leftrightarrow \left\{ \frac{1 + \lambda [1 - p_{TH} (\hat{c}_T^A)] + (1 - \lambda) [1 - p_{UU} (\hat{c}_U^A)]}{2} \right\} [W_{t1} - W_{t2}] \frac{\partial p_{TH} (c_t^A)}{\partial c_t} - c_t^A = 0. \]

(B-8)

By definition \( p_{TH} (c_t^A) = p_{TH} (c_t^A; x_0) = 1 - F (x_0; t, c_t^A) \), (B-8) can be written as

\[-\Phi [W_{t1} - W_{t2}] \left[ \frac{\partial F (x_0; t, c_t^A)}{\partial c_t} \right] - c_t^A = 0; \text{ where} \]

\[\Phi = \frac{1 + \lambda [1 - p_{TH} (\hat{c}_T^A)] + (1 - \lambda) [1 - p_{UU} (\hat{c}_U^A)]}{2} > 0. \]

Assumption 4 \( (\frac{\partial^2 F(x_0; t, c_t^A)}{\partial t^2} > 0) \) ensures that the objective function \( U_t^A \) is concave and therefore the optimal level of \( c_t \) is determined by (B-9).

To compare \( c_T^A \) and \( c_U^A \), we write

\[0 = -\Phi [W_{t1} - W_{t2}] \left[ \frac{\partial F (x_0; T, c_T^A)}{\partial c_t} \right] - c_T^A \]

\[\leq -\Phi [W_{U1} - W_{U2}] \left[ \frac{\partial F (x_0; U, c_T^A)}{\partial c_t} \right] - c_T^A. \]  

(B-10)

The inequality in (B-10) is supported by Assumptions 3, 4 and 5. Replacing the left-hand-side of the inequality with the expression (B-9) setting \( t = U \) yields

\[-\Phi [W_{U1} - W_{U2}] \left[ \frac{\partial F (x_0; U, c_U^A)}{\partial c_t} \right] - c_U^A = 0 \]

\[\leq -\Phi [W_{U1} - W_{U2}] \left[ \frac{\partial F (x_0; U, c_T^A)}{\partial c_t} \right] - c_T^A \]

\[\Rightarrow \ c_T^A < c_U^A. \]  

(B-11)

The second inequality in (B-11) is implied as the agent’s objective function is concave in \( c_t \).

Part (ii). Consider the case in which \( \Gamma_1 > \Gamma_1^* \) and \( \Gamma_2 < \Gamma_2^* \) so that the principal prefers
full (flexible) staffing when she observes \( \{L, L\} (\{H, H\}) \). For a given \( x_0 \), the type \( t \) agent conjectures about his peer’s manipulation actions, \( ^c_A T \) and \( ^c_A U \), and chooses his own action \( c^A_t \) to maximize the net expected payoff:

\[
\begin{align*}
\text{Max} U^A_t &= p_{tH} \left( c^A_t \right) \left[ \lambda p_{TH} \left( ^c_A T \right) + (1 - \lambda) p_{UH} \left( ^c_A U \right) \right] W_{1t} \\
&+ \left[ 1 - p_{tH} \left( c^A_t \right) \right] \left\{ \lambda \left[ 1 - p_{TH} \left( ^c_A T \right) \right] + (1 - \lambda) \left[ 1 - p_{UH} \left( ^c_A U \right) \right] \right\} \frac{W_{1t} + W_{2t}}{2} \\
+ p_{tH} \left( c^A_t \right) \left\{ \lambda \left[ 1 - p_{TH} \left( ^c_A T \right) \right] + (1 - \lambda) \left[ 1 - p_{UH} \left( ^c_A U \right) \right] \right\} W_{1t} \\
&+ \left[ 1 - p_{tH} \left( c^A_t \right) \right] \left[ \lambda p_{TH} \left( ^c_A T \right) + (1 - \lambda) p_{UH} \left( ^c_A U \right) \right] W_{2t} - \frac{(c^A_t)^2}{2}.
\end{align*}
\]

\[ (B-12) \]

Differentiating (B-12) with respect to \( c_t \) and setting it to zero while taking \( ^c_A T \) and \( ^c_A U \) as given yields

\[
\frac{\partial U^A_t}{\partial c_t} = \frac{\partial p_{tH} \left( c^A_t \right)}{\partial c_t} W_{1t} - \frac{\partial p_{tH} \left( c^A_t \right)}{\partial c_t} \left[ 1 + \lambda p_{TH} \left( ^c_A T \right) + (1 - \lambda) p_{UH} \left( ^c_A U \right) \right] W_{1t} + \frac{W_{1t} + W_{2t}}{2} \\
- \frac{\partial p_{tH} \left( c^A_t \right)}{\partial c_t} \left\{ \lambda \left[ 1 - p_{TH} \left( ^c_A T \right) \right] + (1 - \lambda) \left[ 1 - p_{UH} \left( ^c_A U \right) \right] \right\} \frac{W_{1t} + W_{2t}}{2} \\
- c^A_t = 0
\]

\[ (B-13) \]
By definition $p_{tH} (c_t^A) = p_{tH} (c_t^A; x_0) = 1 - F (x_0; t, c_t^A)$, (B-13) can be written as

$$\Psi [W_{t1} - W_{t2}] \left[ \frac{\partial F (x_0; t, c_t^A)}{\partial c_t} \right] - c_t^A = 0; \text{ where}$$

$\Psi = 1 - \frac{1}{2} \{ \lambda [1 - p_{RH} (\hat{c}_t^A)] + (1 - \lambda) [1 - p_{UH} (\hat{c}_U^A)] \} > 0.$

Assumption 4 ($\frac{\partial^2 F (x_0; t, c_t^A)}{\partial c_t^2} > 0$) ensures that the objective function $U_t^A$ is concave and therefore the optimal level of $c_t$ is determined by (B-14).

To compare $c_t^A$ and $c_U^A$, we apply the parallel analysis as the proof of Part (i).