Working Paper No. 98

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Working Paper Series, No. 98

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August 1994

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On the Endogenous Determination of Time Preference

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Revised, July 20 1994

Abstract

We model a consumer's efforts to overcome an endowed bias against the future, showing how wealth, mortality, addictions, uncertainty and other variables affect one's ultimate degree of time preference. In addition to working out the implications of our model, we provide micro evidence from the PSID. A PSID family's consumption growth is found to be positively correlated with its own income, but even more closely related to its head's parent's income.

JEL classification number D91. Keywords: time preference. savings. preferences.

We received useful comments from Robert Barro, In-Koo Cho, John Cochrane, Jon Elster, Lars Hansen, Kenneth Judd, Michael Kremer, George Loewenstein, Robert Lucas, Patricia Macellaio, Derek Neal, Sherwin Rosen, Xavier Sala-i-Martin, Nancy Stokey, and Yoram Weiss. This research was supported by a National Science Foundation Graduate Fellowship (Mulligan), National Science Foundation Grant #SES-9010748 (Becker), National Institutes of Child Health and Development Grant #HD-22054-09 (Becker), and by the Lynde and Harry Bradley Foundation and the Sarah Scaife Foundation through the Center for the Study of the Economy and the State at the University of Chicago (Becker).

Time preference plays a fundamental role in theories of savings and investment, economic growth, interest rate determination and asset pricing, addiction, and many other issues that are getting increasing attention from economists. Yet, since Samuelson's (1937) discounted utility model, rates of time preference are almost invariably taken as "given" or exogenous, with little discussion of what determines their level. Uzawa (1968), Lucas and Stokey (1984) and Epstein (1987) postulate a correlation between time preference and consumption flows while Becker and Barro (1988) suppose that a parent's generational discount rate is related to her fertility, yet none of these studies discuss the forces that might lead to a relationship between time preference and other economic variables. A few exceptions are found in the literature on generational discount rates (or degrees of intergenerational altruism) such as Montgomery (1992) and Mulligan (1993) - although these studies have little say about life cycle discount rates - and in the sociobiological literature (eg., Rogers (1993)).

Section I defines our concept of "time preference" and reviews the discussion of time preference by some classical economists. Section II writes down a model of patience formation that combines the classical economists' insights with a particular view of what it means to be rational, a conception of rationality that is consistent with many kinds of human frailties, including defective recognition of future utilities. Rational persons may spend resources in the attempt to overcome their frailties. This simple idea provides the point of departure for our approach to endogenizing time preference. Even rational people may "excessively" discount future utilities, but we assume that they may partially or fully offset this by spending effort and goods to reduce the degree of overdiscounting.

We show in section III how income, mortality, addictions, uncertainty and other variables affect the time and goods spent to overcome a bias against the future. We also derive some implications of the model for the dynamics of inequality, interest rate determination, and choice under uncertainty. Some evidence for one of the more important implications - that wealth leads to patience - is provided in section IV, where we show that a son's adult consumption growth is increasing in his parents' wealth - indicating some intergenerational transmission of patience. Section V concludes, suggesting how our results would generalize to more complicated utility functions and more realistic ways of endogenizing time preference.

I. Time Preference: Definitions and Historical Background

We define the rate of time preference according to the marginal rate of substitution between current and future *utilities*, a definition advocated by Uzawa

(1968). We will interchangeably use the words "rate of time preference," "impatience," and "discount factor" to refer to this marginal rate of substitution.[1] Our definition differs from that of Böhm-Bawerk (1891) and Fisher (1930), which is the marginal rate of substitution between current and future consumption. In terms of Figure 1, their discount factor is the (absolute value of the inverse of the) slope of an indifference curve. Both Fisher and Böhm-Bawerk emphasized that their idea of time preference really combined two distinct effects. First, the relative value placed on present versus future consumption depends upon the relative consumption levels. This is why Fisher's indifference curves (or "willingness lines") are not linear. Second, even along a constant consumption path present and future consumption need not be valued equally. This distinction had led many authors, such as Friedman (1976) and Stigler (1987), to associate time preference with the slope of the indifference curves along the 45° line. In the case that utility is intertemporally separable and all instantaneous utility functions are identical, our definition coincides with this 45° version of Fisher's.

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1 When a distinction is necessary, we will follow the convention of associating a "discount factor" with the inverse of the MRS and a "rate of time preference" with the natural log of the MRS. A patient person therefore has a high discount factor and a low rate of time preference.

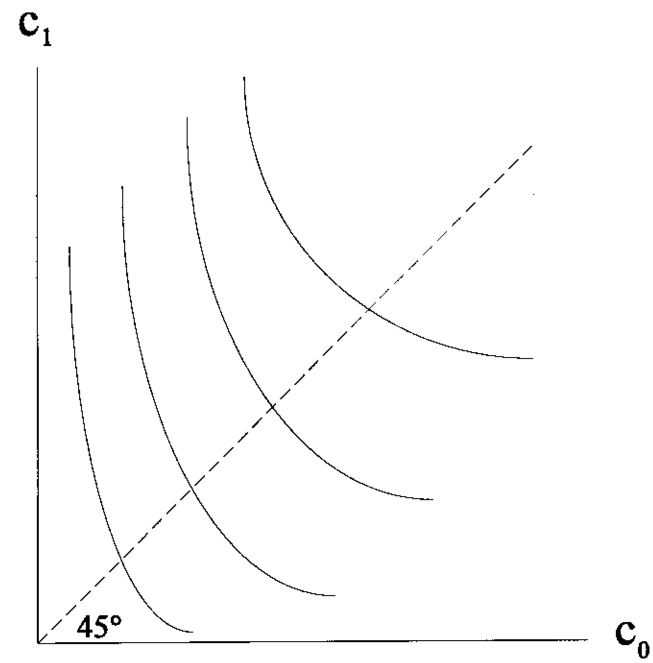

Figure 1 Fisher's Diagram: c_0 vs. c_1

Many economists and philosophers believe that not all people are equally patient and that many of the differences among people are explainable. For instance, Eugen von Böhm-Bawerk felt that patience is part of being civilized,

> The intensity of the preference [for present gratifications] varies widely from person to person, as is attested by that famous scale which ranges from the American Indian who will sell the ancestral hunting grounds for a dram of 'firewater' to the sober, provident and educated scion of Europe's cultured peoples. (1914, pp. 394-5)

Böhm-Bawerk also felt that impatience would "manifest [itself] in extremely different degrees in different individuals, and even in the same individual at different times." (1891, p. 257) Writing earlier, Jevons displayed similar sentiments, "The untutored savage, like the child, is wholly occupied with the pleasures and the troubles of the moment...." (1931, p. 35) Although Fisher allowed for some influence of culture on time preference,[2] he emphasized the contribution of wealth or income: "Poverty bears down heavily on all portions of a man's expected life. But it increases the want for immediate income *even more* than it increases the want for future income." (1930, p. 72) He therefore drew steeper indifference curves nearer to the origin and flatter curves away from it.

Fisher offered some justification for his view, decomposing the influence of poverty into two factors,

> This influence of poverty is partly rational, because of the importance, by supplying present needs, of keeping up the continuity of life and thus maintaining the ability to cope with the future; and partly irrational, because the pressure of present needs blinds a person to the needs of the future. (1930, p. 72)

Although Fisher's rational influence of poverty on impatience is quite reasonable, it does not say anything about time preference as we have defined it. Fisher is simply arguing that, for poor people, utility fails to be intertemporally separable. The level of consumption therefore influences his definition of impatience, the MRS between current and future consumption (even along the 45° line), but not ours - the

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[2] Like Böhm-Bawerk, Fisher also mentions a connection between culture and patience, "In the case of primitive races, children, and other uninstructed groups in society, the future is seldom considered in its true proportions." (1930, p. 81). Other factors mentioned by Fisher are self-control, habit, concern for the lives of other persons, and fashion.

MRS between current and future utility. His "irrational" component does speak to our definition.

Irving Fisher was not the last economist to express a belief that the wealthy are more patient than the poor. This view, which has be designated *decreasing marginal impatience*, has been advocated by many economists from Friedman (1976, p. 64) to Blanchard and Fischer (1989, p. 73). Epstein (1987) appears to be in the minority in his belief in increasing marginal impatience. Other than Fisher's brief justification, all of these views seem to rest upon introspection alone. One contribution of this paper will be to propose a microfoundation for decreasing marginal impatience.

II. A Model of Patience Formation

Fisher wasn't the first economist to argue that impatience is irrational or even morally unjustifiable. John Rae wrote,[3]

> Were life to endure forever, were the capacity to enjoy in perfection all its goods, both mental and corporeal, to be prolonged with it, and were we guided solely by the dictates of reason, there could be no limit to the formation of means for future gratification, till our utmost wishes were supplied. A pleasure to be enjoyed, or a pain to be endured, fifty or a hundred years hence, would be considered deserving the same attention as if it were to befall us fifty or a hundred minutes hence.... (1834, p. 53)

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[3]See Loewenstein (1992) for a review of the history of economic thought on patience and related issues.

However Rae hinted that such patience is perhaps only an ideal. Jevons emphasized that some amount of impatience is an inevitable consequence of human frailties,

> To secure the maximum benefit in life, all future events, all future pleasures or pains, should act upon us with the same force as if they were present, allowance being made for their uncertainty. The factor expressing the effect of remoteness should, in short, always be unity, so that time should have no influence. But no human mind is constituted in this perfect way: a future feeling is always less influential than a present one. (1834, p. 72)

The manifestation of this frailty is Bentham's observation that a pleasure is valued according to "(1) Its intensity. (2) Its duration. (3) Its certainty or uncertainty. (4) Its *propinquity or remoteness*."[4] Since future pleasures are usually more remote that current ones, the marginal rate of substitution between current and future utilities is often less than unity. We will follow Bentham and model an individual as valuing future pleasures or utilities according to their remoteness or vividness in his imagination. Consider the first period objective of a two period lived consumer,

$$V = u_0(c_0) + \beta(S) \cdot u_1(c_1)$$

c_0 and c_1 are consumption levels in the present and future, respectively. The functions $u_0(\bullet)$ and $u_1(\bullet)$ map the present and future consumptions into present and future pleasures. Future pleasures are discounted according to the discount function $\beta(\bullet)$ which, according to Rae, Böhm-Bawerk, and many others, is usually

4 Jevons (1931), p. 28.

less than unity. However, unlike the usual neoclassical approach, we do not assume that β is a fixed parameter, but rather that it adjusts according to the propinquity of future pleasures.

While we admit that human beings have frailties, we believe they are often aware of their weaknesses and may spend resources to overcome them. That people take steps to make future pleasures seem less remote was hinted at by Böhm-Bawerk (1891),

> The present always gets its rights. It forces itself upon us through our senses. To cry for food when hungry occurs even to a baby. But the future we must anticipate and picture. Indeed, to have any effect on the future, we must form a double series of anticipations. We must be able to *form* a mental picture of what will be the state of our wants, needs, feelings, at any particular point of time. And we must be able to *form* another set of anticipations as to the fate of those measures which we take at the moment with a view to the future. (p. 244)

We italicize the word "form" because it hints at the efforts made by people to value future pleasures. Böhm-Bawerk (1891) also mentions that people excessively discount the future because "... it may be that we are not willing to put forth the necessary *effort*." (1891, p. 269, italics added) We model such effort by allowing a consumer to make future pleasures less remote by spending resources (S) on imagining them,[5]

$$\beta(S) > 0\,, \qquad \beta'(S) \geq 0\,, \qquad \beta''(S) \leq 0 \qquad \text{for} \;\; S \geq 0$$

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[5] It is possible, even likely, that effort spent imagining future pleasures distract from current utility. If so, two period lived consumers would seek to maximize:

$$V = u_0(c_0, S) + \beta(S) \cdot u_1(c_1)$$

where u_0 is decreasing in S. We do not utilize this more complicated objective, but our conclusions continue to hold when u_0 is separable in c_0 and s.

More resources spent on imagination increase the propinquity of future pleasures and therefore their value; $\beta(S)$ is an increasing function. The concavity assumption requires that resources spent imagining future utilities become less and less effective in decreasing their remoteness.

Any tendency to undervalue the future can be incorporated with the specification that $\beta(0) < 1$. We call $\beta(0)$ the *endowed discount factor*. According to Böhm-Bawerk and most others who have written on this subject, endowed discount factors are less than unity because of an imperfect ability to imagine the future. But even this assumption does not necessarily imply that *actual* or *equilibrium* discount factors are also less than one since these are affected by the resources put into improving one's ability to imagine the future. That is, even though $\beta(0) < 1$, it is possible that $\beta(S^*) \geq 1$, where S^* is the level of resources spent on imagining the future.

How can a person improve his capacity to appreciate the future? What exactly is S? First, S is partially determined by time and effort spent anticipating future pleasures. Such time and effort may be far from trivial. While forming a mental picture of one's future pleasures may not be incredibly difficult, the process of anticipation is not merely one of image formation but also one of scenario simulation. Even image formation may not be cheap because images of future pleasures have to be repeatedly refreshed in our mind in order to compete with current pleasures. The time and effort spent anticipating the future is time and effort that cannot be spent producing, either at home or in the market.

S is also affected by spending on certain goods. Purchases of particular goods, such as newspapers, can distract one's attention away from current pleasures and towards future ones. People may also purchase disciplinary devices, such as alarm clocks or piggy banks. Financial instruments such as piggy banks involve a cost - foregone interest - but can be beneficial if they are successful at diverting one's attention toward the future. "Tearing up one's credit cards" or making "too" frequent trips to the store to buy ice cream might also fit into this category.

Schooling also determines S partly through the study of history and other subjects, for schooling focuses students' attention on the future. Schooling can communicate images of the situations and difficulties of adult life, which are the future of childhood and adolescence. In addition, through repeated practice at problem-solving, schooling helps children learn the art of scenario simulation. Thus, educated people should be more productive at reducing the remoteness of future pleasures.

Parents often spend resources on teaching their children to better plan for the future. As children become teen-agers and then adults, they experience what had been future utilities, and these experiences also help them to better imagine what the future will be like.

In general, we assume that consumers live more than two periods, although only for a finite number. A consumer maximizes:

$$V = \sum_{i=0}^{T} \beta(S)^i \cdot u_i(c_i) \ , \tag{1}$$

where T is the length of life. We assume that the u_i functions are nonnegative, strictly increasing, and strictly concave for nonnegative consumptions. To guaranteed the interiority of solutions, we also impose some Inada conditions:

$$\lim_{c_i \to 0} u_i'(c_i) = \infty \ , \qquad i = 0, \ldots, T$$

For our analysis to be of interest, $\beta'(0)$ must be large enough to justify some investment in S.

Of course, we do not rule out altruistic interpersonal linkages. Since we index the pleasure functions by age, the age T pleasure function can be interpreted as a bequest function for a parent who lives T-1 periods. Although parents may pass on resources which could be used by a child to purchase lower discount rates on his future utilities, our specification rules out any *direct* intergenerational transmission of patience. In other words, all pleasure functions, including the one at age T, are not functions of S.

Our formulation assumes that the rate of discount per period is the same for all future periods, although Böhm-Bawerk and others who discuss weaknesses in imagining future wants have sometimes argued that the *rate* of discount gets larger as the future gets more remote (see Loewenstein, 1992). We could allow for the rate of discount to depend on age, but to obtain time consistent behavior, we assume that discount rates do not depend on distance into the future. The assumption of a constant rate of discount is not merely sufficient to generate time consistent choices - it may be implied by a certain type of rational reasoning.

Consider a person who discounts utility in the next period by say 2% because he has difficulty in fully imagining the utilities in that period. By how much does he discount utilities in the second period? He may have much greater trouble imagining utilities two periods ahead than those one period ahead, which is the point made in the literature (reviewed by Loewenstein) suggesting that discounting is not exponential. But that is not clearly the relevant question.

A present self who is not in conflict with his future selves needs to know not the rate at which he discounts utilities two periods ahead, but the rate at which his future self will discount utilities that are now two periods ahead, but that next period will only be one period ahead. Of course, the present self does not know the one period discount rate of next period's self, but we see no reason why that future rate would be systematically misestimated in one direction or another. This kind of reasoning will result in a recursive valuation of sequences of future utilities that will lead to time consistent choices by persons who do not believe they have conflicts with future selves. In the special case that one estimates the discount rates of future selves to be identical to one's current one period discount rate, then discounting will be exponential.

In this way we arrive at exponential (or at least recursive) discounting of future utilities, even though it may be almost impossible to imagine directly what utilities will be like many years into the future. It is possible that people do not reason in this recursive fashion because of basic conflicts between present and future selves, perhaps partially because the ability to imagine future pleasures and pains is different from the ability to imagine one's future imagination of the more distant utilities. It is also possible that recursions themselves are psychically costly so that the distant future must be directly imagined if it is to be valued. Therefore, our form of recursive reasoning may be contradicted empirically (see Ainslie and Haslam (1992) present experimental evidence which suggests, but does not prove, that people behave inconsistently over time), but it does show that a limited ability to imagine the distant future *per se* does not weaken the case for recursive discounting and consistent behavior over time.

III. Implications of the Model

The capital market is assumed to be perfect, with a present value of all assets and earnings given by A_0. We assume that the cost of adding to imagination or "future-oriented" capital S is constant, and that the stock does not depreciate over time. With these assumptions, the equilibrium S (S^*) can, without loss of generality, be determined entirely in the initial period. Consumers choose nonnegative investments in patience S and a life profile of nonnegative consumptions $\{c_i\}_0^T$ in order to maximize their objective V, subject to an initial wealth endowment A_0 and the intertemporal budget constraint below.

$$\sum_{i=0}^{T} R_i c_i + \pi S = A_0$$

The R_i's are the usual interest rate factors, and π is the price of S. Although time may well be a major input into the production of future-oriented capital - time spent in school and other learning, in trying to imagine the future, and in other ways - we start out be assuming that π is the same for every individual (and equal to 1), regardless of wealth or preferences. More generally, a person's π would be a weighted average of goods prices and his wage rate.

The first order conditions with respect to consumption at each date are still the familiar ones: the marginal utility of consumption at any age i, adjusted by a discount factor and an interest rate factor, must equal the marginal utility of wealth. Our new condition, which holds when the optimal choice of S is strictly positive, requires that the marginal benefit of investments in time preference also equal the marginal utility of wealth λ_0.

$$\beta'(S)\left[\sum_{i=1}^{T} i[\beta(S)]^{i-1} u_i(c_i)\right] = \lambda_0 = u_0'(c_0) \tag{2}$$

When $\beta(S)u_1(c_1)$ (as well as u_0) is concave, first order conditions are sufficient in the two period case. With more than two periods, the second order conditions require more concavity in the β and u_i functions. See the discussion in the appendix.

The marginal benefit of S depends not only on the rate at which S increases the propinquity of future utilities ($\beta'(S)$) and on the length of life (T), but also on the level of the discount rate and the level of future utilities (the term in large brackets). The effect of future utilities is clearly seen in the two period case (T=1).

$$T = 1 \quad \Rightarrow \quad \beta'(S)u_1(c_1) = \lambda_0 = u_0'(c_0) \tag{3}$$

More generally, future utilities influence the marginal benefit of S according to their distance i and the level of the discount factor.

A. Complementarity between Time Preference and Future Utility

Equations (2) and (3) show the most robust implication of endogenous time preference. An increase in future utilities raises the advantages of low discounts on the future since discount rates are then weighted by larger utilities. In essence, there is a complementarity between future utilities and weighting the future more heavily. Consequently, anything that raises future utilities without raising the marginal utility of current consumption will tend to lower the equilibrium discount on the future. We give several examples of how this complementarity affects the equilibrium rate of discount on the future.

The complementarity between future-oriented capital and future utilities derives from some apparently cardinal assumptions about consumer's utility functions; our formulation presumes that the marginal product of future-oriented capital depends on the *level* of future utilities. This is one way to capture the idea that the marginal rate of substitution of current for future consumption depends on future-oriented capital, but is not the only way. While the functional form (1) cannot be readily justified on *a priori* grounds, the discounted utility model is our starting point because of the frequency of its use in the literature. The behavioral implications of the functional form (1) are distinct from some potential alternative formulations of the time preference problem, but we argue that the former conform with observed behavior.

Without contradicting Samuelson (1937) and others who have noted that it is the indifference map and not a person's utility function per se that determines his choices, we believe that it is meaningful to think about the level of the date t utility flow u_t. By modeling changes in age, health or family size as shifts of the utility function, we are saying that, for a given S, these variables increase the period 0 self's enjoyment of future consumption.

A formulation that places meaning on the level of utility is consistent with the idea that people differ in their ability to produce happiness for a given consumption bundle. For two reasons, such differences are useful for understanding human behavior. First, one's ability to produce happiness is correlated with observable characteristics such as age or health. Second, we believe that intertemporal (and interpersonal) comparisons of happiness influence decisions. We interpret a zero utility level as the utility of unconsciousness. Thus, when a person has zero utility at a given date, he is indifferent between consciousness and unconsciousness at that date. The comparison is relevant for choices: a person who is cannot expect positive utility in the future may choose to commit suicide (see Becker and Kilburn (1993) for an analysis along these lines), people will try to forget painful (negative utility) experiences and try not to anticipate future pain, parents will not bear children whose lifetime utility fails to be positive. Thus our only cardinal assumption is our assignment of zero to the utility of unconsciousness.

A.1 Mortality

Suppose there is an increase in life expectancy from T to T' years that is accompanied by an increase in lifetime earnings that maintains the marginal utility of wealth, λ_0, constant. If the discount rate were unchanged, consumption in the first T years would then be unaffected, but consumption between T and T' would become positive. This increases the marginal benefit from investments in time preference, and hence increases the equilibrium accumulation of future-oriented capital. Longer lifetimes not only directly induce consumers to plan further into the future, but lower mortality also induces them to place more weight on the utility in all future periods - including those periods $t < T$.

Death probabilities affect time preference in a similar way. Consider consumers who survive to the second period with probability p, and do not have access to either life insurance or annuities markets. Like their expected utility maximizing cousins, our consumers increase the weight placed on future consumption in response to a decrease in mortality merely because future consumption becomes more likely. However, our consumers reinforce their increased desire for future consumption by becoming more patient. Even though the cost of investing in S increases because the marginal valuation of current consumption increases, the increased productivity of these investments, due to the higher future utility level and its greater probability, more than compensates.[6] If p also influenced A_0, perhaps because future earnings are more valuable, then there is yet another force encouraging the accumulation of patience.

In the usual analysis - without life insurance markets - differences in the valuation of future consumption resulting from mortality differences are easily calculated. For example, a person with a 10% death probability will have 2% higher time preference (net of mortality) than a person with an 8% death probability. In our analysis, the difference in time preference exceeds the difference in mortality: a 2% difference in survival rates results in time preference rates that differ by more than 2%.

The effect of mortality on S is more ambiguous when fair annuities are available. In the usual analysis, changes in mortality rates induce no change in the valuation of future consumption (relative to current consumption). In our analysis,

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[6] The effects here identical to those that we considered in our discussion of the endowed discount factor $\beta(0)$ (the multiplicative case). Like $\beta(0)$, p influences the productivity of imagination directly - as well as indirectly by stimulating higher future utility levels.

consumers do respond to mortality rates even with access to fair annuities markets, but there are offsetting effects. On one hand, lower mortality increases the price of future consumption. The accumulation of patience can be discouraged because of this reduction in the budget set for a given present value of earnings A_0. On the other hand, investing in future-oriented capital can be more productive because of the greater likelihood of survival, despite the possibility of lower future utility levels (contingent on living). Finally, any increase in the present value of earnings resulting from a higher survival probability will have the same wealth effects shown earlier on the accumulation of patience. None of these reactions are found in the usual analysis.

Fuchs (1982) and others believe that differences in time preference across individuals explain important differences in health related decisions. Our analysis implies the converse, that differences in health cause differences in time preference because greater health reduces mortality and raises future utility levels. We are not saying that investments in health *are* investments in future-oriented capital S (although they might be), just that they affect the incentives to invest in S.

A.2 Immortality

In lifecycle models of choices with certainty, consumers who do not want to leave bequests only consider utilities over their own lifetimes. They are assumed are assumed to neglect any utilities after death unless they believe in an afterlife that is affected by what they do while alive. In the conventional setup this is easily justified even if the utility from each year after death is not zero. If $D = \Sigma \beta^i u_i$ is the discounted value of the utilities after death, and if β is fixed, then the *constant* D can simply be subtracted from the utility function V to get another utility function V' that has the same indifference curves as V. However, V' only considers utility streams while the person is alive.

If β is endogenously determined by the accumulation of future-oriented capital, this transformation is no longer legitimate since D is no longer constant. Then the value of utilities after death would affect behavior so long as $u_i \neq 0$, for some $i > T$. To the extent that future-oriented capital is "general" - it facilitates the imagination of events at a variety of distances into the future - a higher utility of death will encourage consumption growth *before* death.

If consumers expect positive utility after death - perhaps because they believe they will go to "heaven" - this raises their investments in future-oriented capital because their future utilities are increased. In equilibrium, therefore, consumers who expect to go to heaven will discount the future less. Conversely, consumers who expect negative utilities after death will reduce their investments in future-oriented capital, so that consumers who are afraid they will go to "hell" endogenously place a high discount on the future.

Differences across adults in opportunities of their descendants to produce happiness or in their rate of intergenerational altruism (ie, a given happiness of descendants is appreciated to different degrees) are another reason why the "utility at death" might vary across individuals and are therefore a source of differences in lifecycle rates of time preference.

A.3 Time Preference and Rates of Return

The complementarity between future utilities and the incentive to invest in future-oriented capital also helps explain why increases in the rate of return to saving tend to induce more patience. Consider first an easy case where an increased rate of return is compensated to hold the marginal utility of wealth constant. Then for a given rate of time preference, all future consumptions rise since the rate of return is higher, and current consumption is unchanged by the marginal utility of wealth assumption. The induced rise in future utilities induces an increased investment in future-oriented capital since the marginal cost of the investment - given by the marginal utility of wealth - is unchanged.

The argument is more complicated but the conclusion is basically the same when the present value of wealth is held constant as the rate of return increases. To see this, consider the two period version of our problem,

$$\max_{c_0, c_1, S} \left[u_0(c_0) + \beta(S) \cdot u_1(c_1) \right]$$

$$\text{s.t.} \qquad c_0 + \frac{c_1}{(1 + r_1)} + S = A_0$$

The consumer has two ways to "invest in the future." First, he can save more so that he can enjoy greater utility in period 1. Second, he can increase S, making his future pleasures seem less remote. For an optimal plan, the rate of return to both forms of investment in the future must be equal the payoff to a unit of current consumption,

$$\beta'(S)u_1(c_1) = (1+r_1)\beta(S)u_1'(c_1) = \lambda_0$$

We expect c_1 to increase in response to a higher interest rate; c_1 is relatively cheaper and the consumer has a larger budget set. If c_0 remains constant or increases, then λ_0 does not increase; the equation of rates of return requires that S increase as r_1 does. If instead c_0 decreases but the elasticity of period 1 utility with respect to period 1 consumption is approximately constant, then S still increases as r_1 does. To see this, notice that, in the constant elasticity case, the equation of the rates of return to both forms of "investment" becomes:

$$\sigma\frac{\beta(S)}{\beta'(S)} = \frac{c_1}{1+r_1} \quad ,$$

where σ is the elasticity of u_1 with respect to c_1. Suppose, contrary to fact, that both S and c_0 fall in response to a higher interest rate. We see above that the present value of c_1 must fall. The budget equation implies that S must increase - a contradiction.

This effect of the interest rate on time preference is important because it helps in thinking about general equilibrium models inhabited by consumers who invest in patience. The interest rate is an important link between the production and consumption sides of a dynamic general equilibrium model. The distribution of wealth in an economy will effect an individual's rate of time preference through the interest rate. Thus, a person will tend to be more patient when most other people in the economy are poorer. This is because the representative consumer, when poorer, will choose to remain impatient. With impatience widespread, the rate of return will be high, motivating even more patience in the richer (untypical) guy. In this way, borrowing and lending in an economy promotes specialization in the production of future-oriented capital.

Economists attempt to link asset prices to consumers' intertemporal marginal rates of substitution. They compare empirical measures of the LHS and RHS of the consumer's Euler equation below.

$$(1+r_1)^{-1} = \beta\big(S(A_0, r_1)\big)\frac{u_1'(c_1)}{u_0'(c_0)}$$

Various asset prices are used to measure the LHS, while data on consumption (either macro aggregates or for households) are used to measure the RHS. Of course, these analyses treat β as a constant. Our analysis suggests that such exercises may suffer from an identification problem: changes in the interest rate and wealth not only change consumers' budget sets but also shift their intertemporal marginal rates of substitution. As discussed by Mulligan (1993) in the context of intergenerational discount rates, some empirical studies of growth and the distribution of wealth suffer from this identification problem as well. The Euler equation above shows, for example, that a correlation of growth rates with wealth levels across families, or even across countries, need not imply a correlation of rates of return with wealth levels.

Our model of the intertemporal marginal rate of substitution does not appear to help resolve either the "equity premium puzzle" or the "risk-free rate puzzle." But more work is needed to determine whether there is a link between endogenous time preference and these puzzles.

A.4 Time Preference and Uncertainty

The complementarity between future utilities and investments in future-oriented capital also explains why less certain and less pleasant states of nature are discounted more heavily.

Consider again a two period lived consumer, but introduce N states of nature in the second period. We model consumers as valuing pleasure in any single state of nature according to its propinquity and begin with the case when insurance is unavailable to the consumer. A consumer's problem is:

$$\max_{c_0, c_1, \{S_i\}_{i=0}^{N}} \left[u_0(c_0) + \sum_{i=0}^{N} \beta(S_i) p_i u_1(c_1 - x_i) \right]$$
$$\text{s.t} \qquad c_0 + R_1 c_1 + S = A_0 \ ,$$

where the x_i are the hazards in difference states and S is the sum of the S_i.[7] The propinquity of pleasure in state i depends on two factors. First, resources S_i can be spent to reduce the "remoteness" of that pleasure. Second, because part of the process of anticipation involves scenario simulation, more probable states of nature seem more salient; highly probable states of nature are imagined more productively.

It is straightforward to show that, holding constant consumption levels across states, more resources are spent anticipating pleasures in highly probable states. Second, holding constant probabilities, expenditures on S are greater in states with higher utility. These two results obtain because future-oriented capital can be transformed, one-for-one, from one state to another while the payoff to investing in reducing the discount on a state depends on its probabilities and the utility it gives. Equivalently, future-oriented capital is transformed from one state to another according to their relative likelihoods while the benefits depend on the utility levels.

Together the results imply that low probability states tend to be ignored - especially the unhappy ones. For example, consumers should provide more for favorable future outcomes than unfavorable ones, such as death, even when they are

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[7] The problem of allocating S among states of nature is analogous to the problem of allocating altruism among children. See Mulligan (1993) for a study of the optimal allocation of altruism.

equally likely. The relatively high discount on unfavorable future outcomes may help explain why so many people with assets die without a will.

To understand the allocation of S among states of nature, one need not take a stand on the importance of time costs in the production of S. As long as the process of transforming time and goods into S is the same for each state, the importance of time relative to goods in that process does not have any effect on the allocation among states. Nor is it necessary to assume that the discount rate in each state depends only on it own "state-specific future-oriented capital," S_j. It is more reasonable to assume that there is some general future-oriented capital that influences the rate of discount on several or all states of nature (we thank Boyan Jovanavic for the analogy to human capital theory).

Whether or not our results are modified by introducing general future-oriented capital depends on the particular specification, but we can point out specification for which our results do generalize. For example, if the state-specific discount factors are multiplicatively separable functions of general and state-specific future-oriented capital ($\beta_j = \beta(S)\pi(S_j)$), then the general capital can be factored out and our previous results go through exactly as before. Or we can suppose that states of nature that have similar utility levels and similar likelihoods share a stock of general future-oriented capital. However, if improbable states share a stock of general capital with highly probable ones or if happy states share a stock with sad ones, then results may be qualitatively different from the case where there is no general future-oriented capital.

The results that obtained in the absence of insurance opportunities are reinforced if consumers have access to actuarially fair insurance. To see this, consider a consumer who had an equal allocation of the S_i's across states ($S_0 = S_1 = \ldots = S_N$). An expected utility maximizer with access to fair insurance will smooth consumption, and therefore utilities, across states of nature. But with a smooth consumption allocation, the consumer wants to reallocate more S to the more probable states. And an unequal distribution of the S_i's encourages a reallocation of consumption toward the more probable states - since they are less discounted. This consumption reallocation reinforces the initial incentive to move S to the more probable states.

If the time preference function $\beta(S)$ and the utility function $u_1(c)$ are not too concave, then the optimal allocation of S and c across states of nature will not only be unequal, but also degenerate: unlikely states will be completely unprovided for, even though fair insurance is available. Our consumers always *underinsure* when they have access to actuarially fair insurance, and even when there is no fixed cost of providing for a state.

By linking the problems of intertemporal choice and choice under uncertainty, our analysis also confirms some of Adam Smith's insights on the problems of gambling and insurance. Smith believed that consumers systematically overvalued good states of nature and undervalued the unhappy ones:

> That the chance of gain is naturally over-valued, we may learn from the natural success of lotteries. (1776, I.x.b.27)

> That the chance of loss is frequently under-valued, and scarce ever valued more than it is worth, we may learn from the very modest profit of insurers. (1776, I.x.b.28)

Our departure from Smith is that we view such "undervaluation" and "overvaluation" as consistent with rational behavior.

So far we have compared the demand for insurance by consumers who make attempts to overcome any bias against the future to the demand by expected utility maximizers. Uncertainty also influences the incentives to invest in future-oriented capital. Suppose that all N states are equally probable and equally salient, and that fair insurance is available. Consumption and S will be equally distributed across states and each state will have the same discount factor.[8] However, the amount of S allocated to any particular state of nature will depend on the number of states (N) because the probability of any particular state is decreasing in N. Then the rate of time preference will be increasing in the number of states, which is one reason why uncertainty may discourage patience.

A.5 Addictions

The complementarity between future utilities and discount rates helps gain insights into the effect of addictions on time preference. Increased consumption of harmfully addictive substances, such as drugs, alcohol, and cigarettes, tends to raise present utility at the expense of lowering future utility. Since a decline in

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[8] Second order conditions must hold. Without them, focusing on one particular state may be optimal even though it is no more probable than the others.

future utility reduces the benefits from a lower discount on future utilities, greater consumption of harmful substances would lead to higher rates of time preference by discouraging investments in lowering these rates - assuming such consumption does not lower the marginal cost of these investments. Becker and Murphy (1988) show that people who discount the future heavily are more likely to become addicted. This result establishes the converse, that harmful addictions induce even rational persons to discount the future more heavily, which may in turn lead them to become more addicted.

Drug use, heavy drinking, and certain other addictions are supposed to create urges and telescopic vision that causes future needs to be under estimated. However, the analysis of endogenous discount rates implies that even fully rational utility-maximizing individuals who become addicted to drugs and other harmful substances or behavior are induced to place less weight on the future.

B. Time Preference and Wealth

A richer (in terms of A_0) Fisherian consumer who is not optimizing with respect to time preference ($S = 0$) will have higher future utilities and a lower marginal utility of wealth, given the interest rate and the discount factor. If this consumer is allowed to optimize with respect to time preference, the first order condition with respect to S implies that the rich will have the greatest incentive to increase S because they have both a low marginal utility of wealth (the cost of investing in time preference) and high future utilities (the returns to investing in time preference).

As long as the price of S fails to be correlated with wealth, and if second order conditions are satisfied, wealthier individuals will invest more heavily in the attempt to appreciate future utilities. Our *decreasing marginal impatience* results as well as a discussion of the second order conditions are enumerated by Propositions 1 and 2 and their corollaries in the Appendix. The proofs solve the problem recursively, noticing that after the initial period time preference is given - our problem is identical to Irving Fisher's from period 1 on.

If S has an important time component, higher income people may not be more patient because their higher opportunity cost of time raises the price of S. However, our analysis still implies that persons who are richer because they have more assets would be more patient than equally rich persons with higher earnings.

When the rich choose to be more patient, our analysis has implications for the dynamics of intragenerational inequality. Inequality in wealth and consumption within a cohort should rise as the members age as long as the price of S is not too correlated with wealth. The evidence on the dynamics of inequality within a cohort is somewhat mixed. Deaton and Paxson (1992) find a tendency for the distribution of consumption within a cohort to become more disperse as that cohort ages. Lansing and Sonquist (1969) and Projector et. al. (1969) find little change in the distribution of household net worth as the head ages. Mincer (1974) and others have shown that the distribution of earnings - and therefore human wealth - becomes more disperse as a cohort ages.

Mace (1991), Townsend (1994) and others emphasize the role of insurance markets for the dynamics of inequality. These authors have shown that, with full insurance and identical time additively separable constant relative risk aversion preferences for all households, every household should have the same consumption growth. This has led to an empirical literature that tests that proposition. According to our model of intertemporal choice, households can have systematically different rates of consumption growth even when they have access to a complete contingent claims market. Of course, one can easily weaken econometric tests by introducing a separate preference parameter for every family, but this is not the nature of our criticism. We envision households that have *identical* preferences; different rates of consumption growth arise because households have different incomes or face different prices. We predict that people who are more educated, wealthier, or from wealthier families should have higher rates of consumption growth. A more appropriate rejection of full insurance is, in our opinion, a demonstration that poorer or less educated people sometimes enjoy more rapid consumption growth.

C. Equilibrium vs. Endowed Discount Factors

Consider again consumers who are not optimizing with respect to time preference (each with $S = 0$). Suppose that consumers' discount functions $\beta(S)$ differ by an additive constant; some are more patient than others for a given S. For given assets and a given interest rate, a more patient consumer will have higher future utilities, although a higher marginal utility of wealth. From the first order condition with respect to S, we see that patience has conflicting effects on the incentives to accumulate time preference. On one hand, higher future utilities encourage investments in S. On the other hand, a high marginal utility of wealth discourages investments in S. In general either force can dominate; the relationship between investments in future-oriented capital and endowed discount factors is ambiguous.

There is another way to see the ambiguity in the two-period case. A higher endowed discount factor favors an reallocation of resources away from current consumption and towards future-oriented capital and future consumption ($R_1c_1 + S$). However, for given ($R_1c_1 + S$), higher endowed discount factors encourage a reallocation away from S and towards future consumption. The response of c_1 to an increase in the endowed discount factor is therefore not ambiguous but the response of S is.

If instead consumers' discount functions (or future utility functions) differ by a multiplicative constant (ie, consumer i's endowed discount factor is $B_i\beta(0)$), we can sign the relationship between endowed discount factors and investments in future-oriented capital. As in the additive case, higher endowed discount factors discourage current consumption. However, the endowed discount factor has no effect on the relative allocation of S and c_1 in the multiplicative case. A higher endowed level of β increases the investment in S, which widens the inequality in actual discount rates relative to the inequality in endowed rates. Therefore, the endogeneity in discount rates will tend to exacerbate any inequality in savings, wealth, etc. due to the heterogeneity in tastes regarding the weight attached to the future.

VI. Evidence on Decreasing Marginal Impatience

That wealth *causes* patience one of the most important implications of our model of patience formation. Some empirical work, such as Lawrance (1991), documents a positive *correlation* between proxies for patience, such as consumption growth, and measures of economic status. Other authors, such as Steve Zeldes, have claimed that a family's consumption growth is *negatively* related to its economic status. However, Zeldes (1989) finds a negative correlation only after holding constant family fixed effects; his evidence says nothing about the correlation between consumption growth and income *across* families.

Atkeson and Ogaki (1991) argue that a positive correlation is indicative of differences in intertemporal elasticities of substitution (IES), not differences in time preference. We are not so eager to adopt their interpretation of their findings. First, since they only allow income to influence time preference in a very specific way; it is not clear that they can distinguish a "variable IES hypothesis" from a hypothesis that allows for a more general relationship between time preference and income. Moreover, they "seek to find systematic differences of consumption growth that are not simply explained by differences in household income growth" (p. 18). It also is possible that more patient families and choose not only a steeper consumption profile but also a steeper income profile. Holding constant income growth may be partly holding constant time preference. Third, we note that identification of an IES requires variation in the real interest rate and that most variation in micro studies is obtained from variations in an estimated marginal tax rate - which is just a function of income. This is yet another reason to believe that Atkeson and Ogaki's results are driven by functional form restrictions. Fourth, while they allow for the possibility that time preference might differ across individuals, Atkeson and Ogaki do not allow time preference to be correlated with the real interest rate. Yet we have argued (in section IV.C) that the forces that lead people to have different rates of time preference in equilibrium also lead time preference to react to changes in the interest rate.

Of course, documenting a simple correlation does not reveal whether wealth leads to patience or patience leads to wealth. We will derive an empirical model that allows for both directions of causality, one that can detect whether wealth influences patience. After examining data, we conclude that either (1) wealthy people become more patient or (2) patience "endowments" are transmitted from parents to children. We discuss ways to sort out these two alternative interpretations of our results.

A. An Empirical Model

A.1. The Status Function

Consider an earnings function for person j of generation t.

$$\ln y_{t+1}^{j} = \gamma_0 + \gamma_1 \ln y_t^{j} + \gamma_2 s_{t+1}^{j} + \gamma_3 \ln \beta_{t+1}^{j} + \varepsilon_{t+1}^{j} \tag{4}$$

Superscripts denote family names and subscripts denote generations. y is a measure of economic status such as income, wages or earnings. A son's status depends on his father's because parents often give endowments to their children. Holding constant father's status, a son's status depends on his own schooling if schooling is determined by factors other than father's status. The son's discount factor β_{t+1} influences son's status because patient people are more likely to invest in both human and nonhuman assets and are more likely to work hard early in life. ϵ is an error term. A father's error term can be correlated with his sons, reflecting, perhaps, genetically determined ability. All of the slope coefficients in (4) are assumed to be nonnegative.

Analogous stories would generate a similar equation for a son's schooling:

$$s^j_{t+1} = \sigma_0 + \sigma_1 \ln y^j_t + \sigma_3 \ln \beta^j_{t+1} + \zeta^j_{t+1} \qquad \textbf{(5)}$$

Both slope coefficients in (5) are assumed to be nonnegative.

An important prediction of the "patient people get rich story" is that, holding constant the son's status or his schooling, his father's status is *negatively* correlated with his son's time preference. This result is obtained merely by inverting the son's earnings function (4) or the son's schooling function (5):

$$\ln\beta^j_{t+1} = \frac{1}{\tilde{\gamma}_3}\ln y^j_{t+1} - \frac{\tilde{\gamma}_0}{\tilde{\gamma}_3} - \frac{\tilde{\gamma}_1}{\tilde{\gamma}_3}\ln y^j_t - \frac{1}{\tilde{\gamma}_3}\tilde{\epsilon}^j_{t+1}$$
$$\tilde{\gamma}_k \equiv \gamma_k + \sigma_k \qquad \textbf{(6)}$$

$$\ln\beta^j_{t+1} = \frac{1}{\sigma_3}s^j_{t+1} - \frac{\sigma_0}{\sigma_3} - \frac{\sigma_1}{\sigma_3}\ln y^j_t - \frac{1}{\sigma_3}\zeta^j_{t+1} \qquad (7)$$

The tilded coefficients in (6) denote both direct and indirect effects (through the schooling equation (5)) of a father's economic status on his son's.

(6) and (7) say that rich sons tend to be patient, not because wealth leads to patience, but because patience leads to wealth. When a father is rich relative to his son, the econometrician can guess that the son is impatient; hence the father's status is negatively correlated with the son's consumption growth once the son's status is held constant. If a son's error term (either in his earnings equation or in his schooling equation) is positively correlated with his fathers (so that the error terms in (6) and (7) are correlated with the father's status), the prediction that father's status enters negatively is only strengthened.

Since, in the absence of liquidity constraints, a CIES consumer's consumption growth is a function of his time preference and an interest rate, (6) and (7) can be written in terms of observables.

$$\Gamma^j_{t+1} = \frac{1}{\theta}\left[\ln(1+r^j_{t+1}) + \frac{1}{\tilde{\gamma}_3}\ln y^j_{t+1} - \frac{\tilde{\gamma}_0}{\tilde{\gamma}_3} - \frac{\tilde{\gamma}_1}{\tilde{\gamma}_3}\ln y^j_t - \frac{1}{\tilde{\gamma}_3}\tilde{\varepsilon}^j_{t+1}\right] \qquad (6)'$$

$$\Gamma^j_{t+1} = \frac{1}{\theta}\left[\ln(1+r^j_{t+1}) + \frac{1}{\sigma_3}s^j_{t+1} - \frac{\sigma_0}{\sigma_3} - \frac{\sigma_1}{\sigma_3}\ln y^j_t - \frac{1}{\sigma_3}\zeta^j_{t+1}\right] \qquad (7)'$$

Γ^j_{t+1} is a measure of family j's son's consumption growth sometime during his adult lifetime, r^j_{t+1} is a corresponding interest rate, and $1/\theta$ is the intertemporal elasticity of substitution which is constant across families and generations.

In the absence of correlation of the error terms in the son's and father's earnings functions, estimation of (6)' or (7)' by OLS should yield consistent estimates of the coefficients on father's status, although estimates of the coefficient on son's status are biased downward.[9] Even when a son's error term is positively correlated with his father's, the theory predicts that (the probability limits of) OLS estimates of the coefficients on father's status are negative.

A.2. The Preference Formation Equation

Our theory of endogenous time preference incorporates the earnings and schooling functions (4) and (5) but adds an equation (8) reflecting the son's choice of time preference.

$$\ln\beta^j_{t+1} = b_0 + b_1 \ln y^j_t + b_2 s_{t+1} + b_4 \ln y^j_{t+1} + A^j_{t+1} \qquad (8)$$

All slope coefficients are positive. Father's status appears in the time preference equation (8) because many of a person's attitudes toward the future are formed

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9 This downward bias is the standard "reverse regression effect."

during childhood. A parent's ability to invest in his child's resources will depend on family resources. Schooling can also influence patience. The son's status would influence his patience if he acquired resource during adulthood and used them to invest in patience.

A^j_{t+1} is an error term. To allow explicitly for the possibility that a son's time preference error term is correlated with his father's, we write:

$$A^j_{t+1} = \alpha_0 + \alpha_1 A^j_t + \eta^j_{t+1} \quad \textbf{(9)}$$

η is uncorrelated across generations. If $\alpha_1 > 0$, the error term in the time preference equation (8) is positively correlated with father's status because, according to the earnings equation (4), a patient father will be rich and, according to the endowment transmission equation (9), a patient father will have a patient son. Thus, OLS estimates of (8) could yield a positive coefficient on father's status even if $b_1 < 0$, as in the "patient people get rich story." Nevertheless, a nonnegative coefficient would indicate some intergenerational transmission of time preference - whether the mechanism be automatic or economic.

A theory of endogenous time preference presents a simultaneous equations estimation problem. The three "structural" equations are (6)', (7)' and the consumption growth version of (8).

$$\Gamma^j_{t+1} = \frac{1}{\theta}\left[-\frac{\gamma_0}{\gamma_3} + \ln(1+r^j_{t+1}) - \frac{\gamma_1}{\gamma_3}\ln y^j_t - \frac{\gamma_2}{\gamma_3}s^j_{t+1} + \frac{1}{\gamma_3}\ln y^j_{t+1} - \frac{1}{\gamma_3}\varepsilon^j_{t+1}\right] \quad \textbf{(6)'}$$

$$\Gamma_{t+1}^{j} = \frac{1}{\theta}\left[\ln(1+r_{t+1}^{j}) + \frac{1}{\sigma_3}s_{t+1}^{j} - \frac{\sigma_0}{\sigma_3} - \frac{\sigma_1}{\sigma_3}\ln y_t^{j} - \frac{1}{\sigma_3}\zeta_{t+1}^{j}\right] \qquad \textbf{(7)'}$$

$$\Gamma_{t+1}^{j} = \frac{1}{\theta}\left[b_0 + \ln(1+r_{t+1}^{j}) + b_1\ln y_t^{j} + b_2 s_{t+1}^{j} + b_4\ln y_{t+1}^{j} + \ln A_{t+1}^{j}\right] \qquad \textbf{(8)'}$$

We will call (6)' the "earnings equation", (7)' the "schooling equation" and (8)' the "preference formation equation."

Even in the absence of an intergenerational correlation of time preference (α_1 = 0), the coefficients on father's status, son's status and son' education in equations (6)', (7)' and (8)' are unidentified. Later we'll discuss identification of the parameters of the preference formation equation, but first we consider whether the hypothesis of no preference formation (ie, $b_1 = b_2 = b_4 = 0$) can be rejected on the basis of reduced form estimates.

Consider the simple regression of a son's consumption growth on his father's status. The (probability limit of) regression coefficient can be expressed in terms of the parameters of the model:

$$\frac{cov(\Gamma^j_{t+1}, \ln y^j_t)}{var(\ln y^j_t)} = \frac{1}{\theta}\left[\frac{b_1 + b_2\sigma_1 + b_4(\gamma_1 + \gamma_2\sigma_1)}{1 - b_2\sigma_3 - b_4(\gamma_3 + \gamma_2\sigma_3)} + \frac{(\gamma_3 + \gamma_2\sigma_3)\alpha_1\Pi}{1 - b_2\sigma_3 - b_4(\gamma_3 + \gamma_2\sigma_3)}\frac{var(A^j_{t+1})}{var(\ln y^j_t)}\right]$$

$$\Pi \equiv \left[1 - b_2\sigma_3 - b_4(\gamma_3 + \gamma_2\sigma_3) - \alpha_1[\gamma_1 + \gamma_2\sigma_1 + b_1(\gamma_3 + \gamma_2\sigma_3) + b_2(\gamma_3\sigma_1 - \gamma_1\sigma_3)]\right]^{-1}$$

The algebra above assumes that, within a given generation, a son's interest rate is not correlated with his schooling, status, patience, or his father's status. It also assumes that a son's earnings function error term ϵ_{t+1}, his schooling function error term ζ_{t+1}, and his endowment error term η_{t+1} are uncorrelated with his father's status.[10]

In words, a patient child will not make his parents rich. A correlation between a child's consumption growth (a measure of his patience) and his parents' economic status exists only in two cases: either a child's patience is indicative of his parents' because patience is developed in the household ($b_1 \neq 0$ or $b_2 \neq 0$ or $b_4 \neq 0$) or because it is automatically transmitted ($\alpha_1 \neq 0$).

B. Our Data

We utilize intergenerational data in the *Panel Study of Income Dynamics* (PSID). The adult economic status of sons is measured as averages of log total family income in the son's family (including earnings of all family members as well as unearned income) over adjacent years. Status of a father is constructed from information on his schooling and occupation. Schooling is reported in brackets for

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[10] A similar result obtains if the son's earnings and schooling error terms are correlated with his father's. A final assumption is that $cov(A_{t+1}, \ln y_{t+1}) = cov(A_t, \ln y_t)$.

sons, fathers and mothers, and are converted to years according to the midpoint of the interval. These data also allow for three generations to be linked, although we will not do so in this version of the paper.

The PSID provides data on a household's expenditures on food eaten at home and in restaurants, rent, mortgage payments, property taxes and utilities.[11] The sum of expenditures on these six categories is one measure of consumption. Another measure takes a weighted average of food at home, food away from home, rent, and the value of the family's house. Since the weights are taken from a regression, using data from the Consumer Expenditure Survey, of nondurable consumption on food at home, food away from home, rent, and the value of the family's house, the weighted average is a measure of nondurable consumption.[12] We report results for this weighted average (using the 1972-73 weights) as well as the simple sum of food eaten at home and in restaurants.

Both of these measures significantly differ from an individual's consumption since they relate to family expenditures. It is not obvious how to correct for family

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[11] Data are also available on money saved with food stamps, but this is not included in the measure of consumption.

[12] The Consumer Expenditure Survey (CEX) regressions are reported in Skinner (1987). Skinner provides two sets of weights, one set obtained from the 1972-73 CEX and the other from the 1983 CEX. We use the 1972-73 weights and note that the two sets are quite similar. Using the 1983 weights instead does not change our results.

The two regressions of nondurable consumption using Consumer Expenditure Surveys have R^2 = .724 (1972-73) and R^2 = .708 (1983). Skinner's definition of nondurable consumption is: "total consumption expenditure *minus* housefurnishings and equipment, purchases of automobiles, motorcycles and boats, and mortgage payments, *plus* the imputed rental value of the house, assumed to be 6 percent of its market value." (Skinner, p. 214) composition. Sons are at different stages of their life-cycles: only some are married, only some have children. Some studies have divided family expenditure by family size, by "adult equivalents," or by "family needs." However, such an adjustment ignores the possibility that sons choose to be married and have children.

Sons in our sample are heads of households in one of eleven waves of the PSID (interview years 1975-85). We eliminated sons who did not report their father's schooling and occupation. Since consumption is measured at the household level, we also eliminated those households whose composition changed between years τ and $(\tau+1)$.[13] To measure a family's economic status, we average a family's real income for the years $(\tau-4)$ to $(\tau-1)$.[14] We include only those households who had no change in head or wife between $(\tau-4)$ and τ so that the family income concept is a consistent one. In each of the years $(\tau-4)$ to $(\tau+1)$ the head had to be either working, temporarily laid off, looking for work, or sick; we exclude heads who are retired, in school, permanently disabled, "keeping house" or "other". Members of the Survey of Economic Opportunity are excluded.[15] Families must have either paid rent or owned a house in years τ and $(\tau+1)$, had nonzero food consumption, and

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[13] τ is an interview year. We assume that the food consumption, house value and rent questions pertain to the interview year (as opposed to the previous year).

[14] Family income includes both the labor and nonlabor income of the household head, his wife (if present), and children (if present). Incomes are deflated using the CPI for all items reported in the February 1992 *Economic Report of the President*.

[15] The 1968 and subsequent waves of the PSID has two samples: the Survey of Economic Opportunity and the Survey Research Center sample. The latter was designed to be representative of the U.S. (in 1968) while the former oversamples the poor.

must not have changed regions of the country between those years. Finally, a family must have responded to the 1987 PSID interview. As a result of these selection criteria, some PSID families do not appear in any of our eleven cross-sections while others appear in several of them.

C. Regression Results

Table 1 regresses a family's consumption growth on the log of its own average income, as well as region dummies, the head's age, and its square. In seven of eleven cross-sections, we find that a positive coefficient on family income - although the coefficient is significantly difference from zero in only one. When we pool the eleven cross-sections and add year dummies, the last line of Table 1 shows that log family income has a coefficient of 0.0092 (s.e.=0.005), significant at the 10% significance level. Adding one to a family's log income is associated with 0.9 percentage points more consumption growth per year. Assuming that the intertemporal elasticity of substitution is 0.5 (ie, a coefficient of relative risk aversion of 2), the income difference can be associated with a 1.8 percentage point difference in annual rates of time preference. A negative relation between mortality and own income could also lead to a positive correlation between consumption growth and own income, but the effect reported in Table 1 seems too large to be entirely attributed to mortality differences.

Even though a consumer's Euler equation dictates that his consumption growth is a function of his rate of time preference *and* an interest rate factor, we do not include an interest rate factor in our consumption growth regressions. We believe that, in our sample, there is not enough variation in available measures of an interest rate factor. Lawrance (1991) uses variations in marginal tax rates to identify a coefficient on an interest rate factor. However, with opportunities for tax free savings like pension funds and individual retirement accounts, it is not obvious that Lawrance's procedure is appropriate. Moreover, the omission of an interest rate factor biases estimates on own income *downward* as long as rates of return are negatively correlated with income. A negative correlation would arise, for example, if variations in interest rate factors were primarily due to a progressive tax rate on interest income or if investments in human capital yielded lower returns for richer families. The omitted variable bias would go in the other direction if the rich enjoy higher rates of return, perhaps because they invest their money more wisely or because there are fixed cost to participating in certain markets.

Table 1: Consumption Growth vs. Own Income

Consumption Measure:		Nondurable (est.)		Food	
Included Cross-Sections	N	Log Own Avg. Income	$\bar{R}^2$	Log Own Avg. Income	$\bar{R}^2$
'75	1,108	-0.023 (0.016)	.009	-0.022 (0.019)	.012
'76	1,169	0.071 (0.018)	.019	0.137 (0.025)	.024
'77	1,237	-0.011 (0.019)	.025	-0.048 (0.025)	.014
'78	1,278	0.035 (0.019)	.017	-0.013 (0.024)	.000
'79	1,361	0.006 (0.017)	-.000	0.009 (0.021)	-.001
'80	1,461	-0.005 (0.018)	-.001	-0.042 (0.023)	.003
'81	1,516	0.012 (0.016)	.003	0.017 (0.022)	-.001
'82	1,566	0.016 (0.014)	.005	0.063 (0.021)	.009
'83	1,666	-0.010 (0.014)	.009	-0.037 (0.021)	.004
'84	1,669	-0.015 (0.015)	.000	-0.033 (0.021)	-.002
'85	1,773	0.019 (0.014)	.012	0.032 (0.019)	-.001
'75-'85	15,804	0.0078 (0.0049)	.013	0.0060 (0.0066)	.007

Notes: (1) For cross-section τ, the dependent variable is $\log(c_{\tau+1}/c_\tau)$ where c_τ is family consumption expenditure in year τ.
(2) Standard errors in parentheses
(3) A constant, region dummies, the household head's age and its square are included in every regression. Year dummies were also included in the pooled regression.
(4) Equations estimated with OLS.

It is possible that a positive correlation between a family's income and its consumption growth arises because patient families accumulate more assets (both human and nonhuman). Table 2 therefore displays estimates of equation (6)', the inverse of the status equation. Consumption growth is regressed on log own average income and an estimate of log parent's average income. Log parent's average income is obtained by regressing log own average income on own years of schooling, ten dummies for own occupation, and year dummies in a sample that pools the 1975-79 cross-section.[16] The resulting coefficients, together with each *father's* schooling and occupation, are used to construct parent's income. Notice that, for a given family, our measure of parent's income is a function of the father's schooling and occupation only. A given family's own income and own schooling influence our measure of parent's income only through its negligible effect on the coefficient estimates in a sample of 6,146.

We are not able to find a coefficient on parent's income that is significantly less than zero together with a coefficient on own income that is significantly greater than zero, yet the "patient get rich" alternative predicts that the partial correlation of a family's consumption growth with its own income be positive while that on parent's income be negative (or perhaps zero). Instead we find in the pooled regression that both variables enter a consumption growth regression with a

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[16] Own schooling and occupation are for the (male) head of household. After 1979, the occupation codes reported for heads of house and their fathers are not readily comparable (the former are 3 digit census codes and the latter are 1 digit PSID categories). When pooled, the 1975-79 cross-sections yield a sample size of 6,146. About 10 observations have no information on either the head's schooling or his occupation.

positive sign. The fact that parent's income enters with a larger coefficient (one that is significantly greater than zero at the 10% significance level) may indicate a formation of time preference during childhood or adolescence, instead of during adulthood.[17]

The greater importance of parent's income seems troublesome for a "liquidity effects" explanation of our results. A liquidity effects advocate would have to argue (1) that richer families are *more* liquidity constrained and (2) that parent's income (not own income) is a better predictor of the lagrange multiplier on a liquidity constraint.[18] We are skeptical of this joint hypothesis.

Our model implied causation in both directions: rich families become patient and patient families become rich. Table 3 estimates the reduced form of our simultaneous equations model: regressions of consumption growth on region dummies, age, age squared and log parent's average income. In seven of eleven cross-sections, the point estimate on log parent's income is positive, although only two are significantly different from zero. In the pooled regression, log parent's income enters with a coefficient of 0.021 (s.e.= 0.009) - different from zero at the 5% significance level. Adding 1 to log parent's income (ie, almost tripling income), "adds" 2.4 percentage points to a family's consumption growth. With an intertemporal elasticity of substitution of 0.5, this is a subtraction of almost 5

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[17] The F-statistic for the hypothesis that both income coefficients (own and parent's) are zero is 3.45 (19 d.f. for the numerator, 15865 for the denominator). The p-value of the test is 0.00.

[18] See Zeldes (1989) for a discussion of liquidity effects and tests using the PSID.

percentage points from the rate of time preference - an effect that is too large to be entirely attributed to the positive effect of parent's income on the life expectancy of their adult children.

The final line of Table 3 uses years of father's schooling as a measure of parent's economic status in a pooled regression. The coefficient on father's schooling is 0.0014 (s.e.=0.0008): four more years of father's schooling adds 0.6 percentage points to consumption growth or subtracts 1.2 percentage points from the annual rate of time preference (assuming an intertemporal elasticity of substitution of 0.5).

Table 2: OLS Estimates of the Status Equation

Consumption Measure:		Nondurable (est.)			Food		
Included Cross-Sections	N	Log Parent's Avg. Inc. (est.)	Log Own Avg. Income	$\bar{R}^2$	Log Parent's Avg. Inc. (est.)	Log Own Avg. Income	$\bar{R}^2$
'75	1,108	-0.012 (0.031)	-0.021 (0.017)	.008	0.016 (0.038)	-0.025 (0.020)	.012
'76	1,169	0.011 (0.035)	0.069 (0.019)	.018	0.067 (0.049)	0.125 (0.027)	.025
'77	1,237	-0.036 (0.036)	-0.005 (0.021)	.025	-0.032 (0.046)	-0.042 (0.026)	.014
'78	1,278	0.037 (0.036)	0.028 (0.020)	.017	0.061 (0.045)	-0.023 (0.025)	.001
'79	1,361	0.049 (0.032)	-0.002 (0.018)	.001	0.020 (0.040)	0.005 (0.022)	-.002
'80	1,461	0.010 (0.033)	-0.007 (0.018)	-.001	0.010 (0.043)	-0.043 (0.024)	.002
'81	1,516	0.064 (0.030)	-0.009 (0.017)	.005	0.018 (0.041)	0.014 (0.023)	-.002
'82	1,566	-0.011 (0.028)	0.018 (0.015)	.005	0.003 (0.041)	0.063 (0.023)	.008
'83	1,666	-0.012 (0.027)	-0.008 (0.014)	.008	-0.010 (0.042)	-0.036 (0.022)	.004
'84	1,669	0.049 (0.030)	-0.022 (0.016)	.001	0.069 (0.041)	-0.043 (0.021)	-.001
'85	1,773	0.027 (0.028)	0.015 (0.015)	.012	0.049 (0.039)	0.024 (0.020)	-.001
'75-'85	15,804	0.018 (0.009)	0.0048 (0.0051)	.013	0.026 (0.013)	0.0018 (0.0070)	.007

Notes: (1) For cross-section τ, the dependent variable is $\log(c_{\tau+1}/c_\tau)$ where c_τ is family consumption expenditure in year τ.
(2) Standard errors in parentheses
(3) A constant, region dummies, the household head's age and its square are included in every regression. Year dummies were also included in the pooled regression.
(4) Equations estimated with OLS.

Table 3: Consumption Growth as a Function of Parent's Status

Consumption Measure:		Nondurable (est.)			Food		
Included Cross-Sections	N	Log Parent's Avg. Inc. (est.)	Father's Schooling (years)	$\bar{R}^2$	Log Parent's Avg. Inc. (est.)	Father's Schooling (years)	$\bar{R}^2$
'75	1,108	-0.025 (0.029)		.007	0.000 (0.036)		.011
'76	1,169	0.053 (0.033)		.008	0.143 (0.047)		.007
'77	1,237	-0.038 (0.034)		.026	-0.057 (0.043)		.013
'78	1,278	0.053 (0.034)		.016	0.048 (0.043)		.001
'79	1,361	0.048 (0.031)		.002	0.023 (0.039)		-.001
'80	1,461	0.007 (0.031)		-.001	-0.013 (0.041)		.001
'81	1,516	0.059 (0.028)		.005	0.026 (0.039)		-.001
'82	1,566	-0.000 (0.026)		.004	0.040 (0.039)		.004
'83	1,666	-0.016 (0.026)		.009	-0.029 (0.040)		.003
'84	1,669	0.038 (0.029)		.001	0.047 (0.039)		-.002
'85	1,773	0.035 (0.027)		.012	0.063 (0.037)		-.001
'75-'85	15,804	0.021 (0.009)		.013	0.027 (0.012)		.007
'75-'85	15,804		0.0013 (0.0007)	.013		0.0015 (0.0010)	.007

<u>Notes:</u> (1) For cross-section τ, the dependent variable is $\log(c_{\tau+1}/c_\tau)$ where c_τ is family consumption expenditure in year τ.
(2) Standard errors in parentheses
(3) A constant, region dummies, the household head's age and its square are included in every regression. Year dummies were also included in the pooled regression.
(4) Equations estimated with OLS.

VII. Conclusions and Extensions

We find that a son's adult consumption growth is positively correlated with his parent's income. We propose that either wealth causes patience (perhaps through the mechanism discussed in this paper) or that "patience endowments" are automatically transmitted from one generation to the next. Future empirical work could use three generations of data to distinguish the two explanations.

Second order conditions, while assumed to hold, are somewhat strong: the discount and utility functions have to be "sufficiently" concave. In the two period case, we rule out increasing returns to $\beta(S)u_1(c_1)$. With more than two periods, some intuition for the additional required concavity is given in the appendix. However, all of these restrictions are invoked for their analytical convenience, not for their economic plausibility. When second order conditions fail, multiple optima can exist and our comparative static results will be only local ones. Moreover, as suggested in the appendix, the second order conditions are more likely to fail for large T: the durability of future-oriented capital introduces nonconvexities.

Future-oriented capital is completely durable in our setup. Instead of assuming that S, and hence $\beta(S)$, is fully chosen in the initial period, it is more realistic to permit S to depreciate over time. Then it would be necessary to continue to invest, as with continuing education, if one wants to prevent S from declining over time at the rate of its depreciation. At young ages the incentive to invest in S would be relatively high because many years remain to collect the returns, while the investment incentive would be small at old ages because few years remain. Therefore, S might rise with age for a while, and then fall with the approach of old age; the future would tend to be discounted relatively heavily at both young and old ages. It is often claimed that the young discount the future heavily because they are too impatient (see xx). It is frequently assumed that the old discount the future heavily too. However, it is not explained why the discount on the future falls as children become adults. With the endogeneity of discount rates embedded in a utility-maximizing framework, the life cycle in these rates can be explained.

It might appear that our comparative statics depend on the assumption of an additively-separable function in the utilities over time. However, our first order condition with respect to S depends only on the indifference curve system, and it is unaffected by monotonic transformations of the utility function V. We have assumed separability in the u_0 function between c_0 and S, that the marginal utility of S is positive, and that time preference is multiplicatively separable from the future utilities.

Moreover, it should be obvious that the implications of the analysis generalize to a broader class of preferences than additive-separability in consumption. For example, the comparative statics would be similar when the utility at age i, u_i, depends recursively on the consumption in earlier periods. However, more work is necessary to determine the boundaries of the class of utility functions that give similar implications to the separable case with regard to the effects of wealth and other variables on attitudes toward future utilities.

The popular conclusion that such behavior as not writing a will, or paying "too much" attention to good outcomes, is an indication of irrational behavior is implicitly predicated on the assumption that contemplating the future - in particular, that contemplating unpleasant future outcomes - is costless. If that were true, the failure to do so would indeed be a sign of non-utility-maximizing behavior. However, once it is recognized that effort may be required to overcome the reluctance to think about unpleasant future outcomes, then such behavior is not prima facie evidence of irrationality. By endogenizing discount rates, it appears possible to explain with a model of rational behavior many assertions in the literature that are claimed to imply irrational choices. A good example is Adam Smith's comment concerning the overweening conceit that young people have about their good fortune:

> How little the fear of misfortune is then capable of balancing the hope of good luck, appears still more evidently in the readiness of the common people to enlist as soldiers, or to go to sea, than in the eagerness of those of better fashion to enter into what are called the liberal professions. ...though they [soldiers] have scarce any chance of preferment, they figure to themselves, in their youthful fancies, a thousand occasions of acquiring honor and distinction which never occur. (1776, I.x.b.29-30)

According to our analysis, "common people" are willing to enlist because they "rationally" discount the future misfortunes. The test of rationality in contemplating the propinquity of various future states of nature is whether the effort put into that activity responds, in the direction predicted by utility maximizing behavior, to changes in the level of resources, the likelihood of an event, or to the cost of accumulating future-oriented capital.

Proofs of Propositions

Proposition 0 establishes the equivalence between the sequence problem (1) in the text a recursive formulation of the problem. Proposition 1 enumerates those conditions on the date 1 value function $v_1(A_1,S)$ which, in addition to a second order condition, guarantee positive wealth effects on S in the T period problem. Proposition 2 then shows that, under a particular decreasing returns assumption, those conditions on $v_1(A_1,S)$ do in fact hold for the T period problem; its corollary states that the second order conditions guarantee positive wealth effects. All Lemmas and Propositions assume that the functions $\beta(S)$, $u_0(c_0)$, ..., $u_T(c_T)$ are positive, strictly increasing and strictly concave. Our definition for the date t interest rate factor is:

$$R_i \equiv \prod_{s=1}^{i} (1 + r_s)^{-1}$$

We also define value functions for each age :

$$v(A_0) \equiv \max_{c_0, \ldots, c_T, S} \left[\sum_{i=0}^{T} \beta(S)^i \cdot u_i(c_i) \right] \qquad \text{s.t.} \quad \sum_{i=0}^{T} R_i \cdot c_i + S = A_0$$

$$v_i(A_i, S) \equiv \max_{c_i, \ldots, c_T} \left[\sum_{\tau=i}^{T} \beta(S)^{\tau - i} \cdot u_\tau(c_\tau) \right] \qquad \text{s.t.} \quad \sum_{\tau=i}^{T} R_\tau \cdot c_\tau / R_i = A_i \;, \quad i = 1, \ldots, T$$

<u>Proposition 0</u> The sequence of value functions $v(A_0)$, $\{v_t(A_t,S)\}_1^T$ satisfy a sequence of Bellman equations:

$$v_0(A_0) = \max_{c_0,S}\left[u_0(c_0) + \beta(S)v_1\left((1+r_1)(A_0 - c_0 - S), S\right)\right]$$

$$v_i(A_i,S) = \max_{c_i}\left[u_i(c_i) + \beta(S)v_{i+1}\left((1+r_{i+1})(A_i - c_i), S\right)\right] \qquad i = 1,2,\dots,T-1$$

Furthermore, policy functions $S(A_0)$, $c(A_0)$, $\{c_t(A_t,S)\}_1^T$ that maximize the RHS of the Bellman equations above also generate a sequence of optimal controls for the sequence problem in the text.

<u>Proof</u> See Bertsekas (1987), p. 14. ■

<u>Proposition 1</u> If the RHS of the age 0 Bellman equation is a strictly concave function of c_0 and S (holding fixed A_0), the cross-derivative of the age 1 value function is nonnegative, and the age 1 value function is concave and strictly increasing in A_1 (holding fixed S), then the optimal choice of S is a strictly increasing function of initial wealth A_0. In terms of notation,

$$H(c_0,S) \equiv u_0(c_0) + \beta(S)\cdot v_1[(1+r_1)\cdot(A_0 - c_0 - S), S]$$

$$H(c_0,S) \text{ strictly concave}, \ \frac{\partial^2 v_1}{\partial A_1 \partial S} \geq 0, \ \frac{\partial v_1}{\partial A_1} > 0, \ \frac{\partial^2 v_1}{\partial A_1^2} \leq 0 \quad \Rightarrow \quad S'(A_0) > 0$$

where $S(A_0)$ is the optimal policy function for S.[19]

Proof First, the strict concavity of H implies that the first order conditions are sufficient. Standard comparative statics shows that:

$$\frac{dS}{dA_0} = -u_0''(c_0)\cdot\left[\beta(S)\cdot\frac{\partial^2 v_1}{\partial A_1 \partial S} + \beta'(S)\cdot\frac{\partial v_1}{\partial A_1} - \beta(S)\cdot(1+r_1)\cdot\frac{\partial^2 v_1}{\partial A_1^2}\right]\cdot(1+r_1)/|D| \quad > 0$$

where c_0 and S denote optimal choices given A_0, the partial derivatives of v_1 are evaluated at the optimal choices, and $|D|$ denotes the determinant of the Hessian of $H(c_0,S)$. ■

Corollary If T = 1 and $\beta(S)u_1(c_1)$ is jointy concave then $S'(A_0) \in (0,1)$. In other words, the standard conditions on the time preference function and the "instantaneous" utility functions together with a decreasing returns assumption implies that the wealth effect on S is positive in the two period case.

Proof When T = 1, $v_1(A_1,S) = u_1(A_1)$ and the slope of the policy function for S becomes:

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[19] $u_0(c_0)$ can depend on S without changing our wealth effect result as long as the cross-derivative is nonnegative:

$$\frac{\partial^2 u_0}{\partial S \partial c_0} \geq 0$$

$$\frac{dS}{dA_0} = \left[1 + \frac{(u_1')^2 - [R_1 u_1]^2 \frac{\beta''}{\beta} - \beta'' u_1^2 \frac{u_1''}{u_0''} - (u_1')^2 \frac{\beta' u_1' u_1}{\beta u_0''}}{(u_1')^2 - u_1 u_1''} \right]^{-1} \in (0,1)$$

The joint concavity of $\beta(S)u_1(c_1)$ implies that the second order conditions are satisfied for all feasibile (c_0, c_1, S). ■

Before proving Proposition 2, we prove three Lemmas. Lemmas 2 and 3 exploit the fact that, after the first period, time preference is given. Periods 1 and later influence the optimal choice of S only through their effect on the level and shape of the date 1 value function.

<u>Lemma 1</u> All value functions are positive.

<u>Proof</u> Since the discount function and all utility functions are positive, Lemma 1 follows from the definition of the value functions. ■

<u>Lemma 2</u> If, for some $i \in \{1, 2, ..., T\text{-}2\}$, the age i+1 value function $v_{i+1}(A_{i+1},S)$ has a positive cross-derivative, is strictly increasing in S and, holding fixed S, is strictly increasing and concave in its first argument, then the age i value function $v_i(A_i,S)$ shares those properties. In other words,

$$\frac{\partial^2 v_{i+1}}{\partial A_{i+1}\partial S}, \frac{\partial v_{i+1}}{\partial A_{i+1}}, \frac{\partial v_{i+1}}{\partial S} > 0, \frac{\partial^2 v_{i+1}}{\partial A_{i+1}^2} < 0$$

$$\Rightarrow \frac{\partial^2 v_i}{\partial A_i \partial S}, \frac{\partial v_i}{\partial A_i}, \frac{\partial v_i}{\partial S} > 0, \frac{\partial^2 v_i}{\partial A_i^2} < 0$$

Proof Consider the age i Bellman equation,

$$v_i(A_i, S) = \max_{c_i}\left[u_i(c_i) + \beta(S)v_{i+1}\left((1+r_{i+1})(A_i - c_i), S\right)\right]$$

Since the RHS of the age i Bellman equation is strictly concave for any feasible consumption choice, optimal age i consumption, as a function of A_i and S, is defined by the first order condition:

$$u_i'\left(c_i(A_i, S)\right) = \beta(S)\frac{\partial v_{i+1}}{\partial A_{i+1}}\left((1+r_{i+1})[A_i - c_i(A_i, S)], S\right)$$

Since time preference is given, this is simply a two period version of Fisher's problem; the strict concavity of u_i and v_{i+1} (with respect to its first argument) guarantee the sufficiency of the first order condition.

Taking the partial derivatives of the first order condition above reveals that the optimal policy function $c_i(A_i,S)$ is strictly decreasing in S while strictly increasing in A_i. In fact, $\frac{\partial c_i}{\partial A_i} \in (0,1)$. The policy function $c_i(A_i,S)$ can be used to define a policy function $A_{i+1}(A_i,S)$:

$$A_{i+1}(A_i,S) = (1+r_{i+1})[A_i - c_i(A_i,S)]$$

Notice that the policy function $A_{i+1}(A_i,S)$ is strictly increasing in both arguments.

The policy functions $c_i(A_i,S)$, $A_{i+1}(A_i,S)$ and the Bellman equation define the age i value function:

$$v_i(A_i,S) = u_i\big(c_i(A_i,S)\big) + \beta(S)v_{i+1}\big(A_{i+1}(A_i,S),S\big)$$

The sufficiency of the first order condition permits the application of the Envelope Theorem:

$$\frac{\partial v_i}{\partial A_i}(A_i,S) = (1+r_{i+1})\beta(S)\frac{\partial v_{i+1}}{\partial A_{i+1}}\big(A_{i+1}(A_i,S),S\big) \quad > 0$$

$$\frac{\partial v_i}{\partial S}(A_i,S) = \beta'(S)v_{i+1}\big(A_{i+1}(A_i,S),S\big) + \beta(S)\frac{\partial v_{i+1}}{\partial S}\big(A_{i+1}(A_i,S),S\big) \quad > 0$$

Taking partial derivatives,

$$\frac{\partial^2 v_i}{\partial A_i^2} = (1+r_{i+1})\beta(S)\frac{\partial^2 v_{i+1}}{\partial A_{i+1}^2}\frac{\partial A_{i+1}}{\partial A_i} \quad < 0$$

$$\frac{\partial^2 v_i}{\partial A_i \partial S} = \beta'(S)\frac{\partial v_{i+1}}{\partial A_{i+1}}\frac{\partial A_{i+1}}{\partial A_i} + \beta(S)\frac{\partial^2 v_{i+1}}{\partial A_{i+1}\partial S} \quad > 0$$

Note that Lemma 1 was used to sign one partial derivative. ■

Lemma 3 The age T-1 value function $v_{T-1}(A_{T-1},S)$ has a positive cross-derivative, is strictly increasing in S and, holding fixed S, is strictly increasing and concave in its first argument. In other words,

$$\frac{\partial^2 v_{T-1}}{\partial A_{T-1}\partial S}, \frac{\partial v_{T-1}}{\partial A_{T-1}}, \frac{\partial v_{T-1}}{\partial S} > 0, \frac{\partial^2 v_{T-1}}{\partial A_{T-1}^2} < 0$$

Proof The age T-1 Bellman equation is:

$$v_{T-1}(A_{T-1},S) = \max_{c_{T-1}}\left[u_{T-1}(c_{T-1}) + \beta(S)u_T\left((1+r_T)(A_{T-1}-c_{T-1})\right)\right]$$

Notice that this is the age i Bellman equation found in Lemma 1 for $v_{i+1}(A_{i+1},S) \equiv u_T(A_T)$ and $i \equiv T\text{-}1$. Since $u_T(A_T)$ is positive, strictly increasing and strictly concave, the proof of Lemma 3 is merely a special case of the proof of Lemma 2. ■

Proposition 2 All of the value functions $v_1(A_1,S)$, ..., $v_{T-1}(A_{T-1},S)$ have positive cross-derivatives and, holding fixed S, are strictly increasing and concave in their first argument. Each is also strictly increasing in S.

$$\frac{\partial^2 v_i}{\partial A_i \partial S} , \frac{\partial v_i}{\partial A_i} , \frac{\partial v_i}{\partial S} > 0 , \frac{\partial^2 v_i}{\partial A_i^2} < 0 , \qquad i = 1, \dots, T-1$$

Proof Proposition 2 follows by induction from Lemmas 2 and 3. ■

Corollary If the function $H(c_0,S)$ defined in Proposition 1 is strictly concave then the optimal choice of S is a strictly increasing function of initial wealth A_0.

Proof Proposition 2 guarantees that the age 1 value function $v_1(A_1,S)$ has a positive cross-derivative, is strictly increasing in S and, holding fixed S, is strictly increasing and concave in its first argument. When $H(c_0,S)$ is strictly concave, Proposition 1 guarantees that S is a strictly increasing function of initial wealth A_0. ■

With only standard conditions on the discount function and the utility functions, Proposition 2 showed that the value functions (other than the initial one) are strictly concave in their first argument and have a positive cross derivative. However, in order that $H(c_0,S)$ be concave, the value functions must be *sufficiently* concave in S.

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